

An Analysis of the Underground Economy and its Macroeconomic Consequences

by
Era Dabla-Norris
and
Andrew Feltenstein
International Monetary Fund

Abstract

This paper develops a dynamic general equilibrium model to examine the causes and implications of underground economies. We apply the model to stylized data from Pakistan, a country which faces severe problems from tax evasion. In the model, optimizing agents produce for current consumption as well as invest, yet they can partially avoid paying taxes on their activities by operating in the underground economy. The cost to avoiding taxes is that the firm finds itself subject to credit rationing from banks. Our model simulations show that raising the tax rates too high can actually increase the budget deficit by driving firms into the underground economy, thereby reducing the tax base. Taxes that are too low will eliminate the underground economy, but will cause an increase in the budget deficit, thereby partially driving out private investment. Thus, the optimal rate of taxation, from a macroeconomic point of view, may lead to some underground activity.

1. Introduction

A substantial part of output in many developing and transition economies goes unreported.¹ There is widespread belief that underground economies arise due to high statutory tax rates and excessive regulation imposed by governments that lack the capability to enforce compliance. The presence of underground economies, however, can impede macroeconomic management and undermine economic growth in several ways. First, the loss of government tax revenues can result in larger budget deficits, which by leading to higher public borrowing requirements can crowd out private investment.² Lower tax revenues can also compromise the quality and quantity of public goods provided, thereby impairing economic growth (Loayza (1996), Johnson, Kaufman and Zoido-Lobatan (1998)). Second, firms operating underground cannot make use of market-supporting institutions like the judicial system and courts (De Soto (1987)) and, as a result, may invest too little. For instance, the inability to enforce legally binding contracts can limit their access to formal capital markets. In addition, efforts to avoid detection can generate distortions, resulting in a misallocation of resources in the economy (Shliefer and Vishny (1993)).³

We develop an intertemporal general equilibrium model that can endogenously

¹Schneider and Enste (2000) provide a survey on the size of the underground economy in different countries. While there are a number of ways of defining the underground economy, as a general rule we may view it as that part of the economy that escapes official registration and taxation.

² It may also cause a decrease in foreign direct investment since it acts as an additional tax on those activities, which cannot directly participate in the underground economy (Wei 1997).

³ For example, firms may be unable to achieve economies of scale or choose an optimal capital-labor mix due to fear of detection (Chaudhari (1989) and Gupta (1993)).

generate an underground economy. In particular, we derive entry and exit into the underground economy as part of optimizing behavior that depends, among other things, on taxes and interest rates. The cost to avoiding taxes is credit rationing by banks who reduce loans in relation to the firm's non-payment of taxes. Since the size of the underground economy depends upon exogenous and endogenous parameters, our model will also have scope for policy changes. Thus, for example, changes in tax rates or monetary policy may influence the size of the underground economy.

We develop numerical applications of our model using data for Pakistan. There have been a number of empirical studies of the scope of the underground economy in Pakistan. Most of these studies use proxies, such as the amount of cash in circulation or electricity consumption to estimate the size of the underground economy. Also, they are not derived from optimizing models, but are based upon ad hoc empiricism. Accordingly, they can at best be used for partial equilibrium analysis and hence may lead to inaccurate conclusions.

Data for Pakistan have a number of problems in both scope and reliability. Nonetheless, the country faces severe problems from tax evasion and parallel markets for both goods and financial assets. Hence economic reform will depend upon policies that reduce the various forms of tax evasion, especially given the country's difficulties with controlling its budget deficit.

The next section will survey the relevant literature on underground economies. Section III will provide a brief overview of our modeling of the underground economy. Section IV presents our dynamic general equilibrium model. Section V will discuss the parameterization of the model, indicating how underground economies may be generated

by changes in tax rates. Section VI concludes.

II. A Brief Survey of the Literature on Underground Economies

There is a vast literature on the determinants and consequences of underground economies.⁴ The benefits of operating partially or completely in an underground economy generally relate to firm's desire to evade taxes, regulations or other institutional distortions.⁵ For instance, higher effective tax rates and other onerous regulations can make operating in the underground economy more attractive by imposing high entry costs to legality (de Soto (1987), Loayza (1996), Rauch (1991)). In many developing countries, taxes on formal firms constitute a major source of government revenues, and narrow tax bases for formal firms have often resulted in governments imposing very high marginal tax rates.⁶ Cutting taxes and red tape are, according to this view, the main ways of bringing firms into the official economy.

A related view is that the unofficial economy may arise due to predatory behavior of corrupt officials, seeking bribes from anyone engaging in officially registered economic

⁴ See Schneider and Enste (2000) and Thomas (1992) for excellent overviews on how to measure it, its causes, and its effects on the official economy. See also Tanzi (1999).

⁵ In an empirical study, Johnson, Kaufmann, and Zoido-Lobatan (1998a, 1998b) find that high a corporate tax burden combined with ineffective and discretionary application of the tax system and other regulations influences the size of the underground economy. Friedman et. al (1999) find that a one point increase in their index of regulation (ranging from 1-5) is associated with a 10 percent increase in the underground economy for 76 developing, transition, and developed countries.

⁶Burgess and Stern (1993) note that in developing countries, corporate income taxes represent 17.8 percent of total tax revenues as opposed to 7.6 percent in industrialized countries.

activity (Shliefer and Vishny (1997), Kaufman (1994), Johnson et al (1998)). In this view, the problem that needs to be addressed is bureaucratic corruption. In reality, it may be difficult to determine the direction of this causality as enterprises may be able to profitably avoid paying taxes by bribing officials, resulting in higher level of corruption in the economy.

However, informality can impose significant costs to firms operating in the underground economy. If, for instance, penalties for tax evasion increase with the amount of unpaid taxes, higher tax rates may not create an incentive to hide (Andreoni, Erard, and Feinstein (1998)). In addition, firms in the underground economy may face higher bribes to avoid detection.⁷ An important cost of operating in the underground economy is the inability to enforce contracts. For example, firms may have to deal with a restricted set of trading partners, foregoing gains from trade arise from a broader set of potential trading partners (Johnson, Mc Millan, and Woodruff (1999)). It may also be difficult to raise equity capital or to borrow from capital markets because to do so would require official documentation (Loayza (1996)).⁸ As a result, firms may face higher borrowing rates, thereby reducing incentives to invest.⁹

⁷De Soto (1998) finds that informal entrepreneurs pay between 10 to 15 percent of their gross income in bribes to corrupt government officials, whereas formal entrepreneurs pay an average of only 1 percent of gross income).

⁸ While informal capital markets (Besley (1995)) may be sufficient to fulfill the firms external financing needs at low levels of production, the small scale and undiversified nature of informal capital markets makes them unsuitable for satisfying the firm's financing needs at larger scales of operation.

⁹Huq and Sultan (1991) note that in Bangladesh, while borrowing rates from commercial banks were around 12 percent, firms dependent on noninstitutional sources to meet their financing needs paid rates between 48 to 100 percent.

III. Macroeconomic Background

Our model assumes that firms do not have to operate entirely in the legal, or in the underground economy. That is, they can operate partially in the above ground and partially in the underground economy. That part of their operation that takes place in the legal economy pays taxes and can borrow from the banking system. That part that is underground does not pay taxes and cannot borrow.¹⁰ Admittedly this distinction is artificial, but captures some of the benefits and costs of operating in the underground economy discussed in the literature (see previous section).

We ignore the effects of corruption in generating an underground economy because of the difficulty of deriving relevant quantitative measures for countries. Presumably, the size of the underground economy may itself serve as a proxy for the extent of corruption in the economy. This might be appropriate if the country in question has relatively low statutory tax rates, so the incentives to evade taxes would not exist unless doing so were not easy. Of course, the presence of a large underground economy could indicate high costs of ensuring compliance rather than corrupt officials.

Our approach also does not consider possible penalties faced by firms for evading taxes. From a modeling perspective, it is difficult to determine just what the penalty should be. That is, should it be a criminal penalty or a fine? Should it be proportional to the size of the tax evasion, or should it be a flat rate? In addition, what is the probability of apprehension faced by a tax evader? Presumably this probability should itself be some function of the enforcement technology, as well as the amount of money spend on

¹⁰ In reality, the underground firm may not be able to borrow from official banks but may still be able to invest by borrowing from secondary lenders who charge higher than market interest rates and are willing to incur high risks.

enforcement. Here we have the difficulty of attempting to determine, using real data, just what the enforcement technology is, and how spending affects the probability of being caught. In addition, in order to incorporate some realistic description of apprehension, we would need a stochastic model. As our approach will be to use a modified perfect foresight model, we wish to avoid introducing stochastic elements.

In our model, the decision to operate in the underground economy depends on the firm's present value of the future stream of returns on marginal investment relative to the return on the corporate capital tax rate. If the marginal rate of return is higher than the corporate tax rate, the firm chooses to operate in the above ground economy, since it is profitable to borrow and pay taxes. If, on the other hand, the tax rate is greater than the marginal rate of return on investment, then the firm begins to operate underground. However, the firm does not make a bi-polar choice. That is, it reduces its tax payments and borrowing for investment proportionally to the difference between the rate of return and the tax rate. Hence if the rate of return were 0, then the firm would pay no taxes and carry out no borrowing for investment. If tax rates and rates of return on investment are equal, then the firm pays the full tax rate and invests.

In this framework, one could measure the size of the underground economy by aggregating the value of all lost tax revenues and comparing it to the revenues that would accrue if rates were low enough so as to generate no underground activity. The ratio of the two would then provide a measure of the share of the underground economy in total economic activity. We would thus compare two simulated equilibria.

IV. A General Equilibrium Specification

To analyze this problem we develop the formal structure of a dynamic general equilibrium model that will generate an endogenous underground economy. Much of the structure of our model is designed to permit a numerical implementation. Our model has n discrete time periods. All agents optimize in each period over a 2 period time horizon. That is, in period t they optimize given prices for periods t and $t + 1$ and expectations for prices for the future after $t + 1$. When period $t + 2$ arrives, agents re-optimize for period $t + 2$ and $t + 3$, based on new information about period $t + 2$.

Our model structure is related to a number of earlier papers, starting with Strotz (1956). Here preferences are inconsistent over time, primarily because the future does not turn out as anticipated. Thus it may be optimal for agents to commit themselves for a few periods into the future. They may be better off, however, if they re-optimize at some later date, based on their own changed preferences or changes in economic variables. This is quite different from the notion of time inconsistency of Kydland and Prescott (1977), where rational behavior by economic agents itself leads to inconsistencies in what would otherwise be an optimal government plan.

1. Production

There are 8 factors of production and 3 types of financial assets:

- | | |
|----------------------|---------------------|
| 1-5. Capital types | 9. Foreign currency |
| 6. Urban labor | 10. Rural labor |
| 7. Domestic currency | 11. Land |
| 8. Bank deposits | |

The five types of capital correspond to five aggregate non-agricultural productive sectors.¹¹ An input-output matrix, \mathbf{A}_t , is used to determine intermediate and final production in period t . Corresponding to each sector in the input-output matrix, sector specific value added is produced using capital and urban labor for the non-agricultural sectors, and land and rural labor in agriculture. Assuming that there are more than five sectors in the economy, the different factors would be allocated across the economy so that agriculture uses land and rural labor, and all other sectors use one of the five capital types plus urban labor. Accordingly, capital is perfectly mobile across a given sub-sector, but is immobile across other sub-sectors. Labor, on the other hand, may migrate from the rural to the urban sector.

The specific formulation of the firm's problem is as follows. Let $y_{K_i}^j, y_{L_i}^j$ be the inputs of capital and urban labor to the j th non-agricultural sector in period i . Let Y_{G_i} be the outstanding stock of government infrastructure in period i . The production of value added in sector j in period i is then given by:

$$va_{jt} = va_{jt}(y_{K_t}^j, y_{L_t}^j, Y_{G_t})$$

where we suppose that public infrastructure may act as a productivity increment to private production. Sector j pays income taxes on inputs of capital and labor, given by t_{K_j}, t_{L_j} , respectively, in period i . The interpretation of these taxes are that the capital tax is a tax on firm profits, while the labor tax is a personal income tax that is withheld at the source.

¹¹ We could have any number of capital types without affecting the structure of the model.

We suppose that each type of sectoral capital is produced via a sector-specific investment technology that uses inputs of capital and labor to produce new capital. Investment is carried out by the private sector and is entirely financed by domestic borrowing. Let us define the following notation.

C_{HI} = The cost of producing the quantity H of capital.

r_i = The interest rate in period i .

P_{KI} = The return to capital in period i .

P_{MI} = The price of money in period i .

δ = The rate of depreciation of capital.

Suppose, then, that the rental price of capital in period 1 is P_1 . If C_{HI} is the cost-minimizing cost of producing the quantity of capital, H_1 , then the cost of borrowing must equal the present value of the return on new capital. Hence:

$$C_{HI} = \sum_{i=2}^N \left[\frac{P_{KI} (1 - \delta)^{i-2} H_1}{\prod_{j=1}^{i-1} (1 + r_j)} \right]$$

where r_j is the interest rate in period j , given by:

$$r_j = 1/P_{Bj}$$

where P_{Bj} is the price of a bond in period j . The tax on capital is implicitly included in the investment problem, as capital taxes are paid on capital as an input to production.

If at some point the present value of investment, as given in equation (3), falls below the corresponding value of debt service, then the sector is unable to pay its debt obligations which were incurred to finance this investment. Accordingly, the bank which holds these assets now holds corresponding bad debts. This situation might occur if, after the investment was incurred, the interest rate rose or the rate of return to capital fell, due to some unanticipated event. We assume that a bankrupt firm cannot invest.¹²

The decision to invest depends not only the variables in the above equation, but also upon the decision the firm makes as to whether it should pay taxes. This decision determines the firm's entry into the underground economy. We will suppose that the firm's decision is based upon a comparison of the tax rate on capital with the rate of return on new capital. If the tax rate on capital is less than the corresponding rate of return, then the firm will pay the full tax. If the tax rate is greater than the return to new capital, then the firm pays less than the full capital tax. That is, it withdraws, at least partially, into the underground economy.¹³ Formally, suppose that we were in a two period world. Suppose that:

$$\frac{P_{K2}}{1 + r_1} \geq t_{K1}$$

In this case the present value of the return on one unit of new capital is greater than the

¹²Such an increase in interest rates might occur if the underground economy grows, leading to a loss in revenues and a corresponding increase in borrowing requirements. Hence there will be a direct connection between the size of the underground economy and the solvency of the banking system.

¹³Clearly this is an ad hoc assumption. We wish to capture the notion that the decision to pay or not pay taxes is based on the relationship between the return to investment and the tax on capital.

current tax rate on capital. In this case we assume the investor pays the full tax rate on capital inputs. Suppose, on the other hand, that:

$$\frac{P_{K2}}{1+r_1} \leq t_{K1}$$

Here the discounted rate of return is less than the tax rate, and the firm will attempt to reduce its tax payments by moving into the underground economy. The extent to which the firm goes into the underground economy is determined by the gap between the tax rate and the rate of return to investment. That is, the firm pays a tax rate of \bar{t}_{K1}

where:

$$\bar{t}_{K1} = t_{K1} \left[1 - \left(\frac{t_{K1} - \frac{P_{K2}}{1+r_2}}{t_{K1}} \right)^\alpha \right]$$

Here $0 \leq \alpha$ and higher values of α lead to lower values of taxes actually paid. That is,

the ratio $\frac{\bar{t}_{K1}}{t_{K1}}$ reflects the share of the sector that operates in the above ground economy.

Hence α represents a firm-specific behavioral variable. An “honest” firm would set $\alpha = 0$, while a firm that is prone to evasion would have a high value for α .

If a sector can avoid paying taxes, as above, by going into the underground economy, why does it pay taxes at all? That is, why does it simply not set $\bar{t}_{K1} = 0$? In

the next section we will develop a simple approach that supposes that a firm's refusal to pay taxes reduces its ability to borrow from the commercial banking system. Thus a firm's desire to invest will constrain its' avoidance of tax payments.¹⁴

2. Banking

The banking sector in our model is quite simple and is meant to capture some of the key features and problems in many developing countries. We will suppose that there is one bank for each non-agricultural sector of the economy. There are 5 such sectors, and hence 5 banks. We make this assumption of sectoral specialization of the banking system for several reasons. First, such specialization reflects the reality of many developing countries. Second, if we had only a single bank, then we would not be able to generate the type of heterogeneous failures that are also typical of developing countries. Third, our use of multiple banks permits us to capture the notion of more and less successful banks without introducing elements of risk or private information, which we do not have in our model.

We contend that the underground economy effects different sectors in a non-uniform way. Indeed, tax avoidance in one sector may benefit the sector at a micro level but may be harmful to the macro economy. Tax avoidance varies across sectors not only because of the behavior of firms in that sector, but also because different banks may have varying attitudes towards lending to clients who have avoided paying taxes.

Each bank lends primarily to the sector with which it is associated. The banks

¹⁴We will implicitly suppose that banks know the firm's true tax obligations and its actual payments.

are, however, not fully specialized in the sector they correspond to. We will make the simplifying assumption that each bank holds 50 percent of the outstanding debt of its particular sector. It then holds 12.5 percent of the debt of each of the remaining 4 sectors. Hence bank 3, for example, holds 50 percent of the debt of sector 3, and 12.5 percent of sectors 1,2,4, and 5. Similarly, it makes 50 percent of the loans to sector 3 and 12.5 percent of the loans to the other 4 sectors.¹⁵ We make this assumption of diversification of assets in order to avoid a possible situation in which the insolvency of a particular sector leads to the automatic insolvency of its related bank. At the same time, the firm that avoids taxes and thereby enters the underground economy should receive varying degrees of credit rationing from the different banks to which it applies for loans.

We assume that banks follow a strategy of lending that looks at the risks associated with their borrowers. That is, as their borrowers become more insolvent, the banks ration credit to those borrowers.¹⁶ We will choose a simple functional form that connects credit rationing to borrower insolvency. Suppose that C_{ij} is the demand for borrowing by sector j in period i . Suppose also that bank k has Def_{ik} percent of its total assets in default in period i . Let $\delta_k \geq 0$ be a parameter specific to bank k , and let β_{jk} be the share of borrowing by sector j taken by bank k . Sector j then receives loans L_{jt}

¹⁵Clearly these percentages are arbitrary and should serve only for illustrative purposes. We could have any initial pattern of distribution of bank assets across the different sectors.

¹⁶The rationale for this approach is that banks are aware that depositors will withdraw their deposits if they believe bank assets are risky. In order to reduce these withdrawals the banks, in turn, ration credit to risky borrowers. Our approach is thus a simple version of that presented in Calomiris and Wilson (1998).

where:

$$L_{jt} = \sum_{k=1}^5 \beta_{jk} (1 - \delta_k Def_{ik}) C_{Ejt} \quad (4)$$

Thus if there are no bank assets in default, then no credit rationing takes place. If assets are in default, then the credit demanded by sector j for investment is reduced by each bank proportionally to the share of that bank's defaulted assets in total assets.¹⁷ The parameter δ_k is bank specific and is some measure of the risk aversion of the particular bank. Higher values of δ_k indicate a more rapid contraction of credit in response to bad loans.¹⁸ Our numerical simulations will show that this admittedly ad hoc formulation of optimizing behavior by banks leads, in fact, to reductions in failures of those banks. We will also see that increased tax evasion in a sector may reduce the rate of defaults in that sector, by reducing enterprises' tax obligations. At the same time, the increased borrowing by the public sector will tend to raise real interest rates, thereby harming enterprises' ability to repay their loans. Hence, entry into the underground economy may have effects upon the firm that work in different directions.

We impose a solvency requirement on the banking system. Namely, if " percent of a bank's assets are in default, caused by a corresponding insolvency in its borrowers,

¹⁷We are thus abstracting from any uncertainty across firms, as well as any notion of private information about those firms. The only information banks possess about firms is their stock of defaulted assets.

¹⁸Clearly δ_k is not derived from optimization, but is taken to be exogenous and does not vary over time.

then the bank is declared insolvent. At this point a fraction of the bank's deposits are seized by the government.¹⁹ In particular, depositors in the bank find part of their deposits frozen. We use a simple rule to determine the fraction of a bank's deposits that are seized. If Def_{ik} is the share of bank k 's assets that are in default in period i , as before, then regulators seize $\omega_k Def_{ik}$ of the bank's deposits, where ω_k is a bank specific parameter. This seizure of deposits correspondingly reduces the bank's ability to lend.

There is one further issue in the determination of the supply of bank loans. We will suppose that the banks restrict credit not only because of solvency issues, as above, but because of the perceived "legitimacy" of the potential borrower. Here the borrower is required to show the bank his tax returns in order to obtain a loan. We assume that the bank can estimate the "true" tax obligation of the borrower. This is plausible because the bank uses the reported capital stock as a proxy for the amount of legal business activity the firm conducts. It hence can estimate the degree to which the borrower is evading

taxes as $\frac{\bar{t}_{PI}}{t_{PI}}$ as above. That is, since taxes are linear without exemptions, we would

have $\frac{\bar{t}_{PI}}{t_{PI}} = 1$ if there were no evasion as the effective rate would equal the statutory rate.

¹⁹This figure of " percent is simply taken to correspond to standard bank regulations. That is, if the average ratio of capital to total assets in the banking system is approximately " percent, then an " percent loss of assets would be tantamount to a total liquidation of capital. In practice, a figure of 8 percent is generally used by regulators in the United States.

If $\frac{\bar{t}_{EI}}{t_{EI}} \leq 1$ then evasion is taking place.

Suppose now that the lender curtails the requested loan as the degree of tax evasion rises. In this case the borrower will receive less than the desired loan, C_{HI} . Rather, the loan will be reduced proportionally to the tax payment shortfall. That is, the borrower will be given a loan \hat{C}_{HI} , where:

$$\hat{C}_{HI} = C_{HI} \left[\frac{\bar{t}_{EI}}{t_{EI}} \right]^{\beta}; \beta > 0$$

Here β is a variable specific to a given bank. As β increases, then the bank curtails lending for a particular degree of tax avoidance. Thus the bank's supply of loans, and hence its assets, is determined by the demand for loans from the productive sectors of the economy, as well as the risk imputed to potential borrowers. Additionally, the bank reduces loans to borrowers who have avoided paying taxes. Of course its supply of loans is also restricted by the bank's existing capital. The demand for loans is, in turn, determined by the investment equations described in the previous section. The banks' deposits, and hence liabilities, are determined by the consumers' savings behavior. Since all of these variables are affected by the real interest rate, they will also be affected by the size of the underground economy.

Consumption

There are two types of consumers, representing rural and urban labor. We suppose that the two consumer classes have differing Cobb-Douglas demands. The consumers also differ in their initial allocations of factors and financial assets. The consumers maximize intertemporal utility functions, which have as arguments the levels of consumption and leisure in each of the two periods. We permit rural-urban migration which depends upon the relative rural and urban wage rate. The consumers maximize these utility functions subject to intertemporal budget constraints. The consumer saves by holding money, domestic bank deposits, and foreign currency. He requires money for transactions purposes, but his demand for money is sensitive to changes in the inflation rate. In addition, the consumer's demand for bank deposits is sensitive to his perception of the solvency of the banking system. In particular, as banks increasingly incur bad loans, the consumer's interest elasticity of money declines, causing him to reduce his bank deposits.²⁰ The consumer pays taxes on his consumption, and does not have any direct contact with the underground economy. That is, he pays the full nominal rates under all circumstances.

Here, and in what follows, we will use x to denote a demand variable and y to denote a supply variable. In order to avoid unreadable subscripts, let us let 1 refer to period i and 2 refer to period $i+1$. The consumer's maximization problem is thus:

²⁰This reflects the notion that the consumer worries about the safety of his own deposits as he sees the banks become progressively more insolvent.

$$\max U(x), \quad x = (x_1, x_{L_{u1}}, x_{L_{r1}}, x_2, x_{L_{u2}}, x_{L_{r2}}) \quad (5)$$

such that:

$$(1+t_i)P_{f_i}x_i + P_{L_{ui}}x_{L_{ui}} + P_{L_{ri}}x_{L_{ri}} + P_{M_i}x_{M_i} + P_{B_i}x_{B_i} + e_i P_{B_{F_i}}x_{B_{F_i}} = C_i \quad (5a)$$

$$P_{K_0}K_0 + P_{A_0}A_0 + P_{L_{u1}}L_{u1} + P_{L_{r1}}L_{r1} + P_{M_0}M_0 + r_0B_0 + P_{B_0}B_0 + e_1P_{B_{F_1}}B_{F_1} + TR_1 = N_1$$

$$P_{K_2}(1-\delta)K_0 + P_{A_2}A_0 + P_{L_{u2}}L_{u2} + P_{L_{r2}}L_{r2} + P_{M_2}x_{M_2} + r_1x_{B_1} + P_{B_2}x_{B_2} + e_2P_{B_{F_2}}x_{B_{F_2}} + TR_2 = N_2$$

$$C_i = N_i$$

$$\log(L_{ui}/L_{ri}) = \alpha_1 + \alpha_2 \log \frac{P_{L_{ui}} - P_{L_{ri}}}{P_{L_{ui}} + P_{L_{ri}}} \quad (5c)$$

$$\log P_{B_i}x_{B_i} - \log e_i P_{B_{F_i}}x_{B_{F_i}} = \alpha + \beta (\log r_i - \log \frac{e_{i+1}}{e_i} r_{R_i}) \quad (5b)$$

$$\log P_{M_i}x_{M_i} = a + b \log(1+t_i)P_{f_i}x_i - c \log \pi_i; \quad c = c(DEF/ASSETT) \quad (5d)$$

if $P_{L_{ui}} \leq P_{L_{ri}}$; otherwise $\log(L_{ui}/L_{ri}) = 0$

(if the representative household is rural, otherwise labor holdings are constant)

$$P_{B_2}x_{B_2} = d_0 + d_1(1+t_2)P_2x_2 + d_2 \left[\frac{r_2 - \pi_2}{1 + \pi_2} \right]$$

where:

P_i = price vector of consumption goods in period i .

x_i = vector of consumption in period i .

C_i = value of aggregate consumption in period i (including purchases of financial assets).

N_i = aggregate income in period i (including potential income from the sale of real and financial assets).

t_i = vector of sales tax rates in period i .

P_{Lui} = price of urban labor in period i .

L_{ui} = allocation of total labor to urban labor in period i .

x_{Lui} = demand for urban leisure in period i .

P_{Lri} = price of rural labor in period i .

L_{ri} = allocation of total labor to rural labor in period i .

x_{Lri} = demand for rural leisure in period i .

a_2 = elasticity of rural/urban migration.

P_{Ki} = price of capital in period i .

K_0 = initial holding of capital.

P_{Ai} = price of land in period i .

A_0 = initial holding of land.

$*$ = rate of depreciation of capital.

P_{Mi} = price of money in period i . Money in period 1 is the numeraire and hence has a price of 1.

x_{Mi} = holdings of money in period i .

P_{Bi} = discount price of a certificate of deposit in period i .

B_i = domestic rate of inflation in period i .

r_p, r_{FE} = the domestic and foreign interest rates in period i .

x_{Bi} = quantity of bank deposits, that is, CD's in period i .

e_i = the exchange rate in terms of units of domestic currency per unit of foreign currency in period i .

x_{BFi} = quantity of foreign currency held in period i .

TR_i = transfer payments from the government in period i .

a, b, α, β = estimated constants.

d_i = constants estimated from model simulations.

DEF = The value of non-performing assets in the banking system.

ASSET = Total assets of the banking system.

c = a functional form that depends negatively upon the ratio of non-performing assets to total assets in the banking system.

The left hand side of equation (5a) represents the value of consumption of goods and leisure, as well as of financial assets. The next two equations contain the value of the consumer's holdings of capital and labor, as well as the principal and interest that he receives from the domestic and foreign financial assets that he held at the end of the previous period. The equation $C_i = N_i$ then imposes a budget constraint in each period. Equation (5d) is a standard money demand equation in which the demand for cash balances depends upon the domestic rate of inflation and the value of intended consumption. There is, however, one modification. The inflation elasticity, c , depends upon the share of non-performing bank assets in total assets. If there are no bad assets, then c takes its estimated value. As non-performing assets rise, c declines.

Equation (5b) says that the proportion of savings made up of domestic and foreign

interest bearing assets depends upon relative domestic and foreign interest rates, deflated by the change in the exchange rate. Finally, equation (5c) is a migration equation that says that the change in the consumer's relative holdings of urban and rural labor depends on the relative wage rates.

In period 2 we impose a savings rate based on adoptive expectations, as in equation (5e). The constants (α) are estimated by a simple regression analysis, based on the previous periods. Thus if we are in period t , where t the end of a two period segment, then the closure saving rate for period t is determined by nominal income and the real interest rate. The constants are updated after each two period segment by running a regression on the previous $t - 2$ periods. Thus savings rates are endogenously determined by intertemporal maximization in period t , but are determined by adoptive expectations in period $t + 1$.²¹

The Government

The government collects personal income, corporate profit, and value added taxes, as well as import duties. It pays for the production of public goods, as well as for subsidies. In addition, the government must cover both domestic and foreign interest obligations on public debt. The deficit of the central government in period 1, D_1 , is then given by:²²

$$D_1 = G_1 + S_1 + r_1 B_0 + r_1 e_1 B_{F0} - T_1$$

where S_i represents subsidies given in period i , G_i is spending on goods and services, while the

²¹Since the only information the consumer has about the future is the real interest rate, adoptive expectations is, in this case, equivalent to rational expectations.

²²As before, 1 denotes period i and 2 denotes period $i+1$.

next two terms reflect domestic and foreign interest obligations of the government, based on its initial stocks of debt. T_1 represents tax revenues, which is partially determined by firms' entry into the underground economy.

The resulting deficit is financed by a combination of monetary expansion, as well as domestic and foreign borrowing. If y_{BGI} represents the face value of domestic bonds sold by the government in period 1, and C_{F1} represents the dollar value of its foreign borrowing, then its budget deficit in period 2 is given by:

$$D_2 = G_2 + S_2 + r_2(\Delta y_{BGI} + B_0) + e_2 r_{F2}(C_{F1} + B_{F0}) - T_2$$

where $r_2(\Delta y_{BGI} + B_0)$ represents the interest obligations on its initial domestic debt plus borrowing from period 1, and $e_2 r_{F2}(C_{F1} + B_{F0})$ is the interest payment on the initial stock of foreign debt plus period 1 foreign borrowing.

The government finances its budget deficit by a combination of monetization, domestic borrowing, and foreign borrowing. We assume that foreign borrowing in period i , C_{Fi} , is exogenously determined by the lender. The government then determines the face value of its bond sales in period i , y_{BGi} , and finances the remainder of the budget deficit by monetization.¹⁴ Hence:

$$D_i = P_{Bi} y_{BGi} + P_{Mi} y_{Mi} + e_i C_{Fi}$$

The Foreign Sector

The foreign sector is represented by a simple export equation in which aggregate demand for exports is determined by domestic and foreign price indices, as well as world income. The specific form of the export equation is:

$$\Delta X_{no} = \sigma_1 \left[\frac{\pi_1}{\Delta e_i + \pi_{F1}} \right] + \sigma_2 \Delta y_{wt}$$

where the left hand side of the equation represents the change in the dollar value of exports in period i , π_1 is inflation in the domestic price index, $-e_i$ is the percentage change in the exchange rate, and π_{F1} is the foreign rate of inflation. Also, $-y_{wt}$ represents the percentage change in world income, denominated in dollars. Finally, F_1 and F_2 are corresponding elasticities.

The combination of the export equation and domestic supply responses determines aggregate exports. Demand for imports is endogenous and is derived from the domestic consumers' maximization problems. Foreign lending is assumed to be exogenous. Thus gross capital inflows are exogenous, but the overall change in reserves is endogenous. Finally, we will suppose that the exchange rate is fixed.

The supply of foreign reserves y_{FGi} , available to the government in period i is given by:

$$y_{FGi} = y_{FG(i-1)} + X_i - M_i + x_{F(i-1)} - x_{Fi} + C_{Fi}$$

Here x_{Fi} represents the demand for foreign assets by citizens of the home country, so $x_{F(i-1)} - x_{Fi}$ represents private capital flows. C_{Fi} represents exogenous foreign borrowing by the home government.

Finally changes in the money supply in period i , ΔM_{Si} are now given by:

$$\Delta M_{Si} = \Delta y_{Mi} + \Delta OMO_i + e_i y_{FGi} - e_{i-1} y_{FG(i-1)}$$

where Δy_{Mi} is determined by the government's financing its budget deficit, and ΔOMO_i represents money created via open market operations. The remainder of the right hand side represents the domestic currency value of the balance of payments.

V. Simulations

In this section we will carry out simulations designed to give some qualitative notion of the implications for the economy of tax avoidance and entry into the underground economy. We use data from Pakistan, but we should view this as having only a tenuous relationship to the economy of that country.²³ We will first carry out a base line scenario and then carry out certain counterfactual exercises designed to analyze the effects of alternative tax policies in reducing the size of the underground economy.

In order to use our model for counter-factual simulations, we first generate an equilibrium using benchmark policy parameters. We then run the macroeconomic model for eight years.²⁴ We take tax rates to have their estimated effective values. Government current and capital expenditures are given their historical values for entire 8 years. We also suppose that the Central Bank maintains a fixed exchange rate, with the rate being fixed at the level of the first year.

We will suppose that a sector is unable to repay its debt when the present value of the future stream of earnings from the investment becomes less than the corresponding debt obligations. Finally, we will suppose that the bank solvency requirement, ω , is 8 percent. Thus if a bank's non-performing assets are greater than 8 percent of its total assets, then a portion of the bank's deposits are seized and depositors are unable to retrieve that share of their assets. As before, if the bank's borrowers default on their loans, then the bank loses $\omega \mathbf{DEF}_t$ of its

²³We have used various parameters derived from Iqbal et al. (1998) and Iqbal (1994) in order to implement the functional forms of our model.

²⁴In practice, we take 1993 as the base year. By this we mean that initial allocations of factors and financial assets are given by the Pakistan stocks at the end of 1992. We have data for fiscal and other policy parameters for the next 8 years, that is, through 2000.

deposits, resulting in a wealth shock to depositors. In order to be specific, we will let $\omega_{\mathbf{x}} = \mathbf{1}$ for all banks. Finally, for this benchmark simulation we will assume that banks do not optimize. That is, they do not ration credit when their borrowers begin to default.

Table 1 shows the results of this benchmark simulation. It may be worth making a few remarks concerning the simulated values. We would not wish to make comparisons with actual historical data from Pakistan, given the illustrative nature of this example. In particular, we assume a fixed exchange rate while, in fact, the country has had a managed float. First notice that our model generates moderate rates of growth in real GDP for the first seven periods, after which real growth stagnates. This is primarily the result of the fixed nominal exchange rate, which becomes progressively overvalued. The budget deficit rises and then stabilizes, as the overvalued exchange rate lowers the cost of servicing foreign debt. Similarly, interest rates rise and then stabilize.

It is useful to observe the change in participation of the different sectors in the underground economy. We see that sector 2 and 3 both have a share of their activity in the underground economy during the initial periods. As time passes, their underground activity falls, as a share of their total output. The reason for this decline is that the rate of return to capital slowly rises over time, as real GDP rises more rapidly than does investment. However the rate of change in investment is not uniform across sectors, so underground activity in sector 2 falls more rapidly than in sector 3. We thus see that underground activity may be cyclical.

Table 1. Base Case									
Period	1	2	3	4	5	6	7	8	
Nominal GDP 1/	100.0	109.8	136.5	158.2	174.8	202.2	218.9	254.3	
Real GDP 1/	100.0	105.7	110.1	112.6	117.6	117.5	123.1	121.7	
Price Level	100.0	103.9	124.0	140.5	148.7	172.1	177.9	208.9	
Interest rate	1.5	2.5	6.5	8.9	7.7	6.4	7.0	8.0	
Budget deficit 2/	4.2	4.8	8.2	8.1	10.1	10.0	7.7	7.7	
Trade balance 2/	-7.3	-6.5	-9.2	-8.8	-10.1	-9.7	-10.6	-10.4	
Net capital stock at 3/ end of period 8						Percent of sector in underground economy			
						2	4	6	8
Sector 1	100.0					0.0	0.0	0.0	0.0
Sector 2	100.0					12.6	0.6	0.0	0.0
Sector 3	100.0					20.9	13.3	17.7	10.9
Sector 4	100.0					0.0	0.0	0.0	0.0
Sector 5	100.0					0.0	0.0	0.0	0.0

Suppose now that the government moves to a high tax regime. That is, the government increases the capital tax rate from its current 13 percent to 25 percent. Obviously this is an arbitrary change, but could be viewed as a typical instrument for reducing the budget deficit. Table 2 shows the outcomes of this exercise. As might be expected, the increase in the corporate tax rate has a deflationary impact upon the economy. In addition, there has been a decline in real GDP, due primarily to the sharp decline in investment in all sectors. Thus we note that the final capital stock in all sectors has dropped, as compared to the previous table.

There are, however, certain unexpected outcomes in this simulation. Possibly most

striking, we see that there is only a slight decline in the budget deficits, and by period 6 the budget deficit is actually higher with the higher tax rates than it was before. Indeed, in the first 2 periods of the simulation, the example with higher tax rates has higher budget deficits than does the original example. The deficit falls in the last two periods, relative to the previous example, although only slightly. In fact, these minor improvements are not reflected in the primary deficit, but are due largely to the reduction in the interest rate in this example. The higher tax rate, and corresponding lower rate of return to capital, has driven 4 of the five sectors into the underground economy. Since underground activity does not pay taxes, there is a corresponding reduction in tax revenue and hence the observed lack of improvement in the budget. At the same time, the failure to pay taxes leads to credit rationing on the part of the banks. Hence we see that the underground sectors, 1-4, have much lower rates of capital formation than does sector 5 which remains fully legal, pays taxes, and does not experience credit rationing.

Hence raising the corporate income tax rate has negative consequences beyond those that one might normally expect. The entry of firms into the underground economy has led to a decline in the tax base and a corresponding increase in the budget deficit. At the same time, credit has been rationed to the non-tax paying firms, leading to further reductions in investment.

Table 2. A 25 percent capital tax

Period	1	2	3	4	5	6	7	8	
Nominal GDP 1/	90.1	92.1	117.2	131.7	148.0	161.5	193.5	220.3	
Real GDP 1/	96.4	99.6	105.8	107.2	112.9	112.1	120.0	119.1	
Price Level	93.4	92.5	110.8	122.9	131.1	144.1	161.3	185.0	
Interest rate	-4.5	-4.8	2.6	3.1	3.2	2.6	3.9	4.7	
Budget deficit 2/	4.3	6.3	7.3	8.0	9.1	10.2	5.7	6.4	
Trade balance 2/	-6.9	-4.7	-7.9	-7.1	-8.6	-7.3	-6.6	-5.4	
Net capital stock at 3/ end of period 8						Percent of sector in underground economy			
						2	4	6	8
Sector 1	81.7					60.1	31.0	17.4	0.0
Sector 2	76.5					70.1	61.2	46.7	31.6
Sector 3	74.7					66.8	48.5	44.0	35.1
Sector 4	83.1					57.9	36.8	24.1	2.8
Sector 5	98.3					0.0	0.0	0.0	0.0

Suppose now that the government decides to move in the opposite direction. That is, it lowers taxes. Such a policy might be carried out as an attempt to create something like a Laffer effect that increases tax revenues by increasing economic activity in response to lower taxes, while reducing the attractiveness of entry into the underground economy. As an extreme example, we will reduce the corporate income tax rate to 3 percent, from the 13 percent in the base case. At the same time we will reduce the sales tax from the 11 percent of the base case to 1 percent. Clearly the intent of such a policy would be to stimulate growth by increasing both investment and consumption. At the same

time, lower tax rates would presumably discourage underground activity and therefore enhance the tax base. Table 3 gives the outcomes of the low tax case.

Table 3. A 3 percent capital tax, 1 percent sales tax									
Period	1	2	3	4	5	6	7	8	
Nominal GDP 1/	135.4	147.7	201.7	236.5	286.7	335.4	392.6	455.7	
Real GDP 1/	105.6	111.4	116.1	118.0	123.5	122.7	128.6	126.3	
Price Level	128.3	132.6	173.7	200.4	232.0	273.3	305.2	360.8	
Interest rate	3.0	4.2	10.4	10.6	10.4	7.9	8.8	7.9	
Budget deficit 2/	12.6	12.8	17.4	16.7	18.8	18.1	17.5	17.1	
Trade balance 2/	-8.7	-8.2	-11.0	-11.0	-12.3	-12.3	-13.2	-13.1	
Net capital stock at 3/ end of period 8						Percent of sector in underground economy			
						2	4	6	8
Sector 1	100.6					0.0	0.0	0.0	0.0
Sector 2	96.6					0.0	0.0	0.0	0.0
Sector 3	113.8					0.0	0.0	0.0	0.0
Sector 4	101.3					0.0	0.0	0.0	0.0
Sector 5	101.7					0.0	0.0	0.0	0.0

Again, there are some unexpected changes, as compared to the base case. Thus we see that, although there is no underground activity, the rate of capital formation has increased only slightly in all but one sector. Indeed, it has declined in one sector. This is largely due to the fact that the budget deficit has more than doubled, leading to crowding out of private investment by public borrowing. This occurs even though there is only a small increase in the nominal interest rate, and a decrease in the real interest rate, due to the fact that the deficit is primarily financed by monetization. At the same time, the average annual inflation rate rises from 11.1 percent to 20.1 percent in response to the monetization of the budget deficit. Also, the trade balance

deteriorates sharply as increases in the monetary base combine with the assumed fixed exchange rate regime.

Thus we may conclude that the low tax regime is not sustainable over time, even though it does away with underground economic activity. Accordingly, we may conclude that it might well be possible to have tax rates that induce some underground behavior, yet are nonetheless optimal for the overall economy.

VI. Summary and Conclusion

We have constructed a model that is designed to analyze the causes and effects of an underground economy. We use a dynamic general equilibrium structure in which optimizing firms compare the rate of return on investment with the corporate tax rate. If the tax rate is higher than the return to investment, then the firm begins to move into the underground economy, that is, avoids paying taxes. At the same time, a firm that avoids paying taxes is subject to credit rationing by banks, as failure to pay taxes is taken as a sign of lack of credit worthiness.

We carry out a series of simulations of the model, based on stylized data from Pakistan, a country with a large underground economy. Since we have not estimated any parameters, our results should be viewed as having only a tenuous relationship to Pakistan reality. A benchmark simulation, using actual tax rates, shows that entry into the underground economy can have a cyclical nature, as the rate of return on investment changes. A second simulation raises the corporate tax rate, as a possible anti-budget deficit policy. This turn out to be counter-productive with a large amount of production fleeing to the underground economy, thereby lowering the tax base and actually increasing the deficit. A third simulation reduces the corporate tax rate, with the intent of creating Laffer curve effects. This policy does, indeed, eliminate underground

activity, but at the cost of high rates of inflation, increased budget deficit, and loss of foreign reserves. Hence this scenario is not sustainable in the long run.

We may thus conclude that it is possible that an economy may have to accept some underground activity, that is, tax avoidance as part of an otherwise acceptable tax program. We have not considered the possibility of a government enforcement technology that might reduce the incidence of tax avoidance. Also, we have not looked at the impact of productive government spending on infrastructure in reducing the underground economy. These may represent directions for future research.

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