TAYLOR RULES AND THE DEUTSCHMARK - DOLLAR REAL EXCHANGE RATE

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ABSTRACT

We explore the link between an interest rate rule for monetary policy and the behavior of the real exchange rate. The interest rate rule, in conjunction with some standard assumptions, implies that the deviation of the real exchange rate from its steady state depends on the present value of inflation and output gap differentials. An initial look at German data yields some support for the model.

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1. INTRODUCTION

This paper explores the link between an interest rate rule for monetary policy and the behavior of the real exchange rate.

A large body of research has studied the connection between monetary policy or interest rates on the one hand and exchange rates on the other. The vintage monetary model of the exchange rate takes the money supply as the indicator of policy. (See Frankel and Rose (1995) for a survey.) Some recent literature (Bergin (2002)) also assumes exogenous monetary policy, while estimating a general equilibrium sticky-price open economy macroeconomic model. Yet an ongoing literature has argued, persuasively in our view, that recent monetary policy can be better modeled as taking the interest rate as the instrument of policy, with policy described by a feedback rule. The empirical literature for the United States includes Taylor (1993) as a relatively early contribution, English et al. (2002) as a recent study. Open economy interest rate rules that include exchange rates have been studied quantitatively (e.g., Clarida et al. (1998), Meredith and Ma (2002)) and in terms of welfare properties (e.g., Ball (1999), Clarida et al. (2001), Svensson (2000), and Kollman (2002)). These papers do not, however, consider the positive effects of interest rate rules on the exchange rate.

At least four strands of literature do specifically analyze real or nominal exchange rates in a framework that treats the interest rate - exchange rate interaction in detail. One is the literature on identified VARs. Kim (2002), who includes an exchange rate in his interest rate equation, is an example. A second strand of the literature tests or examines interest parity, in a relatively unstructured way, decomposing real exchange rate movements into components that can be linked to interest rates and those that cannot. Examples include Campbell and Clarida (1987), Edison and Pauls (1993) and Baxter (1994). The papers in these two strands typically, though not always, find a statistically strong connection between interest rates and exchange rates. This encourages us to study the connection. We use an approach that is shared by two other strands of the literature.

These two strands are a long-standing one that models the exchange rate as a present value, and uses traditional econometric techniques, and a more recent one that develops general equilibrium sticky price models, and uses calibration. Examples of the present value approach include Woo (1985), Frankel and Meese (1987) and West (1987). Examples of the calibration approach include Benigno (1999), and Benigno and Benigno (2001). In terms of mechanics, our empirical approach is similar to that of the present value literature, though this literature has yet to consider Taylor rules. Our theoretical approach is congruent with the calibration literature, though in contrast to this literature we use our theory to guide interpretation but not to dictate the specifics of our quantitative work.

Several of the papers cited above have added terms in exchange rates to otherwise standard Taylor rules (e.g., Clarida et al. (1998), and Benigno (1999)). We, too, take this approach, adding the deviation of the real exchange rate from its steady state value to a Taylor rule that also includes standard terms in inflation and output. We do so with the aim of evaluating the effect such a term has on time series properties of aggregate variables, including in particular exchange rates. We show that in conjunction with interest parity, this modified rule delivers a relation between current and expected real exchange rates on the one hand and inflation and output on the other.

Taking steady state values as given, we use a simple, stylized two country model based on Galí and Monacelli (2002) to note that the addition of an IS-like equation and a price adjustment equation (Phillips curve) serve to determine deviations from steady state of real exchange rates and other endogenous variables. As in Benigno and Benigno (2001), the real exchange rate is stationary and the nominal exchange rate has a unit autoregressive root.

A more novel result concerns how the real exchange rate responds to a shock to the Phillips curve. Such a shock pushes inflation up in our model. Some monetary models imply that a country experiencing transitory high inflation will experience a transitory real depreciation of its currency (Frankel (1979), Engel and Frankel (1984)). But in our model this shock results in an appreciation of the currency. This is because of the endogenous response of the monetary authority: a stable Taylor rule requires policy makers to react to an incipient increase in inflation with an increase real interest rates.

In our empirical work we do not, however, study responses of the exchange rate or other variables to this or other shocks. Instead, our empirical work, which is the focus of the paper, uses the model to guide construction of a real exchange rate series that is a function of observable variables, and compares this "model-based" series to the actual real exchange rate. To do so, we use the model's implication that the real exchange rate is the present value of the difference between home and foreign output gaps and inflation rates. The discount factor depends on the weight that the real exchange rate receives in the Taylor rule. The weights on output and inflation are those of the Taylor rule. We attempt to gauge the congruence between the variable implied by this present value and the Deutschmark-dollar real exchange rate, 1979-1998. We focus on Germany because Clarida et al. (1998) found that the real exchange rate entered an interest rate rule for Germany with a coefficient that is statistically significant, albeit small.

To construct this present value–what we call a "model-based" real exchange rate–we compute forecasts of German-U.S. inflation and output gaps differentials from an unconstrained vector autoregression and impose rather than estimate Taylor-rule coefficients. We generate a model-based nominal exchange rate by adding actual inflation to changes in the model-based real exchange rate.

We find that the model-based exchange rates display two well known properties of actual exchange rates. First, they follow processes not too far from random walks. ("Not too far" does not mean *exactly* random walks; changes in each display some (small) autocorrelations.) Second, the correlation between changes in nominal and real exchange rates is nearly one. For the model-based real exchange rate, this follows because discounting a sum of highly persistent series (in our case, output and annual inflation) produces a series far more variable than the series that are being discounted. So constructing the model based nominal exchange rate by adding inflation does little to change the behavior of the series. As well, the confidence interval around the point estimate of the correlation is very tight.

We also compare the implied time series of the model-based real exchange rate with that of the actual series. Here the results are more modest. The correlation between levels of the two variables is about 0.4, between growth rates is about 0.15. The correlation between model-based and actual changes in nominal exchange rates is also about 0.15. Some investigation suggests that the 0.4 figure results because 0.4 is essentially the correlation between the linear combination of inflation and output that enters the Taylor rule on the one hand, and the real exchange rate on the other. The 0.4 correlation between levels comes with a very wide confidence interval that includes zero; the 0.15 figures come with confidence intervals that allow us to reject the null of zero correlation at traditional significance levels.

These correlations admittedly are not particularly large. And whatever success our approach does achieve empirically comes after making many simplifying, and admittedly debatable, assumptions. These include among others: model-consistent (rational) expectations; expectations that are homogeneous across participants in exchange rate markets and monetary policy makers; stable regimes (no bumps from reunification, no bumps–even in 1998–from the prospective introduction of the euro); identical parameters in U.S. and German monetary policy rules. Hence we recognize that these results should be interpreted with caution.

Section two describes our stylized model. Section three outlines data, econometric model and estimation technique. Section four presents basic results. Section five presents additional results and discussion. Section 6 concludes. An Appendix works out the stylized model.

2. MODEL

We consider a New Keynesian open economy model similar to the ones in Benigno (1999), McCallum and Nelson (1999), and, especially, Galí and Monacelli (2002). The model allows us to describe analytically the joint endogenous behavior of the exchange rate, output and inflation. Our aim is not to calibrate but to motivate and guide our interpretation of empirical results. In this section, we try to limit algebraic details to the ones most relevant to understanding our empirical work, with full details in the Appendix.

We imagine a two country world. Each of the countries produces one good and consumes two. The foreign country is large, in the sense that its aggregate price level and consumption is indistinguishable from the price and consumption of the good it produces. The home country, by contrast, is small.

Let "*h*" denote the home country (Germany, in our empirical work), "*" the foreign country (the U.S., in our empirical work.) Also, define:

(2.1) i^h_t = interest rate in home country, i^*_t = interest rate in foreign country, $i_t = i^h_t - i^*_t$ is the difference in between home and foreign interest rates;

y^h_t, y^*_t, $y_t = y^h_t - y^*_t$: deviation of log output from trend;

p^h_t, p^*_t, $p_t = p^h_t - p^*_t$: log price levels;

π^h_t, π^*_t, $\pi_t = \pi^h_t - \pi^*_t$: inflation (first difference of log consumer price level [CPI]);

u^h_{mt}, u^*_{mt}, $u_{mt} = u^h_{mt} - u^*_{mt}$, : shock to monetary policy rule;

s_t: log nominal exchange rate (e.g., DM/\$, when Germany is the home country);

$q_t = s_t-(p^h_t-p^*_t) \equiv s_t-p_t$: log real exchange rate;

E_t: mathematical expectations conditional on a period t information set.

For convenience, we will generally omit the qualifier "difference" when referring to differences between home and foreign variables: y_t and π_t will be called plain old "output" and "inflation."

In our analytical calculations, the monetary rules in the foreign and home countries are:

$$(2.2) \quad i^*_t = \gamma_\pi E_t \pi^*_{t+1} + \gamma_y y^*_t + u^*_{mt},$$

$$(2.3) \quad i^h_t = \gamma_q q_t + \gamma_\pi E_t \pi^h_{t+1} + \gamma_y y^h_t + u^h_{mt}.$$

We omit constant and trend terms here and throughout. Use of expected inflation and current output slightly simplifies some of our calculations below (relative to use of expected output along with expected inflation, or current inflation and current output), but seems unlikely to have important qualitative effects.[1] The equations assumed in our empirical work differ slightly from (2.2) and (2.3) in that the empirical work assumes that the monetary rules include annual inflation–i.e., terms of the form $\gamma_\pi E_t(p^*_{t+12}-p^*_t)$ and $\gamma_\pi E_t(p^h_{t+12}-p^h_t)$, since we use monthly data.

Equation (2.2) is a standard Taylor rule, assumed in our empirical work to apply to the U.S.. In our analytical calculations it applies to a large country. Equation (2.3) is a Taylor rule with the real exchange rate included. This equation is assumed in our empirical work to apply to Germany, and in our analytical calculations to a small country. The assumption that the two countries have the same monetary policy parameters γ_π and γ_y is for convenience; allowing distinct parameters poses no conceptual problems but does complicate the algebra and empirical work. In this section, the parameters are assumed to obey $\gamma_\pi > 1$, $\gamma_y > 0$, and $\gamma_q > 0$.[2]

With $\gamma_q > 0$, the monetary authority is assumed to raise interest rates when the real exchange rate is above (the currency is depreciated relative to) its long-run level. (Recall that since we are omitting constants and trends, we write equation (2.3) in a form that gives the long-run level as zero.) We take this as a reasonable description of monetary policy in some open economies.[3] Perhaps the most pertinent reference is Clarida et al. (1998), who find that a term in the real exchange rate is statistically significant in Taylor rules estimated for Germany and Japan, with $\hat{\gamma}_q \approx 0.1$.

More generally, we take (2.3) to be a specific form of a Taylor rule that includes a term of the form

$$(2.4) \quad \gamma_q(s_t - \bar{s}_t)$$

where s_t is the nominal exchange rate and $\bar{s}_t$ is a target for s_t. In (2.3), the target is $(p_t^h - p_t^*)$ + constant, where the constant has been omitted from (2.3) for simplicity. In Cho and West (2002), the target $\bar{s}_t$ followed an unobserved random walk. The Taylor rules for Italy, France and the U. K. that were estimated by Clarida et al. (1998) fall into this framework with $\bar{s}_t$ set to central parity within the ERM.

Subtract (2.2) from (2.3), obtaining

$$(2.5) \quad i_t = \gamma_q q_t + \gamma_\pi E_t \pi_{t+1} + \gamma_y y_t + u_{mt}.$$

Next, write uncovered interest parity as

$$(2.6) \quad i_t = E_t s_{t+1} - s_t.$$

(We discuss below the implications of a possible presence of a risk premium shock in (2.6).) Upon subtracting the expected value of next period's inflation from both sides of (2.6), and using the definition of q_t, we obtain

$$(2.7) \quad i_t - E_t \pi_{t+1} = E_t q_{t+1} - q_t.$$

Use (2.7) to substitute out for i_t on the left hand side of (2.5). The result may be written

$$(2.8) \quad q_t = bE_t q_{t+1} + bE_t(1-\gamma_\pi)\pi_{t+1} - b\gamma_y y_t - bu_{mt}.$$

In (2.8), $b = 1/(1+\gamma_q)$, $0<b<1$. This equation contains three endogenous variables (q_t, π_t, and y_t). Two additional equations are needed to close the system. In the Appendix, these two equations are (1)a market clearing condition relating the output gap to the real exchange rate and, called the IS curve for convenience, and (2)a price adjustment equation (Phillips curve) that relates inflation and output. Once equilibrium in these three variables is determined, the nominal exchange rate is determined via $\Delta s_t = \Delta q_t + \pi_t$.

In the Appendix, we solve the system assuming stationary AR(1) shocks with nonnegative parameters, along with certain stability conditions. The system has the following properties in terms of responses to shocks:

•A positive monetary policy shock ($u_{mt}\uparrow$–i.e., an exogenous monetary tightening) causes the real and nominal exchange rates q_t and s_t to fall (i.e., appreciation). Inflation π_t and output y_t also fall. The response of exchange rates is consistent with interest parity. The response of inflation and output is consistent with conventional closed economy models.

•Consider a positive Phillips curve shock that, given output and expected inflation, raises inflation transitorily. A real appreciation will result. This follows from the combination of interest parity and the parameter restriction $\gamma_\pi > 1$ in the Taylor rule (2.5). With $\gamma_\pi > 1$, incipient increases in inflation cause the real interest rate to increase. From interest parity, the real exchange rate falls. Output falls as well.

•Consider a positive real shock to the IS curve that, given the real exchange rate q_t, raises output y_t. Then in equilibrium, output rises, the real exchange rate falls and the inflation rate of home produced goods rises relative to that foreign produced goods. The impact on π_t (home CPI inflation π_t^h relative to foreign CPI inflation π_t^*), however, is ambiguous: inflation of home relative to foreign goods rises (pushing π_t up), while the real exchange rate falls (pushing π_t down). But we can say that π_t unambiguously falls for a sufficiently large interest rate response to output γ_y. For in this case, the rise in output will cause an increase in interest rates sufficiently large to dampen inflation.

In our empirical work, we do not attempt to identify and trace through the effects of shocks. The previous discussion is intended to give intuition to how a Taylor rule affects propagation. We do, however, aim to compare certain properties of data generated by our model and the actual data (Germany the home country, U.S. the foreign country). While the model is certainly too simple to reproduce all of the serial correlation and second moments, it is consistent with some notable the features of the data. This includes very high persistence in q_t and y_t but not in π_t, along with very high correlation between Δs_t and Δq_t.

The model's persistence properties are to a certain extent exogenous–in equilibrium the endogenous variables are linear in AR(1) shocks.[4] But if the IS or monetary policy shocks are persistent and volatile while the Phillips curve shock is not, the properties described in the previous paragraph will result if, as well, the Phillips curve is flat and the share of imports is not too large.[5] Low serial correlation in PPI inflation ($\mathring{\pi}_t$, in the notation of the Appendix) follows when PPI inflation is not very responsive to the output gap, and Phillips curve shocks have low serial correlation. CPI inflation π_t will behave much like PPI inflation when import shares are low. Further, high serial correlation in relative output and the real exchange rate will be reproduced in the model when Taylor rule shocks and IS shocks are highly serially correlated and volatile, for such shocks will dominate the behavior of y_t and q_t when inflation is not persistent. As well, there will be high correlation between innovations to real and nominal exchange rates since neither series will be much affected by inflation.

Finally, we note that increasing the weight γ_q put on the real exchange rate in the monetary rule decreases the variance of the real exchange rate. In particular, in the limit, as $\gamma_q \to \infty$, the variance of q_t goes to zero, while those of y_t and π_t stay finite.

We close our discussion of the model with some remarks about the role of uncovered interest parity in the analysis. First, we note that if we add an exogenous risk premium shock to (2.6), then (2.8) still results but with u_{mt} redefined to be the difference between the monetary policy shock and the risk premium shock. (That is, if we call the risk premium shock u_{dt}, rewriting (2.6) as $i_t = E_t s_{t+1} - s_t + u_{dt}$, and rename the monetary policy shock to $\tilde{u}_{mt}$, then (2.8) holds with $u_{mt} = \tilde{u}_{mt} - u_{dt}$). Thus the effects of a risk premium shock are the opposite of those of a monetary policy shock.

Second, after once again suppressing a risk premium shock, we observe that by combining the Taylor rule with uncovered interest parity we have a relationship that is rather richer than uncovered interest parity. To test uncovered interest parity, one needs a model for the exchange rate. Often such a model is supplied in what might be called nonparametric form, with minimal assumptions made about the exchange rate. An example is when one tests or estimates interest parity under rational expectations by replacing expectations with realizations, but one does not spell out the process followed by the exchange rate. See Lewis (1995). Our empirical work not only assumes model-consistent (rational) expectations, but also maintains an additional set of assumptions in the form of the Taylor rule. We expect additional assumptions to on balance be helpful if they are consistent with the data. And in our view, the empirical results are consistent with this expectation.

In our empirical work, we do not estimate a Taylor rule. Instead, relying on (2.8), we focus on the relationship between q_t on the one hand and y_t and π_t on the other. Upon imposing the terminal condition that q_t is non-explosive, the solution to (2.8) may be written[6]

$$(2.9) \quad q_t = b\sum_{j=0}^{\infty} b^j E_t[(1-\gamma_\pi)\pi_{t+j+1} - \gamma_y y_{t+j} - u_{mt+j}].$$

Equation (2.9) directly leads to our empirical work, as described in the next section.

3. DATA, EMPIRICAL MODEL AND ECONOMETRIC TECHNIQUE

A, Data

In our empirical work, Germany is the home country, U.S. the foreign country. We focus on Germany because Clarida et al. (1998) found that a Taylor rule, with a real exchange rate term included, well characterizes German monetary policy. We use monthly data; apart from lags, our sample runs from 1979:10 to 1998:12. The start of the sample is chosen to coincide with the beginning of the Volcker regime shift, the end with the introduction of the euro.

Data were obtained from the International Financial Statistics CD-ROM and from the web site of the Bundesbank. Output is measured as the log of seasonally adjusted industrial production (IFS series 66..c), prices as the log of the CPI (series 64), inflation as the first differences of log prices, interest rates by a money market rate (series 60b), exchange rate as the log of the end of month rate (series ae). (Use of monthly average exchange rates in preliminary work led to little difference in results.) Following Clarida et al. (1998), the output gap y_t was constructed as the residual from quadratically detrended output. U.S. and German output were detrended separately, before the output gap differential was constructed. Output, prices and exchange rates were multiplied by 100 so differences are interpretable as percentage changes.

The IFS data combine data for West Germany (1979-1990) and unified Germany (1991-1998). We obtained a continuous series for West German industrial production and CPI from the Bundesbank. To smooth a break in the price and output levels between 1990:12 and 1991:1, we proceeded as follows. We assumed that West German inflation and growth rates of output in 1991:1 (the first year of reunification) also applied to Germany as a whole, and used that ratio to scale up the level of post-1990 data on German output and price level. Thus growth rates of inflation and output match those for West Germany through 1991:1, Germany as a whole afterwards. The scaling still affects our empirical results, because we use the level rather than change of output (quadratically detrended) and the price level figures into the real exchange rate. One final adjustment was to smooth out a one month fall and then rise of over 10% in industrial production in 1984:6 that was present in both IFS and Bundesbank data. We simply set the 1984:6 figure to the average 1984:5 and 1984:7. (We observed this downward spike in an initial plot of the data. If the spike is genuine rather than a data error, it presumably reflects a strike known to be transitory and our smoothing likely makes our VAR forecasts more reasonable.)

Figure 1 plots the data. The real exchange rate has been adjusted to have a mean of 50. The strong appreciation of the dollar in the early 1980s is apparent. Initially, inflation in the U.S. was higher than that in Germany. The output gap in U.S. relative to Germany peaked in the early 1990s, and fluctuated a good deal both before and after.

B. Empirical Model and Econometric Technique

In accordance with Clarida et al. (1998), we assume that expected annual inflation $E_t(p_{t+12}-p_t)$ appears in the monetary policy rule. The present value relation (2.9) becomes

$$(3.1) \quad q_t = b\sum_{j=0}^{\infty} b^j E_t[\pi_{t+j+1} - \gamma_\pi(p_{t+j+12}-p_{t+j}) - \gamma_y y_{t+j} - u_{mt+j}].$$

We aim to construct a "model-based" or "fitted" real exchange rate, call it $\hat{q}_t$, and compare its properties with those of the actual exchange rate q_t. We proceed by imposing values for γ_q, γ_π and γ_y, setting $b=1/(1+\gamma_q)$, and computing forecasts of monthly inflation and output gaps with a vector autoregression (VAR). This autoregression relies on a vector, call it z_t, that includes the interest rate along with monthly inflation and the output gap: $z_t = (\pi_t, y_t, i_t)'$. Thus, in making the forecasts, we allow for the possibility that past interest rates help predict future inflation and output gaps. We do not, however, allow for a direct effect from the shock u_{mt}. We omit this shock because of lack of an independent time series for it. Our intuition is that this omission biases the results against our model, as discussed in section C below.

Let n denote the order of the vector autoregression, let Z_t denote the $(3n\times 1)$ vector that results when the VAR is written in companion form as a vector AR(1), $Z_t' \equiv (z_t\ z_{t-1}\ \ldots\ z_{t-n+1})$. It is straightforward to show that $\hat{q}_t$ is linear in Z_t,

$$(3.2)\qquad \hat{q}_t = b\sum_{j=0}^{\infty} b^j E[\pi_{t+j+1} - \gamma_\pi(p_{t+j+12}-p_{t+j}) - \gamma_y y_{t+j} \mid Z_t] = \text{(say)}\ c_q{}'Z_t,$$

where c_q is a $(3n\times 1)$ vector that depends on b, γ_π, γ_y and the VAR parameters. As long as inflation and output gap differentials are mean reverting, so, too, is $\hat{q}_t$.

Define a "model based" nominal exchange rate change as $\Delta\hat{s}_t \equiv \Delta\hat{q}_t + \pi_t$. We use c_q and the VAR parameters to compute autocorrelations of $\hat{q}_t$, $\Delta\hat{q}_t$ and $\Delta\hat{s}_t$ as well as their cross-correlations with π_t, and y_t. We compare these to the corresponding values for q_t, Δq_t and Δs_t Finally, we construct time series for $\hat{q}_t$ from (3.2) and then for $\Delta\hat{q}_t$ and $\Delta\hat{s}_t$ ($\equiv\Delta\hat{q}_t + \pi_t$), and compute their correlation with q_t, Δq_t and Δs_t.

We construct 95 percent confidence intervals from the percentile method of a nonparametric bootstrap. To construct confidence intervals for the autocorrelations of $\hat{q}_t$, $\Delta\hat{q}_t$ and $\Delta\hat{s}_t$ and for their cross-correlations with π_t, and y_t, we proceed as follows.

1. As a preliminary, we use Kilian's (1998) procedure to estimate the bias in the least squares estimator of the coefficients of the VAR in $(\pi_t, y_t, i_t)'$. (There is a downward bias because the data are highly positively serially correlated.)

2. We generate 1000 artificial samples, each of size 231 (231 because that is the number of monthly

observations running from 1979:10 to 1998:12). In each of the 1000 repetitions, we:

a. Generate a sample using bias-adjusted VAR coefficients and sampling with replacement from the least squares residuals. We use actual pre 1979:10 data for initial conditions.

b. Estimate the VAR on the generated sample.

c. Compute each of the statistics of interest (for example, the first order autocorrelation of $\hat{q}_t$),

3. Finally, we sort the 1000 values of each of the statistics. We construct 95 percent confidence intervals by reporting the 25^{th} smallest and 25^{th} largest statistics (25 = 2.5 percent of 1000).

To construct confidence intervals for corr($\hat{q}_t$,q_t) and corr($\Delta\hat{q}_t$,Δq_t), we used a bivariate VAR(4) in ($\hat{q}_t$,q_t), proceeding in a fashion analogous to that just described. Finally, to construct a confidence interval for the correlation between $\Delta\hat{s}_t$ and Δs_t, we used a bivariate VAR(1) in ($\Delta\hat{s}_t$,Δs_t), omitting the Kilian (1998) bias adjustment and setting the lag length to 1 because ($\Delta\hat{s}_t$,Δs_t) shows almost no serial correlation.

C. Discussion

We discuss first some technical aspects of our empirical approach, and then some more general issues.

In (3.1), let q_{1t} be the present value of π_t-$\gamma_\pi(p_{t+12}$-$p_t)$-$\gamma_y y_t$, let q_{2t} be the present value of the (negative of the) monetary policy shock u_{mt}:

$$q_{1t} \equiv [b\sum_{j=0}^{\infty}b^j E_t[\pi_{t+j+1}-\gamma_\pi(p_{t+j+12}-p_{t+j})-\gamma_y y_{t+j}],\ q_{2t} \equiv [b\sum_{j=0}^{\infty}b^j E_t(-\ u_{mt+j})].$$

Then (3.1) can be written $q_t = q_{1t} + q_{2t}$. Let us tautologically write

$$(3.3)\qquad q_t = \hat{q}_t - (\hat{q}_t - q_{1t}) + q_{2t}.$$

We note first that taking the model in the Appendix literally, the use of the three variables (π_t,y_t,i_t) to construct forecasts means that the second term on the right hand side of (3.3) is identically zero: $\hat{q}_t$-q_{1t} = 0. (Here, and in the remainder of this section, we: abstract from sampling error in construction of $\hat{q}_t$; assume that linear projections and mathematical expectations coincide; generalize the Appendix assumption of independent univariate AR(1) processes for three exogenous shocks to instead allow arbitrary stationary processes for the three shocks, possibly multivariate, provided the spectral density matrix of the shocks is of rank 3.) For in this model, the endogenous variables inflation and output gap can be written as (possibly infinite order) distributed lags on the three shocks. The conditional expectation of these variables can therefore be computed by their projections onto the space spanned by the three shocks. This projection in turn is identically equal to the projection onto the space spanned by (π_t,y_t,i_t), because one can map back and forth between (π_t,y_t,i_t) on the one hand the three shocks on the other with linear filters.

Even under the conditions of the previous paragraph, which insure that $\hat{q}_t$-q_{1t}=0, we still have $\hat{q}_t \neq q_t$ because our model-based series $\hat{q}_t$ omits the present value of monetary policy shocks q_{2t}. Thus while the conditions of that paragraph insure, for example, that the correlation between q_t and $\hat{q}_t + q_{2t}$ is unity, the correlation between q_t and $\hat{q}_t$ will of necessity be lower, since while $\hat{q}_t$ does capture the role played by monetary policy shocks in forecasting inflation and output gaps (as explained in the previous paragraph), $\hat{q}_t$ omits q_{2t} and thus omits the direct effect of the present value of monetary policy shocks on the real exchange rate. In this sense, the omission of this present value biases the results against our model.

Of course, the conditions in the penultimate paragraph are extreme. Sampling error will drive a wedge between $\hat{q}_t$ and q_t, as will the fact that a finite order VAR approximates but does not fully span the space spanned by lags of (π_t,y_t,i_t). And more fundamentally, we use our model to guide and interpret but not to dictate the exact structure of the empirical work. Our model is too simple and stylized to serve as a direct basis for estimation. For example, since the model only has three shocks, it implies a singularity among q_t, π_t, y_t and i_t, and that singularity is not in the data.

One possible strategy would be to move from our stylized model to one rich enough to be estimated. Bergin (2002) provides an example. We consider our approach complementary to that of papers like Bergin (2002). We expect our results to continue to hold qualitatively even if there are small departures from the model. To take but one example: the model sketched in the Appendix assumes that traded goods obey the law of one price. It is apparent from the derivation in section 2–which had no occasion to comment one way or the other on the law of one price–that equations (2.9) and (3.1) can still hold if there are such deviations. Yet our results would be contingent on maintained assumptions about deviations from the law of one price were we to attempt to tie our empirical work to the specifics of the model in the appendix.

4. BASIC EMPIRICAL RESULTS

To construct $\hat{q}_t$, we set the lag length of the vector autoregression to 4 ($n=4$), and used the following parameters: $\gamma_q=0.1$ ($\Rightarrow b\approx.91$), $\gamma_\pi=1.75$, $\gamma_y=0.25$. The value for the exchange rate parameter γ_q is roughly that estimated for Germany by Clarida et al. (1998). The values for the inflation and output gap differentials are roughly those estimated for Germany by Clarida et al. (1998) and those estimated for the U.S. by a number of authors, including Rudebusch (2001) and English et al. (2002).

Panel A of Table 1 has autocorrelations of model-based data in columns (2)-(4), with figures for actual data in columns (5) to (9). Begin with the actual data. The exchange rate pattern is familiar. The real exchange rate is highly autocorrelated ($\rho_1=0.98$ [column (5), row(1)]). Growth rates of the real and nominal exchange rates are approximately serially uncorrelated (columns (6) and (7)). (Indeed, there is a considerable body of evidence dating back to Meese and Rogoff (1983a, 1983b) that the “approximately” can be dropped for nominal exchange rates.) The output gap is highly serially correlated (column (9); monthly inflation less so (column (8)).

Good news for the model is that the model-based exchange rates display properties similar to those of the actual exchange rate data. See columns (2) through (4) in panel A. The first order serial correlation coefficient of $\hat{q}_t$ is 0.97 (column (2)). The growth rates of the model’s real and nominal exchange rates are essentially serially uncorrelated (see the point estimates and 95 percent confidence intervals in columns (3) and (4)). Now, as stated in equation (3.2), $\hat{q}_t$ is a linear combination of the VAR variables, which include π_t and y_t. As we shall see, the high serial correlation of $\hat{q}_t$ (column (2)) essentially reflects in the high serial correlation of y_t and annual inflation; low serial correlation of $\Delta\hat{q}_t$ and $\Delta\hat{s}_t$ seem to reflect the fact that $\hat{q}_t$ is constructed as a present value with a discount factor near 1.

Panel B presents data on cross-correlations. It is well-known that real and nominal exchange rates are highly correlated with one another. Indeed, in our data, when rounded to two digits, the correlation is 1.00 (row (3), column (7)). The high correlation between real and nominal exchange rates is also captured by the model, with a figure that also rounds to 1.00 (row (3), column (2)). More generally, cross-correlations of $\hat{q}_t$, $\Delta\hat{q}_t$ and $\Delta\hat{s}_t$ are very similar to those of q_t, Δq_t and Δs_t.

On the other hand, the model is less successful in reproducing the correlations between the real exchange rate on the one hand and the output gap and inflation on the other. We see in panel B that $\hat{q}_t$ and y_t are sharply negatively correlated–specifically, -0.94 (row (5), column (1)), with a very wide confidence interval of (-1.00,-0.14) that includes the rather different correlation between q_t and y_t of -0.37 (row (5), column (6)). The correlation between $\hat{q}_t$ and π_t (-0.25) is also below that of q_t and π_t (0.04), though in this case the 95 percent confidence interval of (-0.61,-0.02) excludes the 0.04 estimate. According to our model, the negative correlation between output and the real exchange rate indicates that real shocks have played an important role. The positive correlation between inflation and the real exchange rate indicates a role for monetary policy or risk premium shocks. (See the bullet points and the closing paragraphs of section 2 above.) Recall that in our stylized model, such shocks cause a positive correlation between π_t and q_t, and between y_t and q_t. That omission may therefore help explain the negative correlation between $\hat{q}_t$ and π_t and the far too negative correlation between $\hat{q}_t$ and y_t.

Panel C presents the correlations between actual and model-based exchange rates. These are 0.44 for the real exchange rate, 0.16 and 0.16 for growth rates of real and nominal exchange rates respectively. Figure 2 plots the model-based and actual real exchange rate series. That the two series track each other is apparent, with a better match in the first than in the second half of the sample.

The predicted real exchange rate from the model matches relatively well during the period of the great appreciation of the dollar in the late 1970s and early 1980s. In that period, the U.S. raised interest rates (relative to those in Germany and other industrialized countries) to combat high inflation. This tight U.S. monetary policy has frequently been cited as a cause of the dollar's strength. Although the U.S. may not have been closely following a Taylor rule in that period, the high U.S. relative interest rates that arose in response to high U.S. inflation captures the essence of one element of our model.

The model does not capture the continuing appreciation of the dollar in late 1984 and early 1985. The appreciation of the dollar in 1984 has frequently been labeled a "bubble". (See Frankel (1994).) In 1985, U.S. interest rates began to decline gradually, contributing to the fall in the dollar.

The model also does not well match the data around reunification (1990-92). During this period, output gaps and inflation were increasing rapidly in Germany relative to the U.S., but, nonetheless, and in contradiction to our model, the Deutschmark appreciated (i.e., fell) modestly rather than dramatically. One possible explanation is that special events–specifically, reunification, and ongoing stresses in the EMS–offset the forces that our model incorporates. (See Clarida and Gertler (1997).) But in later periods, our model-based series seems broadly similar to the actual series. During this period, German growth lagged U.S. growth. U.S. interest rates rose relative to German rates. Our fitted model captures the strong appreciation of the dollar over this period.

In light of the long history of difficulty in modeling exchange rates, our gut sense is that match between model and data is respectable though not overwhelming. We acknowledge, however, that the relevant standard is not clear-cut and there is much movement in actual exchange rates not captured in our model-based series.

5. ADDITIONAL EMPIRICAL RESULTS AND DISCUSSION

Table 2 presents results under some alternative specifications. All such alternatives are identical to the baseline specifications whose results are presented in Table 1, apart from the variation described in panel A. Alternative specification *a* raises the coefficients on expected inflation and output ($\gamma_\pi = 2.0$, $\gamma_y = 0.5$ rather than $\gamma_\pi = 1.75$, $\gamma_y = 0.25$), perhaps bringing them closer to the estimates from recent U.S. studies such as English et al. (2002) or Rudebusch (2001). Specification *b* uses West German data on prices and output throughout. (Recall that the baseline uses West German data up to 1990:12, data for unified Germany after 1990:12.) It may be seen in panel B of Table 2 that these variations yield little change in the behavior of the model's exchange rates.

Specification *c* begins the sample at 1982:10, the date of a possible regime shift in U.S. monetary policy. As in specifications *a* and *b*, the model's exchange rate series continue to display own and cross-moments quite similar to those of the actual data (columns (2)-(5) of panel B). Now, however, the correlation between model and actual exchange rates falls roughly in half (columns (6)-(8)). For example, the correlation between $\hat{q}_t$ and q_t falls to 0.20 from 0.44. That excluding the 1979:10-1982:10 period will cause a fall in the correlation is perhaps unsurprising in light of Figure 2; one can see that the link between the two series is particularly tight during this period.[7]

Specifications *d* and *e* vary the variables used in the VAR used to forecast future inflation and output. Specification *d* drops the interest rate from the VAR, while specification *e* adds commodity price inflation. Columns (7) and (8) indicate that dropping the interest rate causes a fall in the correlations between model and actual data. Adding commodity price inflation causes little change in results.

Specification *f* uses a Hodrick-Prescott filter rather than a quadratic time trend to construct the output gap. One may see in Table 2B that the correlations are little changed. The standard deviations of $\hat{q}_t$, $\Delta\hat{q}_t$ and $\Delta\hat{s}_t$ do, however, fall by about a third (not reported in the table). Specification *g* varies the lag length in the VAR, with little change in results.

All the preceding specifications were evaluated because they plausibly should produce results very similar to those of the baseline, and indeed they did. The final specification is not remotely plausible, but is presented to give insight into how the model works and why it yields whatever success it does achieve. In this specification, γ_q, the monetary policy weight on the real exchange rate, was set to 100 rather than 0.1. Such a large weight would unreasonably indicate that German monetary policy was dominated by the desire to keep variability of real exchange rates low, with a desire for inflation or output stability distinctly secondary. We see in the final row of panel B that $\hat{q}_t$ and, especially, $\hat{s}_t$ are no longer obviously random-walk like because the autocorrelations of their differences are no longer very near zero ($\rho_1 = 0.13$ for $\Delta\hat{q}_t$, $\rho_1 = 0.36$ for $\Delta\hat{s}_t$).[8] As well, the correlation between $\Delta\hat{q}_t$ and $\Delta\hat{s}_t$ is essentially zero. Nonetheless, the correlations between $\hat{q}_t$ and q_t, and between $\Delta\hat{q}_t$ and Δq_t, remain largely unchanged–indeed, these correlations have risen slightly, from 0.44 to 0.46 and from 0.16 to 0.17.

To understand this pattern, observe that with $\gamma_q = 100$, the discount factor b ($= 1/(1+\gamma_q)$) is essentially zero. Our model then becomes static. That is, with such a small discount factor, equation (3.2) essentially becomes

$$\text{(5.1)} \qquad \hat{q}_t \approx bE[\pi_{t+1} - \gamma_\pi(p_{t+12}\text{-}p_t) - \gamma_y y_t \mid Z_t] = bE[\pi_{t+1} - 1.75(p_{t+12}\text{-}p_t) - 0.25 y_t \mid Z_t].$$

Z_t is the list of variables in the VAR (so to prevent confusion we note explicitly that $E(y_t|Z_t) = y_t$ since y_t is an element of the VAR). The value for $b \neq 0$ does not affect the correlation between (1)a random variable constructed as $bE[\pi_{t+1} - 1.75(p_{t+12}\text{-}p_t) - 0.25 y_t \mid Z_t]$, and (2)$q_t$. Thus the fact that b is small is irrelevant to understanding why $\hat{q}_t$ constructed according to (5.1) has a correlation of 0.46 with q_t. So the correlation evidently indicates that a particular linear combination of output and expected inflation is positively correlated with the real exchange rate. We note that this correlation (0.46) is larger than the (absolute value of the) correlation of y_t and q_t ($= -0.37$) or between π_t and q_t ($= 0.04$) (Table 1B, column (6), rows (5) and (4)). The correlation is also larger than the 0.09 correlation between annual inflation $p_t\text{-}p_{t\text{-}12}$ and q_t (not reported in any table). Thus, the correlation between the model-based and actual real exchange rates seems to reflect a correlation between a *particular* linear combination of inflation and output, namely, the linear combination suggested by the Taylor rule.

In our view, this high correlation results because our model captures two essential features of the data: a Taylor rule well describes monetary policy, and the mark tends to strengthen relative to the dollar when German interest rates rise relative to U.S. rates. Our model can therefore account for the finding that when German inflation and output are relatively high, the mark tends to appreciate. It is a notable feature of our model that it captures the correlation in the data that high German inflation is associated with a strong mark. As we noted above, traditional monetary models (Frankel (1979), for example) have tended to predict the opposite–that high inflation is associated with a weak currency. Thus, there are two reasons to think that the correlation between q_t and $E[\pi_{t+1} - 1.75(p_{t+12}-p_t) - 0.25y_t \mid Z_t]$ is supportive of our model. First, as we have already noted, the correlation is high for the particular linear combination of inflation and output implied by the Taylor rule. And, second, the Taylor rule easily rationalizes the negative relation between the real exchange rate and inflation.

To return to Table 3: The reason the static specification delivers unsatisfactory behavior for nominal exchange rates is straightforward. In particular, the standard deviation of $\Delta\hat{q}_t$ happens to be 0.01, a tiny fraction of the 0.33 standard deviation of π_t (not reported in a table). Accordingly, $\Delta\hat{s}_t$ is dominated by the behavior of π_t. Indeed, a comparison of the figures for $\Delta\hat{s}_t$ in row *h* of Table 2B, and those for π_t in Table 1, shows that $\Delta\hat{s}_t$ behaves just like inflation.

The much closer match between model-based and actual behavior when $\gamma_q = 0.1$, and $b \approx 0.9$, reflects two forces. The first is that with the discount factor near 1, $\hat{q}_t$ is far more variable than are inflation and output. Some relevant figures are in Table 3. We see in row (1), column (9), that the standard deviation of $\hat{q}_t$ is 20.53, far above the standard deviations of y_t, $\pi^e_{at} \equiv E_t(p_{t+12}-p_t)$ and $E_t\pi_{t+1}$ (row (1), columns (1), (3) and (5)). Indeed, this standard deviation is above the 17.33 value for actual q_t (not reported in the table). Our baseline specification has the unusual problem of producing an exchange rate series that is *too* volatile, though what we consider important is that our model, like all present value models, can produce a series whose volatility exceeds that of the variables being discounted.[9] In any event, when we construct $\Delta\hat{s}_t$ by adding π_t to $\Delta\hat{q}_t$, the resulting series is dominated by movement in $\Delta\hat{q}_t$. Hence the high correlation between $\Delta\hat{q}_t$ and $\Delta\hat{s}_t$.

The second force producing a closer match when $\gamma_q = 0.1$ than when $\gamma_q = 100$ is one developed in Engel and West (2002). That paper shows that a present value model such as ours produces random-walk like behavior if (1)the fundamentals are very persistent, and (2)the discount factor is near one. It is to be emphasized that this result does *not* require that the fundamentals follow a random walk, merely that they be persistent. We see in rows (2) to (4), columns (1) and (3), that both y_t and annual inflation are quite persistent, with autocorrelations above 0.9.[10] Their first differences, however, also show some autocorrelation and in this sense the levels of these variables are not random-walk like. We see in columns (7) and (8) that if we just take the linear combination of these variables suggested by the Taylor rule, we end up with a variable whose autocorrelations in both levels and differences are more or less a weighted average of the autocorrelations of each of the variables that feed into the combination. But we see in columns (9) and (10) that if instead we compute a discounted sum of the linear combination, with a discount factor of around 0.9, we end up with a variable that is random walk like, with autocorrelations of differences that are near zero.

6. CONCLUSION

We view our study as a promising initial approach to investigating the empirical implications of Taylor rules for exchange rate behavior. Our model reproduces many significant features of the real and nominal dollar/DM exchange rates: both are very persistent, with differences that are nearly serially uncorrelated; both are very volatile; differences of the two are highly correlated. We had less success in reproducing the correlations of exchange rates with output and inflation, perhaps because our empirical work omitted shocks to the Taylor rule itself and to interest parity. Finally, the model-based real and nominal exchange rates we construct as functions of output and inflation are correlated with the actual real and nominal exchange rates. The correlation is modest, but perhaps is acceptable by industry standards.

Our empirical model takes the time-series process for inflation and output as given. A priority for future work is to explicitly interpret these variables in terms of behavioral equations. One advantage of doing so is that we would be able to measure the Taylor-rule shocks. In order to improve the fit for exchange rates, the structural model would probably need to do a good job explaining inflation and output as well.

The focus of much of the existing quantitative literature on Taylor rules in open economics is on optimality properties of different rules. We believe there is promise in further exploration of the implications of such models for the empirical behavior of exchange rates.

APPENDIX

We consider a New Keynesian open economy model similar to the ones in Benigno (1999), McCallum and Nelson (1999), and, especially, Galí and Monacelli (2002). Our aim is not to calibrate but to motivate and guide our interpretation of empirical results.

We imagine a two country world. Each of the countries produces one good and consumes two. The foreign country is large, in the sense that its aggregate price level and consumption is indistinguishable from the price and consumption of the good it produces. The home country, by contrast, is small. The price of the home country's domestically produced good is p_{dt}, of the imported good is p_{ft}. Corresponding inflation rates are π_{dt} and π_{ft}. Preferences are logarithmic in a Cobb-Douglas aggregate over the two goods. In the Cobb-Douglas aggregate, the weight on the home produced good is $1\text{-}\alpha$, on the foreign produced good α. From familiar logic, then, the aggregate inflation rate π_t^h obeys

(A.1) $\quad \pi_t^h = (1\text{-}\alpha)\pi_{dt} + \alpha\pi_{ft}.$

For the foreign country, $\alpha \approx 1$. Otherwise, all parameters are identical in the two countries. The law of one price holds, so

(A.2) $\quad \pi_{ft} = \Delta s_t + \pi_t^*.$

Adjustment of prices of the domestically produced good takes place according to

(A.3) $\quad \pi_{dt} = \beta E_t \pi_{dt+1} + \kappa y_t^h + u_{ct}^h,\ \pi_t^* = \beta E_t \pi_{t+1}^* + \kappa y_t^* + u_{ct}^*.$

In (A.3), $0 < \beta < 1$, and $\kappa > 0$, and u_{ct}^h and u_{ct}^* are cost shocks . We note that such cost shocks are absent in the model of Galí and Monacelli (2002); we allow them for empirical relevance and consistency with the broader literature on open economy macroeconomics. Thus,

$$\pi_{dt} - \pi_t^* = \beta E_t(\pi_{dt+1} - \pi_{t+1}^*) + \kappa(y_t^h - y_t^*) + u_{ct}^h - u_{ct}^*$$

which, for notational simplicity, we write as

(A.4) $\quad \mathring{\pi}_t = \beta E_t \mathring{\pi}_{t+1} + \kappa y_t + u_{ct}, \quad \mathring{\pi}_t \equiv \pi_{dt} - \pi_t^*,\ y_t = y_t^h - y_t^*,\ u_{ct} \equiv u_{ct}^h - u_{ct}^*.$

The output gap differential y_t is linearly related to the real exchange rate and an exogenous disturbance

(A.5) $\quad y_t = \theta q_t + u_{yt},$

with $\theta > 0$. Equation (A.5) can be motivated from first principles, as in Galí and Monacelli (2002), in which $\theta = 1/(1-\alpha)$, for α defined in (A.1), and u_{yt} is the difference between home and foreign productivity shocks. Alternatively, it can be taken as a textbook IS curve in an open economy. (To prevent confusion, we note that (A.5) and the rest of our model is consistent with an open-economy dynamic IS curve relating expected output growth to a real interest rate; see Galí and Monacelli (2002).) Finally, as in the text, interest parity and Taylor rules lead to

(A.6) $\quad (1+\gamma_q)q_t = E_t q_{t+1} + E_t(1-\gamma_\pi)\pi_{t+1} - \gamma_y y_t - u_{mt}$

where, as in the text, $q_t \equiv s_t - p_t^h + p_t^*$ is the real exchange rate, $\pi_t \equiv \pi_t^h - \pi_t^*$ is the inflation differential, and u_{mt} is the exogenous monetary policy shock.

Identities (A.1), (A.2) and $\Delta q_t \equiv \Delta s_t - \pi_t^h + \pi_t^*$ imply that $\mathring{\pi}_t$ and π_t are related via

(A.7) $\quad \mathring{\pi}_t + \dfrac{\alpha}{1-\alpha}\Delta q_t \equiv \pi_{dt} - \pi_t^* + \dfrac{\alpha}{1-\alpha}\Delta q_t = \pi_t^h - \pi_t^* \equiv \pi_t.$

Use (A.5) to substitute out for y_t in (A.4) and (A.6). Use (A.7) led one period to substitute out for

π_{t+1} in (A.6). Upon defining

$$\gamma = \gamma_q + \gamma_y\theta,\ \eta \equiv 1 + \frac{(1-\gamma_\pi)\alpha}{1-\alpha} = \frac{(1-\alpha\gamma_\pi)}{1-\alpha}$$

one may write the two resulting stochastic equations in the two variables q_t and $\mathring{\pi}_t$ as

(A.8a) $\beta E_t\mathring{\pi}_{t+1} - \mathring{\pi}_t + \kappa\theta q_t = -\kappa u_{yt} - u_{ct}$,

(A.8b) $(1-\gamma_\pi)E_t\mathring{\pi}_{t+1} + \eta E_t q_{t+1} - (\gamma+\eta)q_t = u_{mt} + \gamma_y u_{yt}$.

Use (A.8a) and (A.8a) led one period to substitute out for q_t and $E_t q_{t+1}$ in (A.8b). The result is a second order stochastic difference equation in $\mathring{\pi}_t$. Given restrictions on parameters, the difference equation has a unique stationary solution, with $\mathring{\pi}_t$ the present value of future shocks. (We do not write the restrictions because they are not particularly enlightening. But we do note that there can be a unique stationary solution even if $\gamma_q=0$.) This solution can be put into (A.8a) to solve for q_t. The other variables in the system, including π_t and y_t, can then be constructed.

In particular, when the shocks follow AR(1) processes, $\mathring{\pi}_t$ and q_t are linear in the current values of these shocks, say

(A.9) $\mathring{\pi}_t = c_{pm}u_{mt} + c_{py}u_{yt} + c_{pc}u_{ct},\ q_t = c_{qm}u_{mt} + c_{qy}u_{yt} + c_{qc}u_{ct}$.

To write out c_{pm}, c_{py}, c_{pc}, c_{qm}, c_{qy}, and c_{qc}, define the following notation. Write the AR(1) processes as

(A.10) $u_{mt} = \phi_m u_{mt\text{-}1} + \epsilon_{mt},\ u_{yt} = \phi_y u_{yt\text{-}1} + \epsilon_{yt},\ u_{yc} = \phi_c u_{ct\text{-}1} + \epsilon_{ct},\ 0 \le \phi_m, \phi_y, \phi_c < 1$.

Define

(A.11) $d_m = (1-\beta\phi_m)[\gamma + \eta(1-\phi_m)] + \kappa\theta(\gamma_\pi-1)\phi_m,$

$$d_y = (1-\beta\phi_y)[\gamma + \eta(1-\phi_y)] + \kappa\theta(\gamma_\pi-1)\phi_y,$$

$$d_c = (1-\beta\phi_c)[\gamma + \eta(1-\phi_c)] + \kappa\theta(\gamma_\pi-1)\phi_c.$$

Then

(A.12) $c_{pm} = -\kappa\theta/d_m,\ c_{py} = [\kappa\eta(1-\phi_y)+\kappa\gamma_q]/d_y,\ c_{pc} = [\eta(1-\phi_c)+\gamma]/d_c$

$$c_{qm} = -(1-\beta\phi_m)/d_m,\ c_{qy} = -[(1-\beta\phi_y)\gamma_y+\kappa(\gamma_\pi-1)\phi_y]/d_y,\ c_{qc} = -(\gamma_\pi-1)\phi_c/d_c.$$

Two closing notes: First, the discussion of signs of the impulse response functions in section 2 presumes $\alpha\gamma_\pi < 1$, a condition consistent with our small country assumption. Second, the model can be generalized easily to the case in which the foreign country consumes a nontrivial amount of the home country good. Assume a Cobb-Douglas utility function for consumption, in which the foreign residents put a weight of ε on foreign goods (i.e., the goods produced in the foreign country). Equations (A8.a) and (A8.b), which describe the dynamics of $\mathring{\pi}_t$ and q_t, still hold, except η is now defined as: $\eta = 1 + [(1-\gamma_\pi)(1+\alpha-\varepsilon)/(\varepsilon-\alpha)]$. Equation (A.7), relating relative PPI inflation $\mathring{\pi}_t$ to relative CPI inflation π_t becomes: $\mathring{\pi}_t + [(1+\alpha-\varepsilon)/(\varepsilon-\alpha)] = \pi_t$.

FOOTNOTES

1. Of course, the details of the solution will be different when actual rather than expected inflation appears in the monetary rule, and, since conditions for stability and uniqueness are different in the two cases, it is possible that qualitative characteristics of the solution will be radically different. The statement in the text merely means that for a range of parameters, nothing central turns on the use of expected rather than actual inflation.

2. The Appendix relaxes these conditions, in particular allowing $\gamma_q = 0$.

3. Here and throughout, we abstract from operational difficulties in identifying the long run level of the real exchange rate, just as we abstract from the raft of other practical problems involved with implementing a monetary policy rule (e.g., data availability).

4. Thus our model shares the property of many New Keynesian models of generating little endogenous persistence. For example, to generate plausible persistence in real exchange rates, Benigno (1999) requires that a lag of the interest appear in the monetary rule with a large coefficient. While we have not checked the details, it appears that such a lag would also generate persistence in our model.

5. To prevent confusion, we note that the purpose of this paragraph is to emphasize that our stylized model is consistent with certain important properties of the data. We do not claim that the model is adequate to explain these data in the sense of the calibration literature: as just noted, the model is too simple to be put to that task.

6. As noted in an earlier footnote, the Appendix allows $\gamma_q = 0$. In this case, the solution for the exchange rate is still as stated in (2.9), with $b = 1$.

7. That the correlation would fall was not certain, however, because in this and all other specifications, we re-estimated the model from scratch. So VAR coefficients are different. Even the output series differs a bit from that of the baseline, because the detrending regression used to compute the output gap ran from 1982:10 to 1998:12 rather than 1979:10 to 1998:12.

8. To prevent confusion, we observe that in this and all other specifications, $\Delta\hat{s}_t$ is stationary and $\hat{s}_t$ has a unit root. The sense in which this specification is unusual is that $\hat{s}_t$ is not well approximated by a particular unit root process, namely, a random walk.

9. Recall the discussion above that with Hodrick-Prescott detrending (specification *f*), the standard deviation of model-based series is considerably lower. For example, in specification *f*, the standard deviation of $\hat{q}_t$ is 13.8.

10. The intuition of the random walk result is: When the discount factor is near to one, the present discounted value puts relatively large weight on the values of expected fundamentals far into the future. Only the persistent (random walk) component of the fundamentals affects their expectations in the distant future. Note that this result does *not* say that the monetary authority will make q_t follow a near random walk by pushing γ_q to zero, even though that will also push the discount factor to 1. For changing γ_q will also change the stochastic properties of y_t and π_t, possibly making them less persistent. See Engel and West (2002) for further discussion.

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Table 1

Moments of Model-Based and Actual Data

A. Autocorrelations

	Model-based series			Data				
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Lag	$\hat{q}_t$	$\Delta\hat{q}_t$	$\Delta\hat{s}_t$	q_t	Δq_t	Δs_t	π_t	y_t
1	0.97	-0.02	-0.01	0.98	0.05	0.06	0.36	0.94
	(0.95,1.00)	(-0.05,0.06)	(-0.04,0.08)					
2	0.94	-0.00	-0.00	0.96	0.06	0.04	0.12	0.92
	(0.89,0.99)	(-0.05,0.08)	(-0.05,0.09)					
3	0.92	-0.01	-0.01	0.94	0.04	0.04	0.21	0.90
	(0.85,0.99)	(-0.06,0.09)	(-0.06,0.10)					

B. Cross-correlations

	Model-based series (actual π and y)						Data				
	(1)	(2)	(3)	(4)	(5)		(6)	(7)	(8)	(9)	(10)
	$\hat{q}_t$	$\Delta\hat{q}_t$	$\Delta\hat{s}_t$	π_t	y_t		q_t	Δq_t	Δs_t	π_t	y_t
(1)$\hat{q}_t$	1.00					q_t	1.00				
(2)$\Delta\hat{q}_t$	0.12	1.00				Δq_t	0.07	1.00			
	(0.05,0.20)										
(3)$\Delta\hat{s}_t$	0.10	1.00	1.00			Δs_t	0.07	1.00	1.00		
	(0.01,0.17)	(0.99,1.00)									
(4)π_t	-0.25	-0.00	0.06		1.00	π_t	0.04	-0.08	0.02	1.00	
	(-0.61,-0.02)	(-0.25,0.25)	(-0.16,0.33)								
(5)y_t	-0.94	-0.19	-0.18	0.15	1.00	y_t	-0.37	-0.03	-0.01	0.15	1.00
	(-1.00,-0.14)	(-0.28,-0.05)	(-0.27,-0.03)	(-0.11,0.55)							

C. Correlations Between Actual and Model-Based Exchange Rate Series

$\text{corr}(\hat{q}_t,q_t)$	$\text{corr}(\Delta\hat{q}_t,\Delta q_t)$	$\text{corr}(\Delta\hat{s}_t,\Delta s_t)$
0.44	0.16	0.16
(-0.25,0.87)	(0.04,0.28)	(0.04,.028)

Notes:

1. Variable definitions: q = real exchange rate (DM/$), s = nominal exchange rate, π = inflation differential (CPI, U.S. - Germany), y = output gap differential (industrial production, U.S. - Germany, constructed by quadratic detrending). All data are monthly. The sample period is 1979:10 - 1998:12. West German data are used 1979-1990, with price and output levels adjusted post-1990 to smooth the break in the series caused by reunification. See text for details.

2. A "^" over a variable indicates that it was constructed according to the model described in the paper. See equation (3.2). Parameters of the monetary policy rule: $\gamma_q = 0.1$, $\gamma_\pi = 1.75$, $\gamma_y = 0.25$. In constructing $\hat{q}_t$, a fourth order VAR in (π,y,i) was used to construct the present value defined in the text, where i = U.S.-German interest differential.

3. 95 percent bootstrap confidence intervals are in parentheses.

Table 2

Results of Alternative Specifications

A. Description of How Alternative Specifications Vary from Baseline

Mnemonic and Description	
a	$\gamma_\pi = 2.0$, $\gamma_y = 0.5$
b	West German data
c	sample period = 1982:10 - 1998:12
d	fourth order VAR in (π, y) used to forecast (π, y)
e	fourth order VAR in $(\pi, y, i$, commodity price inflation) used to forecast (π, y)
f	Hodrick-Prescott detrending
g	sixth order VAR in (π, y, i) used to forecast (π, y)
h	$\gamma_q = 100$

B. Summary of Results Under Alternative Specifications

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	-------ρ_1------------			--------------cross-correlation---------------			
Mnemonic	$\hat{q}_t$	$\Delta\hat{q}_t$	$\Delta\hat{s}_t$	$(\Delta\hat{q}_t, \Delta\hat{s}_t)$	$(\hat{q}_t, q_t)$	$(\Delta\hat{q}_t, \Delta q_t)$	$(\Delta\hat{s}_t, \Delta s_t)$
Baseline	0.97	-0.02	-0.01	1.00	0.44	0.16	0.16
a	0.97	-0.03	-0.02	1.00	0.43	0.15	0.15
b	0.97	-0.03	-0.02	1.00	0.44	0.14	0.15
c	0.97	-0.03	-0.03	1.00	0.20	0.08	0.09
d	0.97	-0.01	-0.01	1.00	0.38	0.09	0.06
e	0.97	-0.02	-0.01	1.00	0.41	0.14	0.15
f	0.97	-0.02	-0.01	1.00	0.42	0.14	0.14
g	0.98	-0.01	-0.01	1.00	0.40	0.16	0.15
h	0.97	-0.13	0.36	0.02	0.46	0.17	0.04

Notes:

1. The parameters, data, sample period and VAR variables for the baseline specification are reported in the notes to Table 1. The alternative specifications differ from the baseline only in the indicated fashion.

2. The panel B figures for the baseline specification are repeated from Table 1.

3. In panel B, ρ_1 is the first order autocorrelation coefficient.

Table 3

Variability and Autocorrelations of Fundamentals and of $\hat{q}_t$

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	-------------		Data		--------------		--Spec. h--		--Baseline–	
	y_t	Δy_t	π^e_{at}	$\Delta\pi^e_{at}$	$E_t\pi_{t+1}$	$\Delta E_t\pi_{t+1}$	$\hat{q}_t$	$\Delta\hat{q}_t$	$\hat{q}_t$	$\Delta\hat{q}_t$
(1)Std. dev.	4.84	1.64	1.08	0.31	0.15	0.38	0.03	0.01	20.53	4.90
(2)ρ_1	0.94	-0.29	0.96	-0.05	0.63	-0.38	0.97	-0.13	0.97	-0.02
(3)ρ_2	0.91	-0.01	0.92	-0.10	0.53	0.14	0.94	-0.01	0.94	-0.00
(4)ρ_3	0.90	0.07	0.89	-0.06	0.34	-0.29	0.91	0.01	0.92	-0.01

Notes:

1. In columns (3) and (4), π^e_{at} is expected annual inflation, $E_t(p_{t+12}\text{-}p_t)$, with the expectation computed from the VAR in (π,y,i). In columns (5) and (6) expected monthly inflation is also computed from the VAR.

2. In rows (2)-(4), ρ_i is the i'th order autocorrelation coefficient.

3. In columns (7) and (8), "Spec. h" refers to specification *h*, as defined in Table 2. The figures in these columns for ρ_1 are repeated from that Table. In columns (9) and (10), the figures for ρ_1, ρ_2 and ρ_3 are repeated from Table 1.

Figure 1: Real Exchange Rate, Annualized Output Gap, and Annualized Inflation

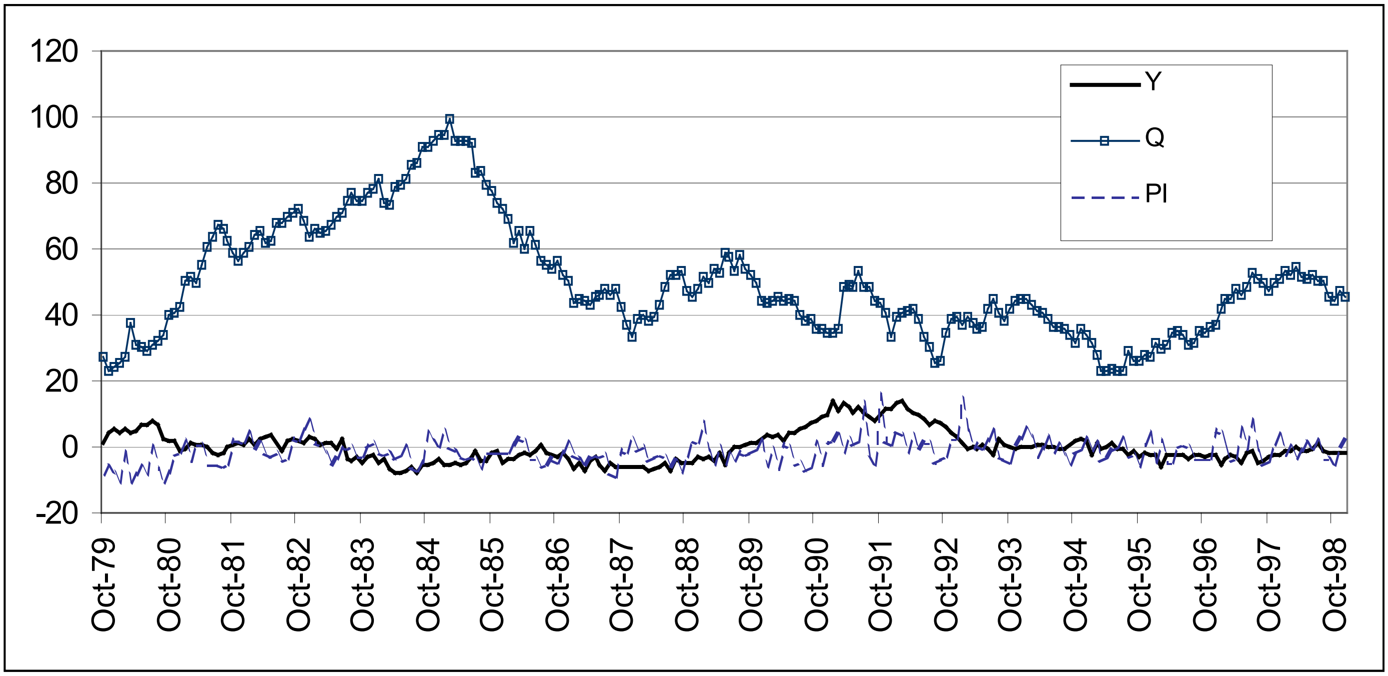

Figure 2: Actual and Model-Based Real Exchange Rates