Preliminary draft

Interpreting Euro Area Inflation at High and Low Frequency

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Abstract

Several authors have recently interpreted the ECB's two-pillar framework as separate approaches to forecast and analyse inflation at different time horizons or frequency bands. The ECB has publicly supported this understanding of the framework. This paper presents further evidence on the behaviour of euro area inflation using cross-spectral analysis and band spectrum regressions which allow for a natural definition of the short and long run in terms of specific frequency bands. The main finding is that movements in inflation are well explained by low-frequency movements of money and real income growth and high-frequency movements of the output gap.

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1. Introduction

On October 13, 1998, the European Central Bank (ECB) announced that its monetary policy strategy would combine a *"prominent role for money with a reference value for the growth of a monetary aggregate"*, later defined to be 4.5% percent growth of M3, and *"a broadly-based assessment of the outlook for future price developments"*.[1] Interpreted by many observers as combining monetary and inflation targeting, the framework quickly became controversial.[2] In particular, it was not clear why the ECB deemed it necessary or even helpful to use "two pillars" – one incorporating "monetary analysis" and another "economic analysis" – in assessing inflation developments and in setting interest rates. This did not necessarily indicate hostility to the reliance on money growth as an information variable for monetary policy purposes, but rather reflected the view that the determinants of inflation, whatever they are, should presumably be included in a single, composite analysis of price developments, as it is the practice in central banks operating with an inflation-targeting strategy.

Recently several authors have sought to formalize the ECB's policy strategy and to rationalize the two pillars by incorporating money growth in empirical Phillips-curve models for inflation in the euro area. Gerlach (2003, 2004) interprets the two pillars as separate approaches to forecast inflation at different time horizons or frequency bands. Under this interpretation, the monetary pillar is seen as a way to predict inflation at long time horizons and to account for gradual changes in the steady-state rate of inflation rate over time. Empirically, the monetary pillar is captured by a geometrically declining, one-sided moving average of M3 growth computed using the simple exponential filter employed by Cogley (2002) to study core inflation. Importantly, Gerlach (2004) finds that filtered money growth contains information useful for forecasting prices that is not already embedded in a similarly filtered measure of inflation. Thus, including money growth in the inflation analysis adds to policy makers' information set.

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[1] See the ECB's press releases of October 13, 1998 and December 1, 1998, which are available at www.ecb.int.

[2] See, for instance, the annual CEPR reports on *Monitoring the ECB,* Svensson (1999 and 2002) or Galí (2003).

Furthermore, the non-monetary pillar, the economic analysis, is understood as the ECB's method to predict short-run variations in inflation around the steady-state level. In the analysis the output gap is identified as the main factor explaining these temporary swings in inflation, but it is recognised that other factors – including oil prices, exchange rates, unit labour costs and tax changes – also play a critical role in the short run.

Neumann (2003) and Neumann and Greiber (2004) present a closely related model, but sharpen the analysis in several ways. In particular, they explicitly incorporate the role of real income growth in determining the trend, or "core", rate of money growth. This is important since it allows for changes in the growth rate of potential to impact on inflation. Furthermore, they use a number of filters to calculate the growth rates of potential and core money growth and investigate what frequency band of money growth has the closest correlation with inflation. The authors find that money growth and output gaps both are significant in empirical inflation equations for the euro area, but that the exact choice of filter is of less importance (although the exponential filter used by Gerlach seems to perform less well than the alternatives considered). One interesting finding is that it is fluctuations of money growth of periodicities greater than 8 years that appears most important in accounting for movements in inflation.

The importance of low-frequency variation in money growth for inflation in the euro area is also studied by Bruggeman et al. (2005), who employ frequency-domain techniques and consider a number of different filters. They also find that longer-term movements of money growth are strongly correlated with inflation, and that the output gap seems to be more important for short-term inflation dynamics. Jaeger (2003) also uses spectral analysis to study the comovements of money and inflation in Europe and notes that these are limited at high frequencies.[3]

One way to think of the papers by Gerlach (2003, 2004), Neumann (2003) and Neumann and Greiber (2004) is that they essentially augment a standard reduced-form, empirical Phillips

[3] Haug and Dewald (2004) also use frequency-domain techniques to study the relationship between money growth and inflation in a sample of eleven countries, using data spanning 1880-2001. Thoma (1994) studies the effect of money growth on inflation and interest rates across frequency bands for the United States.

curve with a measure of the low-frequency component of money growth which they obtain by first filtering money growth.[4] Sustained changes in money growth will therefore shift the Phillips curve vertically, generating changes in the average rate of inflation. By contrast, changes in the output gap, which by construction are temporary, will generate variations in inflation around that average. Interestingly, the ECB in its recent review of the monetary policy strategy attaches a very similar role to money growth in the inflation process. For instance, in an article in the June 2003 Monthly Bulletin on the outcome of its evaluation of the strategy, the ECB (2003, p. 87) writes:

> *"An important argument in favour of adopting the two-pillar approach relates to the difference in the time perspective for analysing price developments. The inflation process can be broadly decomposed into two components, one associated with the interplay between demand and supply factors at high frequency, and the other connected to more drawn-out and persistent trends. The latter is empirically closely associated with the medium-term trend growth of money."*

Furthermore, in commenting on recent studies on the link between money and inflation in the same article the ECB writes (p. 90):

> *"On the basis of statistical methodologies suited to breaking down a time series into the relative contributions of components acting at different time horizons, it has been found that long-term variations in inflation are closely associated with long-term movements in money. Furthermore, it has been found that euro area inflation can be described by a Phillips-curve relationship – i.e. a relationship explaining inflation in terms of indices of economic slack – augmented by a term capturing low-frequency movements in money. This relationship has been interpreted as being indicative in that, whereas fluctuations in inflation in the euro area are driven by factors associated with the state of activity in relation to its long-term potential, the long-term average of inflation is highly correlated with money growth."*

The fact that the ECB has adopted the interpretation that the two pillars refer to the determinants of inflation at different time horizons or frequency bands suggests that further

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4 Gerlach (2004) estimate the long-run trend of money growth jointly with the parameters in the inflation equation.

research on the inflation process in the euro area at different frequencies is well warranted.[5] In this paper we explore the hypothesis that the two pillars, the monetary and economic analysis, contain information useful for understanding inflation at different time horizons using frequency-domain methods. We first employ cross-spectral analysis to understand the co-movements between money growth, real output growth, the output gap and inflation. Next, we go on to use frequency-domain techniques to obtain estimates of potential output and the output gap, and to deseasonalise inflation. Finally, we apply the band spectrum regression approach pioneered by Engle (1974) and later extended by Phillips (1991) for non-stationary time series to estimate reduced-form inflation equations. This approach allows the filtering and estimation to be performed jointly, in contrast to the papers cited above.

The paper is organized as follows. In Section 2 we review the empirical model before we discuss the data in Section 3. Section 4 provides a review of cross-spectral analysis. This method is attractive since it allows us to investigate, by frequency and in a non-parametric manner, the correlations (coherence), lead-lag relationships (phase) and the strength (gain) of the linkages between the variables of interest. We find that money growth and the output gap both precede inflation, but do so in different frequency bands. In Section 5 we present Phillips' (1991) band spectral estimator for cointegrated time series. Section 6 discusses the results from band spectrum regressions for different frequency bands. We show that there is a tight link between money and inflation at low frequencies, and that there is a similarly close relationship between inflation and the output gap at high frequencies. These results are thus fully compatible with the interpretation of the two-pillar framework as applying to different frequency bands.

Section 7 contains our conclusions. Overall the empirical findings are strikingly compatible with the notion that money is useful for understanding the low-frequency variation, and that the output gap contains information about the high-frequency variation, of inflation in the euro area. However, more work remains to be done. First, while money growth appears to

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[5] Interestingly, Jordan, Peytrignet and Rich (2001) describe the new monetary concept introduced by the Swiss National Bank in 2000 as relying on money as a useful indicator for long-run price developments, whereas the output gap is considered as one among other indicators of short-run inflation. The Bundesbank (2005) argues that low-frequency fluctuations in money growth impact on the long-term evolution of inflation, in contrast to high-frequency swings which are much less informative about price developments.

lead movements in inflation, we have not tested the hypothesis that low-frequency movements in money growth contain information useful for forecasting future inflation in addition to that embedded in past inflation. Second, it would be desirable to allow for endogenous shifts in velocity in the econometric work. Since velocity is likely to respond to changes in inflation expectations, it is possible that these effects can be controlled for by introducing a long interest rate in the analysis. Third, while the analysis suggests that money can be used as an information variable for policy purposes, we do not address the question whether this is best done using a two-pillar framework or by integrating the pillars in a single analysis of inflation. Fourth, it would be desirable to extend the analysis by incorporating variables that may capture cost-push shocks to inflation. Fifth, it would be of interest to explore whether the econometric relationships studied in this paper are stable over time.

2. An empirical model for inflation

As noted in the introduction, the ECB has motivated its adoption of the two-pillar strategy by arguing that the determinants of inflation vary by frequency. Under this view, the monetary analysis of the first pillar is intended to help understand and analyse low-frequency movements of inflation, while the economic analysis in the second pillar seeks to understand short-run swings in prices. To formalize this view, we first decompose "headline" inflation, π_t, into low- (*LF*) and high-frequency (*HF*) components:

$$\pi_t = \pi_t^{LF} + \pi_t^{HF} . \tag{1}$$

Following Gerlach (2003), we hypothesise that the high-frequency movements of inflation are related to movements in the output gap, g_t:

$$\pi_t^{HF} = \alpha_g g_t + \varepsilon_t^{HF} . \tag{2}$$

This specification is very simple and it is clear that to fully explain the data a more elaborate model that controls for cost-push shocks arising from unit labour costs, exchange rate changes, value-added taxes etc. is necessary. Unfortunately, the lack of long time series of data on the relevant variables makes this approach impossible. It should also be noted that, by construction, the output gap has no low-frequency variation, which implies that it can at most explain temporary changes in the rate of inflation.

Next, we assume that the low-frequency variation of inflation can be understood in terms of the quantity theory of money, which after taking rates of change and rearranging we can write as:

(3) $$\pi_t^{LF} = \alpha_\mu \mu_t^{LF} + \alpha_\gamma \gamma_t^{LF} + \varepsilon_t^{LF},$$

where μ_t and γ_t denote the growth rate of money and real output, and where ε_t^{LF} captures changes in the growth rate of velocity.[6]

Equation (3) warrants three comments. First, if the average growth rate of velocity is constant but non-zero, which does not appear to be the case in the euro-area data we consider below, a constant will appear in equation (3). For notational simplicity, however, we suppress this. Needless to say, it would be desirable to allow for systematic changes in velocity, for instance by incorporating long-term interest rates in a more developed version of the model. Second, at low frequencies, the growth rate of real output is identical to the growth rate of potential. There are several ways to deal with this in the empirical work that follows. One is to use the actual growth rate of real output in the analysis; another is to first construct a measure of the trend growth rate and use this in the regressions. Since the results will be somewhat different, we use both approaches. Third, under the quantity theory, we expect that $\alpha_\mu = -\alpha_\gamma = 1$.

The full model is given by (disregarding a constant):

(4) $$\pi_t = \left\{\alpha_\mu \mu_t^{LF} + \alpha_\gamma \gamma_t^{LF}\right\} + \alpha_g g_t + \varepsilon_t,$$

where $\varepsilon_t = \varepsilon_t^{LF} + \varepsilon_t^{HF}$. According to this model, the average rate of inflation during some period will be given by the term in curly brackets, { }, that is, by the low-frequency part of money growth relative to real output growth, which we think of as the first pillar. Variation in inflation around that average will be determined by movements in the output gap, which is our short-hand for the second pillar. Under this interpretation of the ECB's monetary policy

6 Lucas (1980) presents frequency-domain evidence for US data in support of this proposition.

strategy, in analysing and forecasting inflation it is indeed appropriate to consider low-frequency (as opposed to "headline") movements in money growth.

The inflation equation proposed above is entirely an empirical model and it is important to understand what it says about the monetary transmission mechanism. Let us first consider the short-run correlation between money growth and inflation. Our view is that movements in money growth induce, or reflect, movements in aggregate demand. In turn, these lead to swings in the output gap and therefore to inflation. However, since money growth is partially due to temporary shifts in money demand and changes in the financial system that may not impact on inflation, perhaps because they are not of sufficient duration to do so, it is an empirical question whether the short-run effects of money are best measured by data on money growth or measures of the output gap, as emphasised by Nelson (2003). An additional reason for why money growth at high frequency need not be significant in the inflation equation is that there may be other factors impacting on aggregate demand. Thus, a finding that the output gap, but not money growth, impacts on high-frequency swings in inflation does not imply that money growth do not trigger short-run swings in inflation.

However, it is much more likely that the effects of money growth on inflation will be clearer at low frequency. First, economic theory suggests that monetary disturbances have at most temporary effects on real variables such as the output gap. It is therefore unlikely that the output gap will capture the long-run effects of a shift in the money growth rate. Second, the output gap is by construction stationary while inflation may display a unit root, perhaps arising from shifts in the inflation regime. This difference in the time-series properties suggests that one would not expect the two variables to be closely related in the long run. Rather, shifts in the money growth rate, which should be tied to changes in the inflation regime, are likely to be informative about changes in the average level of inflation over time.

3. The data

As preliminary step to the formal econometric analysis below we consider the raw data.[7] Since the rate of inflation using the original CPI data displayed quite complicated dynamics and a seasonal factor, perhaps because they are synthetic for a large part of the sample period, we first deseasonalised the series by removing a frequency band around the seasonal peaks.[8] This obviates the need to model the seasonal dynamics in the regressions below. Figure 1 presents a plot of the quarterly rate of inflation using the seasonally adjusted data, the quarterly rate of money growth as measured by M3, and the quarterly rate of real income growth, all for the period 1970Q2 to 2004Q4. In all cases we have demeaned the data.

The figure shows that (deseasonalised) inflation accelerated in the early 1970s and remained high and volatile before declining in the early 1980s. Since the mid-1980s, inflation appears to have fluctuated around a constant level. The fall in inflation was associated with a gradual decline in money growth over the sample as central banks took measures to disinflate after the sharp increase in inflation during the 1970s. Finally, real income growth was quite volatile over the sample. However, there appears to be some evidence that the rate of growth of has output declined, as evidenced by the fact that output growth was below average in most quarters in the 1990s.

Next we turn to the output gap (defined as output relative to a smooth trend). While most researchers use the HP filter to construct a measure of the trend output, we do so by extracting all variation of frequencies of more than 48 quarters from the (demeaned) quarterly growth rate of real output. Converting the resulting series to the time domain and accumulating (incorporating the information in the average growth rate), we obtain a measure of the growth rate of potential.[9] The resulting output gap, which is plotted in Figure 1, is very similar to the

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[7] The output and price level are from the ECB's area-wide model (see Fagan et al. 2001) and have been updated with data from the ECB's Monthly Bulletin. The monetary data were provided by the ECB.

[8] See Sims (1974). The width of the band was chosen to be $\pi/24$, which leads to a loss of 9 degrees of freedom of the total of 139 observations.

[9] Using a HP-filter with the conventional smoothing parameter of 1600, one effectively filters out all fluctuations with a frequency of less than 40 quarters (Kaiser and Maravall, 2001). We chose 48 quarters because this value maximizes the correlation of the spectral-filtered output gap with the HP-filtered output

HP-filtered output gap – the correlation coefficient between the two gaps is 0.95. The main movements seem associated with the large recession around 1974 following the first oil shock, and again in 1992-3.

Some preliminary evidence to assess the model laid out in the previous section is presented in Figures 2 to 4. The two panels in Figure 2 show the low and the high-frequency components of inflation and money growth. The low-frequency components of both series are shown in the left panel. As low-frequency movements we define fluctuations with a periodicity of more than 8 years, while fluctuations with a periodicity of between 8 and 2 years are defined as high-frequency fluctuations.[10] While the low-frequency component of money growth captures the inflation trend well, there is apparently no relation between the high-frequency components of the two series. A similar conclusion emerges for the relation between inflation and output growth shown in Figure 3. By contrast, the output gap is not able to account for the trend-wise decline in inflation whereas the scatter plot suggests a positive relation between the high-frequency components of the output gap and inflation in Figure 4.

The time series characteristics of the data are important for the empirical analysis that follows, and we therefore perform unit root tests for all variables used in estimation, that is, inflation, money growth, output growth, money growth minus our spectral estimate of trend output growth, and the output gap. Since different tests often lead to contradictory results, to get a fuller picture of the unit root behaviour of the variables we perform Augmented Dickey-Fuller (ADF) tests, Elliot, Stock, and Rotenberg (ERS) tests, Phillips and Perron (PP) tests, and the Kwiatkowski, Phillips, Schmidt and Shin (KPSS) test, which in contrast to the other tests considers stationarity as the null hypothesis.[11] The optimal lag length is determined by the Akaike criterion (AIC), under the assumption that it is at most 8 lags.

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gap. The output gap coefficients in the regression remain unchanged when the output gap is defined as containing only fluctuations of less than 32 quarters, which is the frequency often used in business cycle analysis, see Baxter and King (1999).

[10] We exclude fluctuations of below two years from the figures because these turned out to be highly irregular. Of course, we do include these high-frequency fluctuations in the regressions presented below.

[11] The ADF and PP tests are discussed in Hamilton (1994), the ESR test in Elliot, Stock, and Rotenberg (1996), and the KPSS test in Kwiatkowski et al. (1992).

The results, which are shown in Table 1, suggest that inflation and money growth are non-stationary, but that output growth and the output gap are stationary.[12] The finding that output growth is stationary is perhaps somewhat surprising in light of Figure 1. One way to reconcile these findings is to hypothesis that there are both permanent and transitory shocks to the growth rate of output. If the latter have a much larger variance than the former, the unit root tests may falsely indicate that the time series is stationary. This is important because the order of integration of actual output growth and trend output growth, which enter prominently in the analysis below, must be the same. In what follows, we interpret Figure 1 as suggesting that trend output growth, and therefore actual output growth, is non-stationary, despite the results from the unit root tests. Since this assumption can be challenged, we also present results that do not hinge on it.

4. Cross-spectral analysis

To investigate the ECB's view of the inflation process and to better understand the time series behaviour of the variables considered, we explore the relationships between inflation, money growth, real income growth and the output gap in different frequency bands. As noted above, we first do so using cross-spectral analysis. While distributed-lag models and cross-spectral methods offer equivalent ways of investigating dynamic relationships between economic variables, in practice these techniques can usefully be combined, as emphasized by Engle (1976). One important difference between them is that while the distributed-lag models require us to specify or parameterize the model, cross-spectral methods are non-parametric. Of course, parametric methods are more efficient than non-parametric methods if the parameterization is correct. However, if incorrect, the analysis leads to invalid results. It is therefore useful to investigate the relationships using both approaches. As a preliminary, we review briefly some relevant results from the literature.[13]

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[12] The one unexpected result is that the Phillips-Perron test suggests that money growth is stationary.

[13] For a short exposition of spectral analysis see, e.g., Granger and Newbold (1986), Harvey (1993) or Hamilton (1994). A more detailed treatment is provided in Priestley (1981).

The properties of a stationary stochastic process are typically described in the time domain by its autocovariance function $\psi(s) = E(x_t - \mu)(x_{t-s} - \mu)$. In the frequency domain, one thinks of a series as being characterised by a weighted sum of periodic sine and cosine functions. The spectral density function $f(\omega)$ is defined as the Fourier transform of the autocovariance function $\psi(s)$:

(5) $$f(\omega) = \frac{1}{2\pi}\left[\psi(0) + 2\sum_{s=1}^{\infty}\psi(s)\cos(\omega s)\right] = \frac{1}{2\pi}\sum_{s=-\infty}^{\infty}\psi(s)e^{i\omega s}$$

where the frequency, ω, may take any value in the range $[-\pi,\pi]$ and $i = \sqrt{-1}$. Since the spectrum is symmetric about zero, all information is contained in the range $[0,\pi]$. The spectrum is a decomposition of the variance of a series by each frequency, while the area under the spectrum is the variance of the series.

The cross spectrum describes the cross-covariances, ψ_{xy}, of a pair of series. The cross-spectral density function between two series x_t and y_t is defined as

(6) $$f_{xy}(\omega) = \frac{1}{2\pi}\sum_{s=-\infty}^{\infty}\psi_{xy}(s)e^{\ i\omega s}$$

In contrast to the spectrum, the cross spectrum cannot be plotted directly. Instead, the analysis is conducted by investigating the coherence, the gain and the phase, which are all derived from the cross spectrum. The coherence can be thought of as the r-squared in a regression, by frequency, of one of the variables on the other. The gain, which is the ratio of the covariance of y and x to the variance of x, can be thought of as the slope parameter in such a regression. The phase captures the lead-lag relationship between the variables. One particularly useful result is that if the gain is non-zero, then the slope of the phase at the origin will equal (minus) the mean lag of the distribution (Engle, 1976, p. 99).

Figure 5 contains plots of the coherence, gain and phase for the cross-spectrum of inflation and money growth.[14] On the x-axis, we have plotted the frequency, ω, measured in fractions of π.[15] The frequencies can be converted to periodicity as follows: letting ω denote frequency, measured by cycles per quarter, the periodicity, p, measured in quarters, is given by $2\pi/\omega$.[16] Thus, a frequency of 0.5π corresponds to a periodicity of 4 quarters (2/0.5 = 4). Seasonal factors therefore generate at peak in the spectrum at 0.5π. Similarly, a frequency of 0.2π implies a periodicity of 2/0.2 or 10 quarters. The coherence shows that the two series are strongly correlated at frequencies between 0 - 0.05π (that is, periodicity between infinity and 40 quarters), and for business-cycle frequencies around 0.15π. The estimated gain also shows peaks at the same frequency bands. Furthermore, the slope of the phase function is negative at the origin, indicating that the mean lag between inflation and money growth is positive. Overall, the cross-spectral analysis suggests that money growth leads inflation, and that the two series are particularly strongly associated at very low and at business-cycle frequencies.

Turning to the cross-spectral analysis for the output gap and inflation in Figure 6, we note that coherence and gain are strong at business cycle frequencies around 0.15π. At frequency zero the gain is zero. Finally, the estimated phase function suggests that the mean lag between the output gap and inflation is positive, indicating that movements in the output gap lead inflation.

5. Methodology

Next we discuss our approach to estimation of the inflation equations for the euro area. The analysis is complicated by the fact that inflation and money growth are non-stationary, while the output gap and possibly output growth are stationary. We therefore estimate the relationship between inflation, money growth and output growth in a band including the zero

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[14] The time series used contain 139 observations, and we padded the series to the length 1024. We used triangular window with a width of 24. For more information, see the RATS manual.

[15] While we elsewhere in the paper let π denote of the rate of inflation, in discussing frequencies we let it denote the irrational number defined by the ratio of the circumference of a circle to its diameter.

frequency, using Phillips' (1991) spectral estimator which is appropriate in the case of $I(1)$ variables.[17] For frequency bands that exclude the zero frequency, the $I(1)$ variables are stationary. In this high-frequency band, we can also investigate the relation between inflation and the stationary variables.

The main reason why the use of spectral regression techniques is particularly attractive in the present context is that, as indicated above, the ECB has stated that the choice of a two-pillar framework arises from the fact that the determinants of euro area inflation vary across frequencies. Thus, at low frequencies money growth is important, while at high frequencies movements in inflation are *"associated with the state of activity in relation to its long-term potential"*, that is, the output gap. Exploring whether this description of the inflation process is accurate plainly requires us to estimate inflation equations for different frequencies. A further reason why estimation of the inflation equation in the frequency domain is appealing is that, in contrast to the Johansen (1995) estimator, Phillips' spectral estimator does not require us to specify the precise model for the short-run dynamics. Furthermore, it is compatible with different types of error processes.[18] For consistency, with estimation conducted in the frequency domain, we also construct trend output growth and the output gap and perform seasonal adjustment of the inflation data in the frequency domain.

Our model for the low frequency is given by equation (3). To estimate the cointegrating relation between inflation, money growth, and output growth, we start with an error-correction model (ECM) in triangular form, see Phillips (1991):

$$\text{(7)} \qquad \begin{aligned} \pi_t^{LF} &= \alpha_\mu \mu_t^{LF} + \alpha_\gamma \gamma_t^{LF} + u_{1t} \\ \Delta\mu_t^{LF} &= u_{2t} \\ \Delta\gamma_t^{LF} &= u_{3t} \end{aligned}$$

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16 Since ω is measured in cycles per period, the smallest cycle distinguishable in quarterly data is one cycle every two periods, which is also called the Nyquist frequency.

17 Testing inflation and money less trend income growth in a Johansen framework indicates the existence of a single cointegrating relationship between the variables.

18 Two applications of the Phillips estimator are Hall and Trevor (1993), who estimate a consumption function on Australian data, and Corbae et al. (1994) who test the permanent income hypothesis.

where π_t is the inflation rate, μ_t is money growth and γ_t is output growth, all of which, as we have argued, have a unit root.[19] Thus, if the quantity theory held at all frequencies, the income elasticity of money demand, $-\alpha_\gamma$, was unity and if the growth rate of velocity was stationary, the expected value of α_μ would be unity. The error terms u_{1t}, u_{2t} and u_{3t} are assumed to be stationary.

We think of equation (7) as defining the long-run steady state of inflation since in the long-run the output gap is zero and has no influence on the inflation rate. Defining $y_t = (\pi_t, \mu_t, \gamma_t)'$ the cointegrating system has the following ECM representation:

$$\Delta y_t = \kappa\alpha' y_{t-1} + v_t,$$

where $\kappa' = (1,0,0)$, $\alpha' = (1, -\alpha_\mu, -\alpha_\gamma)$, and $v_t = \begin{bmatrix} 1 & -\alpha_\mu & -\alpha_\gamma \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} u_t$.

Next, the time series are transferred into the frequency domain by calculating the finite Fourier transforms,

$$w_\Delta(\lambda) = (2\pi T)^{-1/2} \sum_{t=1}^{T} \Delta y_t e^{i\lambda t},$$

$$w_y(\lambda) = (2\pi T)^{-1/2} \sum_{t=1}^{T} y_{t-1} e^{i\lambda t},$$

$$w_*(\lambda) = (2\pi T)^{-1/2} \sum_{t=1}^{T} y_{*t} e^{i\lambda t},$$

for $\lambda \in [-\pi, \pi]$ and $y'_{*t} = (\pi_t, \Delta\mu_t, \Delta\gamma_t)$ were T denotes the sample length. Next we compute the smoothed periodogram estimates,

[19] In what follows, all growth rates are measured over a quarter.

$$\hat{f}_{22}(\omega_j) = \frac{1}{m}\sum_{B_j} w_2(\lambda_s) w_2(\lambda_s)^* ,$$

$$\hat{f}_{2*}(\omega_j) = \frac{1}{m}\sum_{B_j} w_2(\lambda_s) w_*(\lambda_s)^* ,$$

where $w_y(\lambda)' = (w_1(\lambda), w_2(\lambda)')$ is partitioned into the Fourier transforms of the regressand, $w_1(\lambda)$, and the regressors, $w_2(\lambda)'$, and the summation is over

$$\lambda_s \in B_j = \left(\omega_j - \frac{\pi}{2M} < \lambda \le \omega_j + \frac{\pi}{2M}\right),$$

such that if $\lambda \in B_j$ then - $\lambda \in B_j$ also. This ensures that the resulting estimator β is real valued, see Engle (1974). In effect, the spectra are computed by averaging over $m = T/2M$ neighbouring periodogram ordinates, where M is the total number of frequency ordinates divided by 2.

Phillips' estimator uses $\hat{f}_{vv}^{-1}(\omega_j)$ as weighting function, where $\hat{f}_{vv}$ is the smoothed periodogram estimate of the residuals from an OLS regression of equation (7),

$$\hat{f}_{vv}(\omega_j) = \frac{2M}{T}\sum_{B_j}\left[w_\Delta(\lambda_s) - \gamma\hat{\alpha}' w_y(\lambda_s)\right]\left[w_\Delta(\lambda_s) - \gamma\hat{\alpha}' w_y(\lambda_s)\right]^* .$$

As γ is known by construction, non-linear estimation techniques are not required. To estimate the cointegrating relation, Phillips (1991) suggests computing the estimator over a band around frequency zero, which matters most for long-run estimation. This gives the band-spectral estimator at frequency zero:

$$\text{(8)} \qquad \tilde{\beta}(0) = \left[\frac{1}{2M}\sum_{j=-M+1}^{M} e'\hat{f}_{vv}^{-1}(0) e\hat{f}_{22}'(0)\right]^{-1}\left[\frac{1}{2M}\sum_{j=-M+1}^{M} \hat{f}_{2*}'(0)\hat{f}_{vv}^{-1}(0) e\right],$$

with $e' = \kappa = (1,0,0)$ and variance-covariance matrix:

$$\text{(9)} \qquad V(\beta(0)) = \frac{1}{T}\left[\frac{1}{2M}\sum_{j=-M+1}^{M} e'\hat{f}_{vv}^{-1}(0) e\hat{f}_{22}'(0)\right]^{-1}$$

This estimator for β can be used for cointegrated time series as long as the frequency zero is included in the band. Restricting the spectral regression to the high frequencies makes estimation of β more difficult. Though by excluding the zero frequency the variables in y_t now are stationary, the error term and the regressors will be correlated and this will result in simultaneous regression bias. Corbae, Ouliaris and Phillips (1994) therefore propose a frequency domain Generalized Instrumental Variable Estimator (GIVE). The idea is to instrument $w_y(\lambda)$ to get a consistent estimate of β. If the generating mechanism for Δy is $w_{\Delta y}(\lambda) = \delta\, w_z(\lambda) + w_\zeta(\lambda)$ and z_t is independent of ζ_t and the error term u_{1t}, $w_z(\lambda)$ can be used as instrument in a spectral regression. The resulting band-spectral estimator, β^{GIVE} is given by

$$\beta^{GIVE} = \frac{\left(\sum_{\theta_j \in B_2} f_{xz}(\theta_j) f_{uu}^{-1}(\theta_j)\right)\left(\sum_{\theta_j \in B_2} f_{zz}(\theta_j) f_{uu}^{-1}(\theta_j)\right)^{-1}\left(\sum_{\theta_j \in B_2} f_{zy}(\theta_j) f_{uu}^{-1}(\theta_j)\right)}{\left(\sum_{\theta_j \in B_2} f_{xz}(\theta_j) f_{uu}^{-1}(\theta_j)\right)\left(\sum_{\theta_j \in B_2} f_{zz}(\theta_j) f_{uu}^{-1}(\theta_j)\right)^{-1}\left(\sum_{\theta_j \in B_2} f_{zx}(\theta_j) f_{uu}^{-1}(\theta_j)\right)},$$

with variance-covariance matrix

$$V(\beta^{GIVE}) = \frac{2M}{T}\left[\left(\sum_{\theta_j \in B_2} f_{xz}(\theta_j) f_{uu}^{-1}(\theta_j)\right)\left(\sum_{\theta_j \in B_2} f_{zz}(\theta_j) f_{uu}^{-1}(\theta_j)\right)^{-1}\left(\sum_{\theta_j \in B_2} f_{zx}(\theta_j) f_{uu}^{-1}(\theta_j)\right)\right]^{-1}$$

where f_{xz}, f_{zx}, f_{zy}, f_{zz}, and f_{uu} are the estimated spectra of the respective series, analogous to equations (8) and (9).

6. Results

Tables 2 to 6 present the band-spectrum estimates of equation (4) using the Phillips (1991) estimator. Since we are uncertain about whether there are permanent changes in output growth, we estimate a number of variations of the inflation equation. We define the long run as fluctuations with a periodicity of more than, and the short run as fluctuations with less than, 4 years periodicity. To assess whether this arbitrary, but not unreasonable, definition impacts materially on the results, we also show results when the distinction between the long and the short run is drawn at a frequency corresponding to a periodicity of 2 and 8 years. The first column of each table shows a standard OLS estimate of a regression including all frequencies in order to demonstrate that only money growth is significant, with a parameter that is typically significantly below unity, in the OLS regression. By contrast and as is discussed below, estimating the regression for different frequency bands leads to conclusions that are much more compatible with the ECB's view of the inflation process.

We first study the relationship between money growth, output growth and inflation around the zero frequency. This is the empirical counterpart of the long-run quantity theory in equation (3). The results in Table 2 indicate that while the coefficient on money growth is positive, it is significantly different from both zero and unity irrespectively of the definition of the long run. Interestingly, the coefficient on real income growth is insignificant in Table 2. We suspect that this could be due to output growth being stationary. In this case, we would expect to find two cointegrating vectors, one with $(1, -\alpha_{\mu}, 0)$ and the other with $(0, 0, 1)$ as coefficients (see Johansen, 1995, p. 37). In this case, the coefficient on output growth would not be identified.

We therefore treat output growth as only having a high-frequency component and re-estimate the model. The results in Table 3 show that the coefficient on money growth is significant and close to unity at low frequency, and that the coefficient on output growth is significant and negative at high frequency, although smaller than unity in absolute value.[20] Moreover, the coefficient on output growth moves towards -1, as suggested by theory, when the high-frequency band is defined as containing fluctuations of shorter than 8 years. The output gap is highly significant and positive at high frequency, irrespectively of the definition of the high-frequency band. As evidenced by the t-value, the coefficient is statistically most closely tied to inflation when the high-frequency band is relatively broad. This suggests that in the highest frequency bands the relation in not well defined. Note that money growth is significant and <u>negative</u> when the high-frequency band contains fluctuations of periodicities of shorter than 8 years. One reason for this may be that positive output gaps have raised inflation and, through monetary policy reactions, increases in interest rates which may have depressed money growth.

[20] For the frequency bands excluding the zero frequency we use the instrumental variable estimator. We instrument money growth by its first lag. As output growth and the output gap are stationary, we do not use instruments for them.

Next we hypothesize that output growth is subject to permanent shocks, and use a measure of trend output growth in the low-frequency regressions instead of actual output growth. Trend output growth, which is clearly non-stationary, is filtered to reflect only fluctuations with a periodicity of more than 12 years and thus can be expected to provide more precise estimates of the impact of changes in trend output growth on the rate of inflation.[21]

The results in Table 4 show that money and trend output growth enter with highly significant coefficients of approximately equal size and opposite sign at low frequencies. A Wald-test doesn't reject the hypothesis that the variables enter with coefficients of 1 and -1 when the distinctions between the high and low-frequency bands are drawn at periodicities of 2, 4, and 8 years (test statistics of 0.06, 0.33 and 0.55, compared to a critical value of $\chi^2(2) = 5.99$). At the high frequency, only the output gap is significant, whereas the coefficient on money growth is insignificant (except when the short run is defined as periodicities of less than 8 years, when it is significantly negative).

In Table 5 we present estimates under the assumption that it is money growth less trend output growth that matters for inflation for all frequencies. As could be expected from the results of the Wald tests, the results are very similar to those in Table 4. Finally, in Table 6 we use money growth less actual output growth as a regressor. This imposes a coefficient of $\alpha_\gamma = -1$ while the regressor remains non-stationary and thus does not entail the problems with multiple cointegrating vectors. Still, the coefficient on money growth less trend output growth is insignificantly different from unity at low frequencies and is insignificantly different from zero at higher frequencies. Interestingly, the output gap becomes insignificant when the high-frequency band is defined to include fluctuations of less than 2 years, and output gap coefficients are lower than in the previous tables.

Overall, these empirical results support three conclusions. First, low-frequency movements in money growth lead to equal changes in inflation in contrast to high-frequency variations in money growth that have no impact on inflation. Second, it is unclear whether changes in

[21] As trend output is defined to contain only fluctuations of more than 12 years, it is constant over the high-frequency bands investigated in Table 2.

output growth have impacted on inflation. A main stumbling block is that, as evidenced by the unit root tests, the data is not clear about whether there have in fact been permanent changes in the growth rate of output. Third, short-run movements in the output gap have a strong impact on inflation.

7. Conclusions

In this paper we have analysed the behaviour of inflation in the euro area across frequency bands, using data for the period 1970-2004. The main findings are strikingly compatible with the notion that money is useful for understanding the low-frequency variation, and that the output gap contains information about the high-frequency variation, of inflation in the euro area.

Although the results are supportive of the frequency-band interpretation of the ECB's two-pillar framework for inflation analysis and interest rate setting, five caveats should be emphasised. First, while the cross-spectral analysis indicates that movements in money growth precede movements in inflation, we have not directly tested the hypothesis that low-frequency movements in money growth contain information which is useful for predicting future inflation, but which is not yet embedded in past inflation.[22] Galí (2003) emphasises that while the existence of stable money demand function suggests that inflation and money growth are related, that does not imply that money growth causes inflation. Thus, more work is required to properly understand the inflation process.

Second, the model above has implicitly assumed that changes in velocity are exogenous, or at least unrelated to changes in money and real output growth, and that they follow a random walk with no, or constant, drift. Both these assumptions are likely to be wrong and it would be desirable to allow for such shifts in the econometric work. Since velocity is likely to respond to changes in inflation expectations, which in turn are reflected in long interest rates, it is

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[22] That is, whether money growth Granger causes inflation at low frequencies.

possible that these effects can be controlled for by introducing a measure of the opportunity cost of holding M3.[23]

Third, the finding that money helps forecast inflation suggests that money can be used as an information variable for policy purposes. Whether this is best done using a two-pillar framework or by integrating the pillars in a single analysis of inflation is an important question that goes beyond the scope of this paper. That said, this question appears largely semantic.

Fourth, it would be desirable to extend the analysis by incorporating variables that may capture cost-push shocks to inflation, for instance, exchange rates, import prices, energy prices or food prices.

Fifth, the time-series behaviour of money and inflation appears to have changed over the sample. It would therefore be of interest to explore whether the econometric relationships studied in this paper are stable over time. In doing so it should be kept in mind that the analysis above suggests that low-frequency changes in money growth may be particularly informative about the role of money in the inflation process. In fact, such changes may be critical for detecting the impact of changes in money growth on inflation.[24]

[23] Reynard (2005) shows that accounting for changes in velocity is critical for understanding the relationship between money growth and inflation.

[24] Nelson (2003, pp. 1035-1036) discusses how the relationship between money growth and inflation may appear in a small non-monetary macroeconomic model incorporating a Phillips curve.

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Tables and Figures

Table 1. Unit root tests

	ADF	*PP*	*ERS*	*KPSS*	*AIC lag*
Test with constant					
inflation	-1.03	-1.82	-1.09	1.76*	5
money growth	-1.59	-3.16*	-1.57	1.92*	5
output growth	-5.53*	-7.93*	-2.30*	0.72*	1
money less trend output growth	-1.67	-3.96*	-1.75	1.78*	5
output gap	-4.07*	-3.04*	-3.92*	0.05	4
Test with trend and constant					
inflation	-2.75	-4.04*	-1.80	0.16*	5
money growth	-2.62	-6.70*	-2.05	0.20*	5
output growth	-5.77*	-8.25*	-4.31*	0.11	1
money less trend output growth	-2.93	-6.97*	-1.96	0.16*	5
output gap	-4.09*	-3.07	-4.08*	0.05	4
Test of 1st differences					
inflation	-6.90*	-17.81*	-6.41*	0.08	4
money growth	-6.09*	-23.71*	-2.29*	0.08	8
output growth	-6.59*	-24.90*	-6.51*	0.05	7
money less trend output growth	-5.95*	-23.75*	-0.01	0.08	8
output gap	-8.85*	-8.85*	-4.70*	0.07	0

Note: The last column indicates the number of lags included in the test, which were chosen by the AIC criterion. The 5% critical values for the tests including a constant only are -2.89 for the Augmented Dickey-Fuller (ADF) and the Phillips-Perron (PP) test, -1.95 for the Elliot, Stock and Rotenberg (ERS) test and 0.46 for the Kwiatkowski, Phillips, Schmidt and Shin (KPSS) test. The 5% critical values for the test including a constant and a trend are -3.45 for the ADF and the PP test, -2.89 for the ERS and 0.15 for the KPSS test. The tests of the first differences include a constant but no trend. The sample period is 1970Q2 to 2004Q4. An asterisk, "*", indicates the rejection of the null hypothesis.

Table 2. Band spectrum regressions

	All frequencies	*Low (> 2 years)*	*Low (> 4 years)*	*Low (> 8 years)*
money growth	0.57** (0.05)	0.69** (0.11)	0.68** (0.14)	0.70** (0.14)
output growth	0.05 (0.08)	-0.09 (0.20)	-0.27 (0.30)	-0.48 (0.37)
output gap	0.13** (0.03)			

Note: The dependent variable is the inflation rate. All regressions include a constant which is not shown. Standard errors in parentheses; * indicates significance on the 5%, ** significance on the 1% level. The sample period is 1970Q2 to 2004Q4.

Table 3. Band spectrum regressions

	All frequencies	*Low (> 2 years)*	*Low (> 4 years)*	*Low (> 8 years)*	*High (< 2 years)*	*High (< 4 years)*	*High (< 8 years)*
money growth	0.57** (0.05)	0.81** (0.12)	0.88** (0.15)	0.96** (0.19)	-0.28 (0.60)	-0.95 (0.60)	-0.59* (0.29)
output growth	0.05 (0.08)				-0.32** (0.08)	-0.61** (0.07)	-0.76** (0.07)
output gap	0.13** (0.03)				1.28** (0.15)	6.62** (0.11)	5.73** (0.05)

Note: The dependent variable is the inflation rate. All regressions include a constant which is not shown. Standard errors in parentheses; * indicates significance on the 5%, ** significance on the 1% level. The sample period is 1970Q2 to 2004Q4.

Table 4. Band spectrum regressions

	All frequencies	*Low (> 2 years)*	*Low (> 4 years)*	*Low (> 8 years)*	*High (< 2 years)*	*High (< 4 years)*	*High (< 8 years)*
money growth	0.77^{**} (0.07)	1.00^{**} (0.14)	1.16^{**} (0.19)	1.34^{**} (0.24)	-0.22 (0.59)	-0.94 (0.56)	-0.63* (0.28)
trend output growth	-0.51 (0.30)	-1.09* (0.56)	-1.45* (0.70)	-1.88* (0.87)			
output gap	0.10^{**} (0.03)				1.33^{**} (0.13)	6.63^{**} (0.09)	5.77^{**} (0.04)

Note: The dependent variable is the inflation rate. All regressions include a constant which is not shown. Standard errors in parentheses; * indicates significance on the 5%, ** significance on the 1% level. The sample period is 1970Q2 to 2004Q4.

Table 5. Band spectrum regressions

	All frequencies	*Low (> 2 years)*	*Low (> 4 years)*	*Low (> 8 years)*	*High (< 2 years)*	*High (< 4 years)*	*High (< 8 years)*
money less trend output growth	0.80^{**} (0.06)	0.99^{**} (0.13)	1.11^{**} (0.16)	1.23^{**} (0.20)	-0.22 (0.59)	-0.93 (0.61)	-0.65* (0.30)
output gap	0.11^{**} (0.03)				1.11^{**} (0.13)	6.38^{**} (0.10)	5.45^{**} (0.05)

Note: The dependent variable is the inflation rate. All regressions include a constant which is not shown. Standard errors in parentheses; * indicates significance on the 5%, ** significance on the 1% level. The sample period is 1970Q2 to 2004Q4.

Table 6. Band spectrum regressions

	All frequencies	*Low (> 2 years)*	*Low (> 4 years)*	*Low (> 8 years)*	*High (< 2 years)*	*High (< 4 years)*	*High (< 8 years)*
money less output growth	0.50** (0.06)	0.76** (0.13)	0.98** (0.19)	1.16** (0.26)	-0.01 (0.14)	-0.32 (0.34)	-0.55 (1.09)
output gap	0.16** (0.04)				0.04 (0.17)	0.68** (0.17)	2.41** (0.25)

Note: The dependent variable is the inflation rate. All regressions include a constant which is not shown. Standard errors in parentheses; * indicates significance on the 5%, ** significance on the 1% level. The sample period is 1970Q2 to 2004Q4.

Figure 1. The data

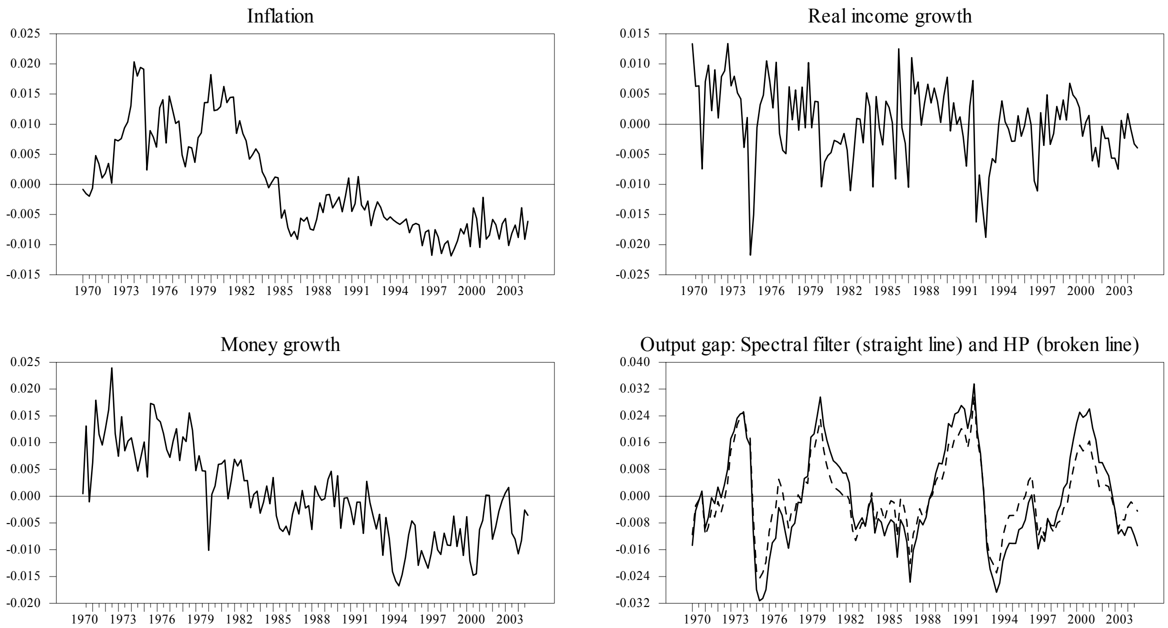

Figure 2. Inflation and money growth: low (left panel) and high frequency (right panel)

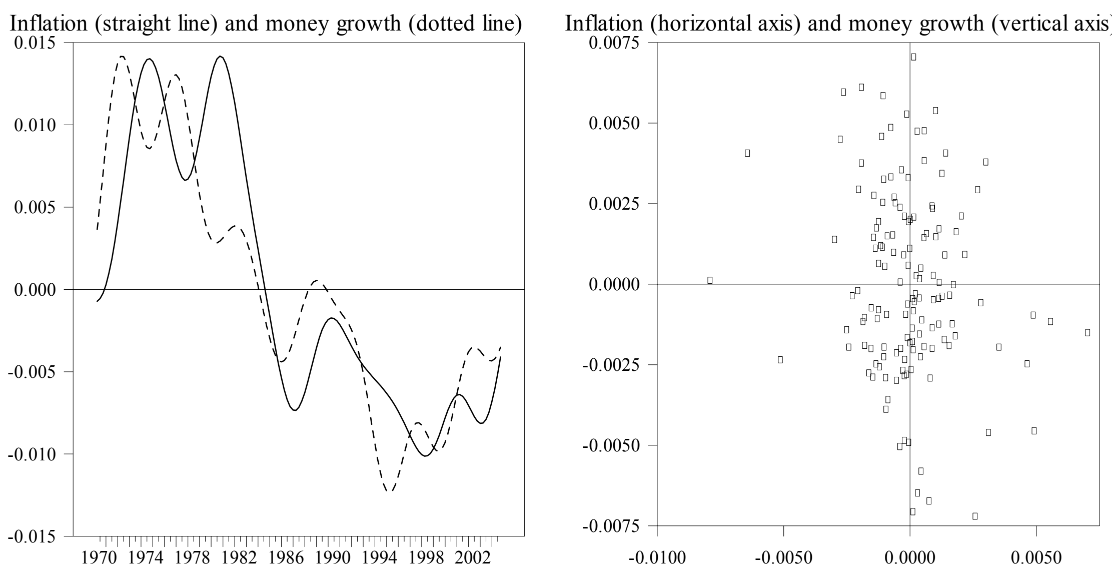

Figure 3. Inflation and output growth: low (left panel) and high frequency (right panel)

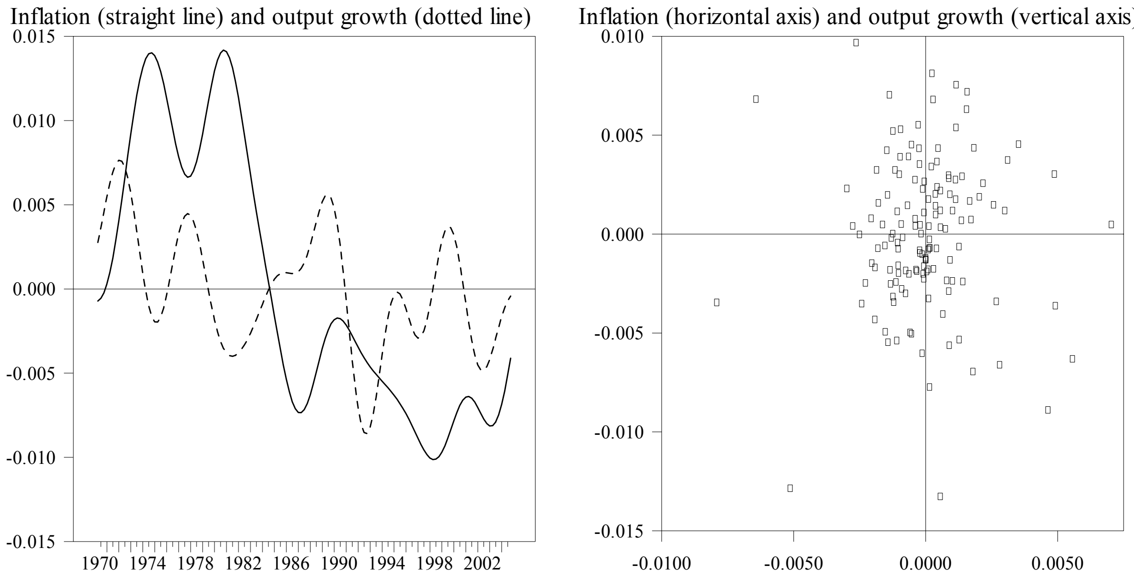

Figure 4. Inflation and output gap: low (left panel) and high frequency (right panel)

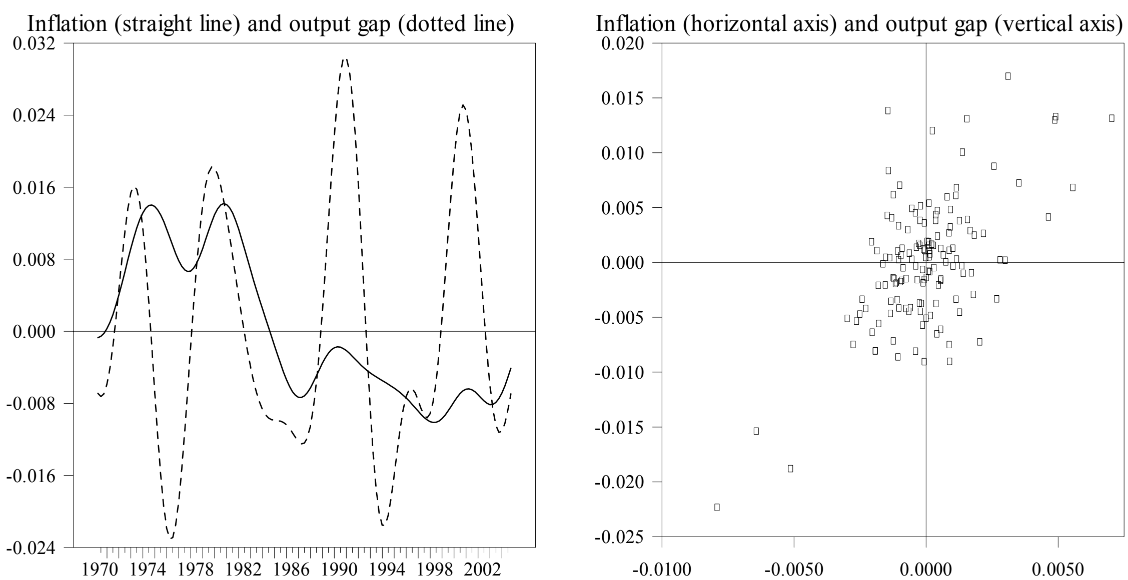

Figure 5. Inflation and money less income growth

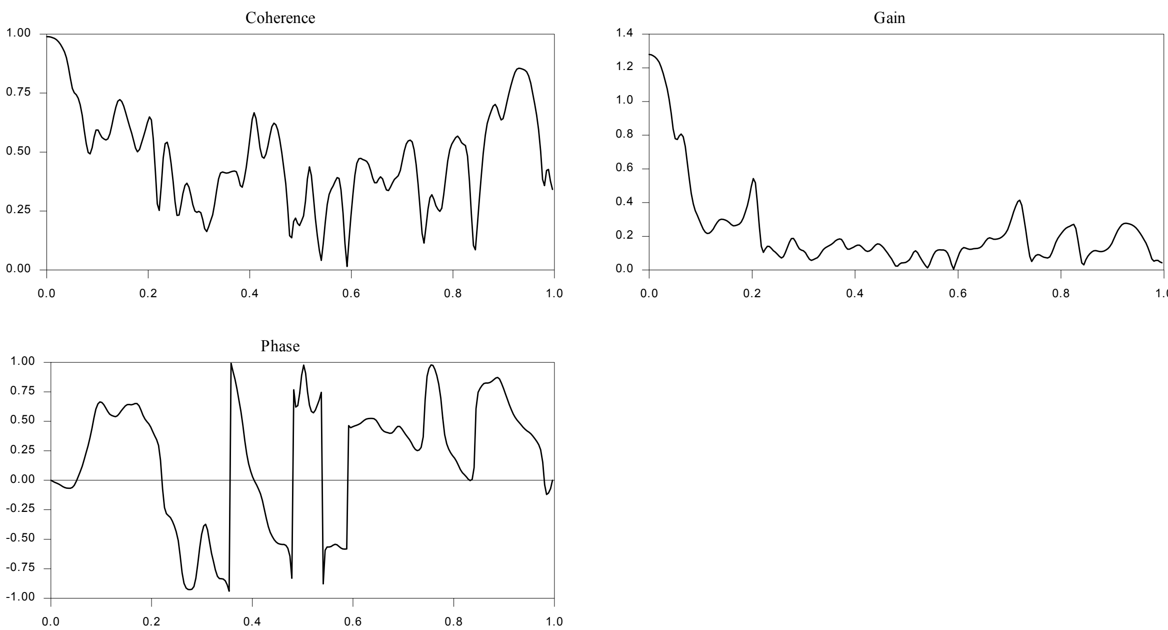

Figure 6. Output gap and inflation