Order Imbalance and the Dynamics of Index and Futures Prices

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Abstract

This study examines empirically with complete transaction records of index futures and of the index stocks, as well as the bid/ask price quotes of the latter, the impact of stock market order imbalance on the dynamic behavior of index futures and the underlying cash index. The study purges spurious correlation in the index by using an estimate of the “true” index with highly synchronous and active quotes of individual stocks. To capture the nonlinear dynamics of the index and futures prices, the study adopts a smooth transition autoregressive error-correction (STAR) process to model the joint dynamics between the two prices. The study finds that order imbalance in the cash stock market significantly affects the error-correction dynamics of index and futures prices. Moreover, order imbalance impedes error-correction when the two forces countervail each other. This finding supports our conjecture that order-imbalance helps explain why real potential arbitrage opportunity may persist over time. The results also show that incorporating order imbalance in the STAR model significantly improves the explanatory power of the framework. Furthermore, the speed of transition increased substantially for the cash index during the crisis period. It can be inferred from the findings that stock market microstructure which allows a speedy resolution of order imbalance promotes dynamic arbitrage efficiency between the futures and the underlying cash stocks.

1. Introduction

Index arbitrageurs endeavor to capture the existing and prospective price discrepancy between index futures and the underlying cash index. Index arbitrage is rather straight-forward to implement; however, many studies document evidence of persistent and apparently real (i.e., exploitable) arbitrage opportunities. Existence of real and persistent arbitrage opportunities, despite the abundance of astute and resourceful arbitrageurs, may imply that these are potential rewards to arbitrageurs for providing a unique financial service. Fung (2005) finds empirical support of Grossman's (1988) conjecture that arbitrage opportunities could be real and are potential compensations to arbitrageurs for providing liquidity in futures when trading is skewed toward one side of the market. In particular, he finds that the level and persistence of the arbitrage basis is related to stock market order imbalance. If arbitrage opportunity can be caused by the liquidity condition in the stock market, then the same market force may also impede the arbitrage mechanisms that are suppose to prevent such opportunities to occur in the first place and to speedily eradicate them if they ever emerge. If order imbalance interferes with error-correction dynamics, then the imbalance may explain the persistence of arbitrage opportunities.

This study extends Fung (2005) and examines how and to what extent stock market order imbalance affects the price dynamics of index and futures. In particular, the study examines the Hang Seng Index (HSI) and futures. The HSI futures have been among the most liquid contracts in the world and the index represents over 75% of the total market capitalization of Hong Kong. We adopt a smooth transition autoregressive error-correction (STAR) process to capture the nonlinear error-correction dynamics of the index and futures prices. The study avoids spurious correlation of the cash index due to infrequent trading and bid-ask bounce by adopting a mid-quote index that is based on synchronous active quotes of all index stocks.

We examine the robustness of the empirical result by comparing findings between periods before and during the 1997 Hong Kong financial market crisis. The results show that there are strong contemporaneous relationships between order imbalance, and index and futures returns. Moreover, incorporating the market impact effect of order imbalance significantly improves the explanation power of the model on the error-correction dynamics of the two prices. The benchmark framework, which does not consider the market impact of order imbalance, provides inconsistent inference on the error-correction dynamics of the two prices for the crisis period sample. Moreover, the responsiveness of the cash index to the basis increased substantially during the crisis period indicating that the intensity of arbitrage-related trades heightened when the market becomes more volatile. Furthermore, the results show that order imbalance dictates price movements of both index and futures when the market impact of order imbalance is opposite to the force of error-correction. This helps explain the observed persistence of index arbitrage opportunities.

2. Literature review

In a frictionless market, arbitrage mechanisms should continuously maintain the equality between the actual futures price F_t and its 'fair' (or theoretical) value F_t^*, where

$F_t^* = S_t(1+r-d)^{T-t}$, S_t is the synchronous index value, r and d represent, respectively, the risk-less rate of interest and the dividend yield of the index portfolio appropriate for the holding period (or time-to-maturity of the futures contract) T-t; T and t are measured in fractions of a year (Klemkosky and Lee, 1991). It follows that, the pricing error or arbitrage basis z_t, herein defined as the difference between the natural logarithm of the actual and fair futures prices (i.e., $z_t = \ln F_t - \ln F_t^*$), should always be close to if not always equal to zero. Early research adopts the Engle and Granger (1987) type linear error-correction framework to model the conditional price dynamics of index and futures. Ignoring the lagged returns, a typical linear error-correction framework is as follows:

$$\Delta f_t = \omega_1 z_{t-1} + \pi_{1t}$$

$$\Delta s_t = \omega_2 z_{t-1} + \pi_{2t}$$

Where $\Delta f_t = \ln F_t - \ln F_{t-1}$ is the futures return between t-1 and t conditional on an observed pricing error z_{t-1} at time t-1. Similarly, $\Delta s_t = \ln S_t - \ln S_{t-1}$ is the conditional index returns. ω_1 and ω_2 are the error-correction coefficients for the futures and index returns, respectively. π_{1t} and π_{2t} are the error terms for the two equations. If z_{t-1} is positive (negative) and the futures is overpriced (underpriced), long-stock short-futures (short-stock long-futures) arbitrage program should cause the futures to drop (rise) and the index to rise (drop). Hence the conditional futures return is expected to be opposite in sign to the price error and ω_1 is expected to be negative. On the other hand, the conditional index return should have the same sign as the pricing error and ω_2 is expected to be positive. The expected error-correction adjustments in index and futures are confirmed by many empirical studies. Studies of the U.S. markets include Garbade and Silber (1983) and Stoll and Whaley (1990). Fung and Jiang (1999) document the results for the Hong Kong market.

However, in reality, arbitrage involves substantial transaction costs in trading stocks and futures. Therefore, the futures price may fluctuate randomly when the magnitude of the arbitrage basis is insufficient to trigger arbitrage (Kawaller, 1987). It follows that the arbitrage basis reverts towards zero only when the deviation of the futures price is sufficiently large to attract arbitrage. To capture the nonlinear pattern of the error correction dynamics, Yadav, Pope and Paudyal (1994) Dwyer, Locke and Yu (1996) and Martens, Kofman, and Vorst (1998) adopt a version of the threshold autoregressive error-correction (TAEC) process. Following Martens, Kofman, and Vorst (1998) and focusing on the error-correction term, a typical TAEC framework is as follows:

$$
\begin{aligned}
z_t &= \rho_1 z_{t-1} + e_t^1 && \ln F_{t-1} \le \ln F_{t-1}^* - c_1 \\
z_t &= \rho_2 z_{t-1} + e_t^2 && \ln F_{t-1}^* - c_1 < \ln F_{t-1} \le \ln F_{t-1}^* + c_2 \\
z_t &= \rho_3 z_{t-1} + e_t^3 && \ln F_{t-1} > \ln F_{t-1}^* + c_2
\end{aligned}
$$

where c_1 and c_2 are the costs or required compensations (in percentage of the fair futures price) associated with long-futures short-stock and long-stock short-futures arbitrage, respectively. $\ln F_{t-1}^* - c_1$ is the so-called lower no-arbitrage bound for the (logarithm of) futures price, and $\ln F_{t-1}^* + c_2$ the upper no-arbitrage bound. To trigger arbitrage, the futures price has to be either below $\ln F_{t-1}^* - c_1$ (i.e., in regime 1) or above $\ln F_{t-1}^* + c_2$ (i.e., in regime 3). Hence, the arbitrage basis is mean reverting and the AR(1) coefficients in regime 1 and 3 (i.e., ρ_1 and ρ_3) are expected to be significantly less than unity. However, if the futures price is within the no-arbitrage bounds, then arbitrage does not take place and the futures price may move randomly and the AR(1) coefficient in regime 2 (i.e., ρ_2) is expected to be close to unity.

This specification allows for asymmetries in the error-correction process of the index and futures prices in response to positive and negative pricing errors. The asymmetry may arise if institutional restrictions and the cost and risk associated with short-selling of equity stock are significant.[1] These effects dampen the speed of error-correction when the futures is underpriced relative to that when the futures is overpriced. If the constraints, costs, and risks against short-selling pose significant impact on the arbitrage relationship, then c_1 will be larger than c_2. However, Chan (1992) argues that quasi-arbitrage that could be conducted by institutional investors that hold sizable equity portfolio may reduce the impact of the constraints and costs against short-selling. In particular, when the futures is underpriced (which means that the cash stocks are relatively overpriced), these institutions may sell part of their stock portfolio and substitute the position by going long the underpriced futures. However, Kempf (1998) and the research on European markets cited there in shows that the constraints against short-selling impede arbitrage. For the Hong Kong market, Fung and Draper (1999) show that both the magnitude and frequency of underpricings are reduced after the Stock Exchange of Hong Kong (i.e., today's HKEx) lifted its restriction against stock short-selling. Moreover, Jiang, Fung, and Cheng (2001) find that the contemporaneous relationship strengthened between index and futures when short-selling is allowed. The result is particularly strong in falling market situations and when the index is overpriced.

Dwyer, Locke and Yu (1996) examine the nonlinear dynamics between the S&P500 futures price and the spot index. Their results show that the model better explain the price dynamics than the linear error-correction model. Martens, Kofman and Vorst (1998) applied a similar framework to estimate the band around the theoretical S&P500 futures price within which arbitrage is not profitable for most arbitrageurs. Their results showed that the arbitrage thresholds are different given positive and negative pricing errors.

However, the TAEC model has implicitly assumed that the arbitrage triggers or cost thresholds are common for all market participants. Differential trading costs among traders imply existence of arbitrage activities for various levels of mispricings and the intensity of arbitrage intensities is positively related to the magnitude of the arbitrage basis. For instance, institutional investors who have large and diversified stock holdings may avoid the high equity trading costs and the constraints, cost and risk against short-selling. To capture a positive pricing error, these institutions may short futures and hedge the position with their existing equity portfolio to lock-in a high riskless return. On the hand, if the futures is underpriced, they may liquidate part of their equity portfolio and substitute the position with long futures positions (Chan, 1992). Moreover, traders may also undertake risky dynamic arbitrage strategy which does not require sufficient "upfront" compensation (or mispricing) if they expect to early unwind their position

[1] Please refer to Draper and Fung (2003) for a detail discussion of the cost and risk associated with

when a large reversal or widening of the basis occur in the future (Brenan and Schwartz, 1990; Mackinlay and Ramaswamy1988). Kawalla (1991) also show that at any price, the futures can be advantageously used by a potential user of the contract. Hence, dynamic arbitrage strategy and arbitrageurs with heterogeneous trading costs could make arbitrage activities a continuous function of the arbitrage basis.

To model the potential arbitrage induced price dynamic for all levels of arbitrage basis, Taylor et al. (2000) and Tse (2001) apply the following form of a Smooth Transition Error-Correction Model (STECM).

$$\Delta f_t = \alpha_1 z_{t-1} F(z_{t-1}; \gamma_1) + \theta_{1t}$$

$$\Delta s_t = \alpha_2 z_{t-1} F(z_{t-1}; \gamma_2) + \theta_{2t}$$

The $\alpha_i's$ are the error-correction coefficients; and θ_i's are the error terms of the two equations. $F(z_{t-1}; \gamma_i)$ is the transition function with the form $1 - \exp(-\gamma_i z_{t-1}^2)$, and γ_i measures the slope of the transition function which indicates how quickly traders in market *i* react to a mispricing. The value of the function $F(z_{t-1}; \gamma_i)$ is monotonically increasing over the magnitude of the pricing error and have values bounded between 0 and 1. If the pricing error is small, arbitrage activities are expected to be low and the value of the transition function is then close to zero, and as a result, error-correction adjustments of both futures and index are small, and vice versa.. The error-correction coefficients (i.e., $\alpha_i's$) thus represent the “maximum” adjustment speed in a particular market.

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conducting short-stock long-futures arbitrage in the Hong Kong market.

Taylor et al. (2000) adopts the framework to examine how the introduction of SETS, an electronic trading system, affects the dynamic arbitrage efficiency between the FTSE-100 and the underlying cash index. They find that adjustment in the spot market is considerably larger, in absolute terms, than adjustment in the futures market during the post-SETS period. Tse (2000) applies the framework to study the dynamics of the Dow Jones Industrial Average (DJIA) futures price and the underlying cash index. His results show that investors respond more rapidly when futures is underpriced than when it is overpriced.

On the other, order imbalance has been found to have significant impact on stock returns. “Executed” order imbalance is defined as the difference between the dollar volume crossed at ask prices and that crossed at bid prices. Positive order imbalance indicates that buying is more active than selling; while negative order imbalance indicates that selling is more active than buying. Blume, Mackinlay, and Terker (1989) find that the correlations between the aggregate order imbalance and the concurrent 15-minute market returns were at .81 and .86, respectively, on October 19 and 20. The correlations are also significant at individual stock level. Easley, O’Hara and Srinvas (1998) show that order imbalance in CBOE options provides information on price movement of the underlying stock. This shows that order imbalance in a particular market may also be associated with the price movement in related securities.

Order imbalance in the cash market may also impact the corresponding futures market. There are two possible effects of order imbalance on the futures prices. Firstly, there is a liquidity effect: institutions may short index futures to substitute for selling off their equity portfolio when large negative order imbalance makes it costly or impossible to unload sizable stock positions. The institutions may be willing to short futures at a discount to induce greater supply of liquidity from the arbitrageurs. This widens the negative basis by pushing down the futures. Similarly, the basis strengthens when institutional buying spills over to the futures market when there are large positive order imbalance in the cash market. Secondly, there is a signaling effect: positive order imbalance signals a rise in the cash market and traders may buy futures ahead of the impending stock price movement. Similarly, negative order imbalance signals a potential drop in the market and traders short futures ahead of the cash market decline. Hence, in this respect, the informational effect reinforces the liquidity effect.

Locke and Sayers (1993) have examined the relation between order imbalance and the stock market volatility. Chan and Fong (2000) show that the volume-volatility relation could be explained by order imbalance. They find that, on a daily basis, the order imbalance is highly correlated with the total number of trades in both the NYSE and Nasdaq stocks; moreover, the volume-volatility relation is weaker after capturing the impact of order imbalance on the intraday stock return. However, the research did not explore whether the direction of price movement is related to the sign of the order imbalance.

A number of studies have examined how market condition affects the dynamics between index and futures. In a study of the U.K. FTSE-100 index futures, Yadav and Pope (1994) fail to find a significant relationship between market returns and the arbitrage basis . Fung and Jiang (1999) and Jiang, Fung, and Cheng (2001) find that the futures lead over the cash index is strengthened in falling market situations and when the futures is underpriced. These results indicate that the hurdle against short-selling impedes the short-stock long-futures arbitrage process when the futures is underpriced. Fung (2005) show that the arbitrage basis is positively related to (signed) order imbalance. In particular, large positive (negative) order imbalance is associated with large positive (negative) arbitrage basis. The relationship is also found to be asymmetrical and the impact on the basis is stronger with negative order imbalance than with positive order imbalance. Moreover, negative order imbalance in particular significantly increases the convergence time when the futures is underpriced. This study extends Fung (2005) and examines the impact of order imbalance on the dynamics of index and futures prices.

A number of studies have examined the index and futures price during the crisis periods. Harris (1989), Kleidon (1992) and Kleidon and Whaley (1992) have examined the large negative basis during the October 1987 U.S. market crash. Draper and Fung (2003) have examined the behavior of the arbitrage basis during the Hong Kong financial crisis and find that the index and futures price remained closely aligned until the Hong Kong government intervened in both the stock and index derivatives markets. On the other hand, Harris, Sofianos, and Shapiro (1990) find that program trading activities is positively related to market volatility. Hence, it is expected that arbitrage-related trading should be intensified during the crisis period and traders should respond faster to mispricing signals.

3. Model and hypotheses

3.1 Smooth Transition Autoregressive Error-Correction (STAR) model - the benchmark case

As a benchmark for measuring the significance of the impact of order imbalance on the error-correction dynamics, following Taylor et al (2000), we adopt the following STAR error-correction model as a benchmark.

$$\Delta s_t = a_{20} + \sum_{n=1}^{p} a_{2(2n-1)} \Delta f_{t-1} + \sum_{n=1}^{p} a_{2(2n)} \Delta s_{t-1} + \left[\sum_{n=1}^{p} b_{2(2n-1)} \Delta f_{t-1} + \sum_{n=1}^{p} b_{2(2n)} \Delta s_{t-1} + (\alpha_{21} + \alpha_{22} D) z_{t-1} \right] * F(z_{t\text{-}1};\gamma) + \eta_{2t}$$

D=1 when $z_{t-1} < 0 ; = 0$ otherwise. Following previous discussions, the error-correction coefficient for futures returns α_{11} is expected to be negative, the coefficient for α_{12} should be positive if the error-correction process is weakened when the futures is underpriced due to the hurdles against short-selling stocks. The error-correction coefficient for index returns α_{21} is expected to be positive and the coefficient for α_{22} should be negative if the hurdles against short-stock arbitrage applies. However, the magnitudes of α_{12} and α_{22} should be less than the corresponding error-correction coefficient α_{11} and α_{21}, respectively, to preserve the convergence between the index and futures prices. a_{ij} are the auto and cross correlation coefficients. $F(z_{t-1};\gamma_t)$ is the transition function with the form $1 - \exp\left(-\gamma_i z_{t-1}^2 / \sigma_{z_{t-1}}^2\right)$. The value of the function $F(z_{t-1};\gamma_t)$ is monotonically increasing over the magnitude of the pricing error and have values bounded between 0 and 1. If the pricing error in the previous period is small, arbitrage is expected to be low and the value of the transition function is then close to zero. As a result, error-correction adjustments of both the futures and index prices are small. γ_i measures how quickly investors in market i respond to the mispricings. z_{t-1}^2 represents the squared pricing error in pervious period and $\sigma_{z_{t-1}}^2$, variance of the pricing error. Following DLY and TDFL, z_{t-1}^2 is normalized with $\sigma_{z_{t-1}}^2$ to make γ_i scale-free measures.

3.2 Impacts of order imbalance on the error-correction dynamics

The market impact of order imbalance may enhance or impede the error-correction process. We examine the following 4 possible scenarios.

Case 1: Both order imbalance and error are positive ($z_{t-1} > 0$ *and* $OI_t > 0$ *)*

In this case, order imbalance has a positive market impact on both index and futures returns. However, positive pricing error triggers short-futures long-stock arbitrage that exerts downward pressure on futures and upward pressure on index. The error-correction dynamics of the index would be enhanced by the market impact of order imbalance; and the conditional return to index is positive. However, the error-correction force of futures is countervailed by the opposite market force of order imbalance. The conditional futures return is ambiguous and depending on the relative dominance of the two forces. If the market impact of order imbalance is stronger than the error-correction force in futures, then the conditional futures return is positive, and vice versa.

Exhibit 1

Case 1: Positive order imbalance and positive pricing error ($OI_t > 0$ and $z_{t-1} > 0$)

Variables	Δf_t	Δs_t
Market Impact $OI_t > 0$	Positive	Positive
Error adjustment $z_{t-1} > 0$	Negative	Positive
Overall direction	Ambiguous	Positive

Case 2: Negative order imbalance and positive pricing error ($OI_t < 0$ and $z_{t-1} > 0$)

In this case, the conditional return to futures is negative since the market impact of order imbalance and error-correction dynamics affect futures at the same direction. However, the conditional return to the index is positive only if the error-correction force dominates the market impact of negative order imbalance. The conditional index return can be negative if the market impact force of order imbalance overwhelms the effect of error-correction.

Exhibit 2

Case 2: Negative order imbalance and positive pricing error ($OI_t < 0$ and $z_{t-1} > 0$)

	Δf_t	Δs_t
$OI_t < 0$ Market Impact	Negative	Negative
$z_{t-1} > 0$ Error-correction	Negative	Positive
Overall direction	Negative	Ambiguous

Case 3: Positive order imbalance and negative pricing error ($OI_t > 0$ and $z_{t-1} < 0$)

In this case, the conditional futures return is positive since the error-correction adjustment for futures price is enhanced by positive order imbalance. However, the conditional index return could be positive if the market impact of positive order imbalance exceeds the error-correction force.

Exhibit 3

Case 3: Positive order imbalance and negative pricing error ($OI_t > 0$ *and* $z_{t-1} < 0$ *)*

	Δf_t	Δs_t
$OI_t > 0$ Market impact	Positive	Positive
$z_{t-1} < 0$ Error-correction	Positive	Negative
Overall direction	Positive	Ambiguous

Case 4: Negative order imbalance and negative pricing error ($OI_t < 0$ *and* $z_{t-1} < 0$ *)*

In this case, the conditional index return is expected to be negative since the market impact of order imbalance enhances the error-correction mechanism for the index. However, the conditional futures return may become negative if the magnitude of the market impact of negative order imbalance exceeds the error-correction force.

Exhibit 4

Case 4: Negative order imbalance and negative pricing error ($OI_t < 0$ *and* $z_{t-1} < 0$ *)*

	Δf_t	Δs_t
$OI_t < 0$ Market impact	Negative	Negative
$z_{t-1} < 0$ Error-correction	Positive	Negative
Overall direction	Ambiguous	Negative

3.3 Modeling the impact of order imbalance - the 4-regime STAR model

To test whether order imbalance significantly impede order correction dynamics, we extend Taylor et al.'s model as follows:

$$\Delta f_t = a_{10} + \sum_{n=1}^{p} a_{1(2n-1)} \Delta f_{t-1} + \sum_{n=1}^{p} a_{1(2n)} \Delta s_{t-1} + \left[\sum_{n=1}^{p} b_{1(2n-1)} \Delta f_{t-1} + \sum_{n=1}^{p} b_{1(2n)} \Delta s_{t-1} \right.$$
$$\left. + \left(\beta_{11} D_1 + \beta_{12} D_2 + \beta_{13} D_3 + \beta_{14} D_4 \right) z_{t-1} \right] * F\left(z_{t-1}; \gamma_1\right) + \eta_{1t}$$

$$\Delta s_t = a_{20} + \sum_{n=1}^{p} a_{2(2n-1)} \Delta f_{t-1} + \sum_{n=1}^{p} a_{2(2n)} \Delta s_{t-1} + \left[\sum_{n=1}^{p} b_{2(2n-1)} \Delta f_{t-1} + \sum_{n=1}^{p} b_{2(2n)} \Delta s_{t-1} \right.$$
$$\left. + \left(\beta_{21} D_1 + \beta_{22} D_2 + \beta_{23} D_3 + \beta_{24} D_4 \right) z_{t-1} \right] * F\left(z_{t-1}; \gamma_2\right) + \eta_{2t}$$

$$D_1 = \begin{cases} 1 \text{ when } z_{t-1} \geq 0 \,\&\, OI_t \geq 0 \\ 0 \text{ otherwise} \end{cases} \qquad D_2 = \begin{cases} 1 \text{ when } z_{t-1} \geq 0 \,\&\, OI_t < 0 \\ 0 \text{ otherwise} \end{cases}$$

$$D_3 = \begin{cases} 1 \text{ when } z_{t-1} < 0 \,\&\, OI_t \geq 0 \\ 0 \text{ otherwise} \end{cases} \qquad D_4 = \begin{cases} 1 \text{ when } z_{t-1} < 0 \,\&\, OI_t < 0 \\ 0 \text{ otherwise} \end{cases}$$

The coefficient for the first dummy variable (i.e., β_1) depicts the conditional response of futures and the cash index in case 1, and so on for the other cases. For the futures equation, β_{12} and β_{13} are expected to be (unambiguously) negative since the market impact of order imbalance enhances error-correction in both cases. β_{11} and β_{14} allow us to test whether order imbalance significantly impede the error-correction mechanism for futures when the two forces drive the futures price to opposite directions. If the market impact of order imbalance dominates, the β_{11} and β_{14} are positive, and vice versa. For the index equation, β_{11} and β_{14} are expected to be unambiguously positive since the market impact of order imbalance and error correct affect the index price in the same direction. If β_{21} and β_{22} are negative, then the result will show that the market impact of order imbalance exceeds the error-correction force, and vice versa.

3.4 Modeling the impact of order imbalance - the 3-regime STAR model

To examine the robustness of the system, we combine regime 1 and regime 4 for the futures equation to test the significance of the impact of order imbalance in the two cases combined. Similarly, we combine regime 2 and regime 3 in the index equation. This allows us to examine the overall impact of order imbalance when it is against the error-correction mechanism.

$$\Delta f_t = a_{10} + \sum_{n=1}^{p} a_{1(2n-1)}\Delta f_{t-1} + \sum_{n=1}^{p} a_{1(2n)}\Delta s_{t-1} + \left[\sum_{n=1}^{p} b_{1(2n-1)}\Delta f_{t-1} + \sum_{n=1}^{p} b_{1(2n)}\Delta s_{t-1} \right.$$
$$\left. + \left(\lambda_{11}D_{11} + \lambda_{12}D_{12} + \lambda_{13}D_{13}\right)z_{t-1}\right]F\left(z_{t-1};\gamma_1\right) + \eta_{1t}$$

$$\Delta s_t = a_{20} + \sum_{n=1}^{p} a_{2(2n-1)}\Delta f_{t-1} + \sum_{n=1}^{p} a_{2(2n)}\Delta s_{t-1} + \left[\sum_{n=1}^{p} b_{2(2n-1)}\Delta f_{t-1} + \sum_{n=1}^{p} b_{2(2n)}\Delta s_{t-1} \right.$$
$$\left. + \left(\lambda_{21}D_{21} + \lambda_{22}D_{22} + \lambda_{23}D_{23}\right)z_{t-1}\right]F\left(z_{t-1};\gamma_1\right) + \eta_{2t}$$

For the futures equation:

$$D_{11} = \begin{cases} 1 \text{ when } z_{t-1} \geq 0 \;\&\; OI_t \geq 0; \\ \quad \text{and when } z_{t-1} < 0 \;\&\; OI_t < 0 \\ 0 \text{ otherwise} \end{cases}$$

$$D_{12} = \begin{cases} 1 \text{ when } z_{t-1} \geq 0 \;\&\; OI_t < 0 \\ 0 \text{ otherwise} \end{cases}$$

$$D_{13} = \begin{cases} 1 \text{ when } z_{t-1} < 0 \;\&\; OI_t \geq 0 \\ 0 \text{ otherwise} \end{cases}$$

For the index equation

$$D_{21} = \begin{cases} 1 \text{ when } z_{t-1} \geq 0 \;\&\; OI_t \geq 0 \\ 0 \text{ otherwise} \end{cases}$$

$$D_{22} = \begin{cases} 1 \text{ when } z_{t-1} \geq 0 \;\&\; OI_t < 0; \\ \quad \text{and when } z_{t-1} < 0 \;\&\; OI_t \geq 0 \\ 0 \text{ otherwise} \end{cases}$$

$$D_{23} = \begin{cases} 1 \text{ when } z_{t-1} < 0 \;\&\; OI_t < 0 \\ 0 \text{ otherwise} \end{cases}$$

For the futures equation, β_{12} and β_{13} are expected to be negative (the normal sign of the error-correction coefficient for the futures returns). The coefficient for the combined regime depicted by D_{11} (i.e., β_{11}) will be positive if order imbalance significantly impede the error-correction mechanisms. Similarly, for the index equation, β_{21} and β_{23}

are expected to be positive. However, a positive β_{22} will show that order imbalance dominates the price movements if it is opposite to the force of error-correction.

4. Data

Time stamped bid/offer quotes for the 33 constituent stocks of the Hang Seng Index (HSI) and the transaction records of the stocks and of the Hang Seng Index futures are obtained from the Hong Kong Exchange and clearing for the period May 1996 to April 1998. The stocks have been traded electronically in a screen-based Automatic Matching System (AMS) system. The futures were traded via open outcry.2 The spot month futures contract is the most liquid among the four concurrently traded maturity months, except on its expiry day. We substitute the next month contract for the spot month contract on the contract expiration day of each month. The sample covers the extreme market conditions during the Asian financial crisis; hence, the data provide records of large intraday and interday variations in the stock and futures prices, as well as their trading volumes. Dividend information including the ex-date, the payment day, and the actual amount of dividend for the constituent stocks are also obtained from the Exchange. For the construction of the market value weight for each index stocks, we obtain market capitalization information of outstanding shares of the stocks as well as the closing index quotes from Hang Seng Index Services Limited. Hong Kong Inter-bank Offer Rates (HIBORs) for maturity of 1-day to 1-month are retrieved from Datastream.

4.1 Construction of the mid-quote index

Studies of the index-futures relationship have been plagued by the measurement problem of the index caused by infrequent trading and bid/ask bounce. Miller, Muthuswamy, and Whaley (1994) show that part of the negative correlation in the basis can be explained by the effect of infrequent trading which delayed the adjustment of the cash index relative to the futures price. The problems are especially pronounced in highly volatile periods (Harris, 1989). However, large variations in the basis only occur during stressful market situations to allow for a meaningful test of its dynamic behavior. Following Blume, Mackinlay, and Terker (1989), the study purges the effects of infrequent trading by adopting a reconstructed time series of the index based on the middle quote synchronous active bid/offer prices of the index constituent stocks. The approach also controls for the bias in the index returns directly due to order imbalance (Lease, Masulis, and Page, 1991). The bias could also be induced by arbitrage itself as the index could be swayed to either side of the spread as a result of index arbitrage (Harris, Sofiano, and Shapiro, 1994). Moreover, Chan, Chung, and Johnson (1993) also show that the use of mid-quotes reduces the impact of the discreteness in the tick size on the responsiveness of the traded price.

The Hang Seng Index (HSI) is a value-weighted index. The current index value is the ratio of the current total market value of the index stocks divided by the total market capitalization at the previous day's close, multiplied by the value of the index at the

[2] Futures began electronic trading on June 5, 2000 via the Hong Kong Atomated Trading System (HKATS).

previous day's close. Following the index construction method, the mid-quote index at time τ on day t is equal to:

$$S_{t_\tau}^{m} = \sum^{33} W_{it}\left(P_{it_\tau}^{a} + P_{it_\tau}^{b}\right)/2$$

where $S_{t_\tau}^{m}$ and the mid-quote index at time τ on day *t*, W_{it} is the market value weight for security *i* on day *t*. $P_{it_\tau}^{a}$ and $P_{it_\tau}^{b}$ are, respectively, the ask and bid price for stock *i* at time τ on day *t*. Since the quotes are refreshed every 30 seconds, a mid-quote index is obtained for a 30 seconds interval when all 33 pairs of bid/offer quotes are available.[3] The study adopts the minute-by-minute sample data. The mid-quote index is adopted in the calculation of index returns and the fair futures price. Returns for the overnight non-trading hours and the lunch break are excluded from the analysis.

4.2 Construction of the fair futures price series

To filter out discrete interday changes in the index-futures relationship due to uneven dividend payments to the index, the actual (ex-post) dividend payments accrued to the index during the remaining life of the contract is factored into the cost-of-carry framework. Let $F_{t_\tau}^{*}$ be the 'fair' (or theoretical) futures price,

$$F_{t_\tau}^{*} = S_{t_\tau}^{m}(1+r)^{T-t} - \sum_{j=t}^{T-t-1} W_{it} D_{ij}\left(1+r_j\right)^{T-j}$$

[3] Please refer to Draper and Fung (2003) for details on the methodology for the construction of the quote based index prices.

where t and T (as fractions of a year) denote the initiation and expiration day of the contract, respectively; r_j is the overnight interest rate; r is the riskless rate for the holding period between day t and T; and D_{ij} is the per share cash dividend for stock i at time j.

We measure the size of the pricing error or arbitrage basis in percentage of the fair futures value - i.e., $z_t = \ln F_t - \ln F_t^*$.

4.3 Measurement of order imbalance

Following Blume, Mackinlay, and Terker (1989), the order imbalance of an individual stock is equal to its dollar volume crossed at the asked price minus the dollar volume crossed at the bid price within a particular interval. The study largely follows Lee and Ready's (1991) approach to identify whether a trade is executed at bid or at ask. A trade is identified as a bid (an ask) trade if the traded price is below (above) the midpoint of the nearest previous bid and ask quotes. If that fails to identify a trade, the nearest quotes following the trade would be used. The reason is that HKEx retrieves the quote by taking snapshots of the limit order book every 30 seconds. Hence, the quotes following the trade could have been the quotes at which that the trade was executed. In cases where the traded price falls exactly on the mid-point of the quotes both preceding and after the trade, the following tick test is used.

If the current traded price is above (below) the previous traded price, the trade is an up tick (down tick) and is classified as an ask (a bid) trade. If the current traded price is equal to the previous traded price, then the trade is classified according to the trade before the previous one. A zero-up tick (i.e., the previous trade is traded at an up tick) is classified as an ask trade, and a zero-down tick (i.e., the previous trade is traded at a down tick) is classified as a bid trade. The process stops when there were no changes in the traded price in the last two transactions, and the trade will not be included in the analysis. Moreover, the maximum time difference between the current trade and the oldest transaction or quote used for the purpose of identification is restricted to five minutes.

Aggregate order imbalance for the index within a particular time interval is obtained by summing the individual order imbalance of the constituent stock of the index within the same time interval; that is $O_t = \sum_{i=1}^{33} O_{i\tau}$; where $O_{i\tau}$ denotes the order imbalance of stock *i* measured for the τ^{th} interval. To make the order-imbalance measure free of the level of the market and comparable with the volume of HSI futures contracts, we convert the aggregate dollar order-imbalance into a measure in terms of an equivalent number of index futures contracts. To accomplish that, we divide the aggregate 30-second dollar order imbalance by the mid-quote index prevailing at the end of the interval and by HK$50 (i.e., the contract multiplier). Hence,

$$O_\tau = \sum_{i=1}^{33} \varpi_{i\tau} \Big/ (S_t * 50).$$

To calculate the 1-minute order imbalance, we simply add the two consecutive 30-second order imbalances within the particular non-overlapping 1-minute interval. This procedure is followed to calculate order imbalance for other time intervals.

To focus on the information revealed through the trades executed within the AMS system, we discard all non-AMS transactions. A trade is classified according to a matching quote that occurs nearest to the time of the trade. This criterion causes some trades to be classified according to the quotes following the trade. This is possible since a trade could have been executed against a quote that was being revised within a 30-second interval; and the revised quotes are only reported after the trade occurs.

5. Empirical results and interpretations

5.1 Lead lag relationships

We first adopt a Granger causality test on the relationship between the futures (or cash index) returns and the order imbalance. Following Fung (2005), it is expected that the cash index returns and order imbalance should bear a strong contemporaneous relationship. Moreover, since futures returns usually lead that of the cash index, futures returns should lead order imbalance. Following Fung and Jiang (1999) and Jiang, Fung, and Cheng (2001), we pre-whiten all series with AR processes.

Table 1 shows the results the lead-lag relationship between futures returns and order imbalance. The pre-crisis results show that the two series lead and lag each other; however, the coefficient for the one-period lead term in order imbalance is most significant with a t-value of 13.72. This shows that there is a significant lead of futures returns over order imbalance by one period. Moreover, the two series have strong positive contemporaneous relationship. The contemporaneous and lead-lag relationship between futures returns and order imbalance strengthened during the crisis period. The one-period lead over order imbalance is again most significant. The 2-period lead of futures over order imbalance and the contemporaneous relationship are also strengthened.

The result is also consistent with Blume et al.'s conjecture that an order imbalance leads to a price change and a price change in turn leads to further order imbalance, and so on.

Table 2 shows the lead-lag results between index futures and order imbalance. Results from both periods show that index returns and order imbalance bear very strong contemporaneous relationship. The t-values for the two periods are 38.71 and 51.60, respectively. The results also show that order imbalance generally leads index returns by one period. However, the lead of index returns seems to be more extensive during the crisis period.

5.2 Asymmetrical error-correction mechanism in response to positive and negative errors - the benchmark case

To account for the non-constant error variance in the 2 equations, we adopt the following GARCH (1,1) process to capture the stochastic variance. $\sigma_{it}^2 = \varpi_i + A_i a_{it-1}^2 + B_i \sigma_{it-1}^2$; i=1,2. a_{it-1}^2 is the lag 1 squared residuals and σ_{it-1}^2 the lag 1 residual variance of Δf_t and Δs_t. Equations (1) and (2) are estimated simultaneously via the Full Information Maximum Likelihood (FIML) approach. Table 3 shows the results of the two sample periods of the smooth transition autoregressive process for the futures and index returns. The index returns equation increased in R-square from 16.63% for the pre-crisis period to 32.24% during the crisis period. While the R-square of the futures returns equation also increase from 4.35% to 8.23%. The error variance could be fitted with the GARCH (1,1) for the pre-crisis period. However, the behavior of the error variance is more chaotic during the crisis period and GARCH (1,1) is insufficient to capture the dynamics of the variance. Hence, we adopt the HCCME to obtain consistent estimates of the parameters for the crisis period sample.

The error-correction coefficient of the index returns (i.e, α_{11}) for the pre-crisis period is positive and which is consistent with the theory. The coefficient for the dummy variable (i.e., α_{12}) is negative and is less than α_{11}. These results are consistent with previous findings that hurdles against shorting stock impede the arbitrage mechanism and reduce the speed of error-correction when the futures is underpriced. However, these two variables are of the wrong sign during the crisis period. This result indicates that the inference from the benchmark framework could be significantly distorted during the crisis period.

The error-correction coefficient for the futures returns is negative for both periods. Moreover, the coefficients for the dummy variables (i.e., α_{22}) for both periods are positive and are smaller in magnitudes than the error-correction coefficients (i.e, α_{21}). Hence, the impediment against shorting stock also affects the speed of error-correction in the futures market. Moreover, the coefficient for the dummy variable is much larger in magnitude relative to its pre-crisis period. This indicates the increased significance of the impediment against arbitrage during the crisis period.

Based on the findings of Harris, Sofianos, and Shapiro (1990), it is expected that the responsiveness of arbitrageurs to the pricing errors should be heightened during the crisis period. However, the response coefficients γ_i for both index and futures are greatly reduced during the crisis period. These results are in contrary to previous findings in the

U.S. markets and may indicate that the benchmark framework is insufficient to capture the conditional price dynamics of the two prices under volatile market condition.

5.3 The impact of order imbalance on the error-correction process – the 4-regime case

Table 4 shows the results from the STAR estimations with 4 dummy variables denoting the four different regimes. β_{11} shows the net adjustment coefficient for futures returns when positive pricing error occurs when order imbalance is positive. β_{11} is positive which shows that the market impact of order imbalance dominates the effect error-correction. However, since the two effects countervail each other, the coefficient is relatively small (0.007588). β_{12} shows the total adjustment coefficient when positive pricing error is concurrent with negative order imbalance. In this case error-correction is enhanced by the market impact of order imbalance. β_{12} is negative (-0.01376) and as predicted. β_{13} shows the total adjustment coefficient when pricing error is negative and order imbalance is positive. Again, error-correction should push the futures price up as does the order imbalance. Consistent with expectation, β_{13} is positive (0.00378). However, β_{13} is smaller than β_{12} due to the impediments against short-stock arbitrage. β_{14} shows the net adjustment coefficient for futures returns following negative pricing error and when order imbalance is negative. Error-correction force should push up the futures price but negative order imbalance is against it. β_{14} is positive (0.003339) which shows that the market impact of order imbalance against exceeds the force of error-correction. Results from the crisis period are generally consistent with those from the pre-crisis period expect that all coefficients are larger in magnitude. Hence, the results from the futures equation show that order imbalance can actually jeopardize the error-correction mechanisms in situations as described in regime 1 (i.e., positive order imbalance and positive pricing errors) and regime 4 (negative order imbalance and negative pricing error). In both cases, the error-correction in futures is completely reversed by the opposite market impact effect of order imbalance. The effect is particularly severe in regime 4 where error-correction is further impeded with hurdles against short selling of stocks.

On the other hand, β_{21} shows the total adjustment coefficient for index returns when positive pricing error occurs with positive order imbalance. In this case the market impact of order imbalance should push the index upward as does error-correction. β_{21} is positive (0.010054) and consistent with expectation. β_{22} shows the net adjustment coefficient when positive pricing error is concurrent with negative order imbalance. In this case error-correction should move the index up but against the negative market impact of order imbalance. β_{22} is negative (-0.00698) which means that the market impact of order imbalance exceeds the force of error-correction. β_{23} shows the net adjustment coefficient when pricing error is negative and order imbalance is positive. Error-correction should push the index down but positive order imbalance is moving the index up. β_{23} is negative (-0.01044). Again, the force of order imbalance exceeds that of error-correction when they are against each other. The magnitude of β_{23} is larger than that of β_{22}. This result again shows that the distortion of the error-correction mechanism by order imbalance is particularly severe when error-correction is impeded by hurdles against short-stock arbitrage when the futures is underpriced. β_{24} shows the adjustment coefficient for futures returns following negative pricing error and when order imbalance is negative. In this case, both forces should move the index price lower. The coefficient is negative (0.0133379) and consistent with expectation. Results from the crisis period are generally consistent with those from the pre-crisis period. Hence, incorporating the market impact effect of order imbalance provides a consistent explanation of the price dynamics between index and futures prices even during the stressful market period. Moreover, the results from the index equation also confirm those from the futures equation. In particular, order imbalance may reverse the error-correction effect when the two forces countervail. In regime 2 and regime 3, (i.e., negative order imbalance and positive pricing errors) and regime 3 (positive order imbalance and negative pricing error), the error-correction in index returns is reversed by the opposite market impact effect of order imbalance. The effect is particularly severe in regime 3 where error-correction is further impeded by hurdles against short selling of stocks.

Moreover, consistent with prior expectation, the reaction coefficients γ_i show that both futures and index are more responsive to pricing errors during the crisis period. The γ_i for the cash index is about 5 times that of the pre-crisis period. These results show that incorporating the market impact of order imbalance allows the framework to capture the price dynamics of the index and futures especially under stressful market conditions.

5.4 The impact of order imbalance on the error-correction process – the 3-regime case

Table 5 summarizes the estimation results after merging the two cases where the market impact of order imbalance is against the direction of error-correction. The t coefficient λ_{11} is positive showing that the direction of the conditional futures returns is reversed (versus being negative indicating error-correction). λ_{12} and λ_{13} are consistent with the previous results. They are both positive and are larger and more significant than in the 4-regime framework. λ_{22} shows the net adjustment coefficient in the combined regime (regime 2 in the framework) for index returns. The coefficient is negative which indicates again that the error-correction process is dominated by the market impact of order imbalance. λ_{21} and λ_{23} are consistent in signs to those in the 4-regime formulation; moreover, they are larger and more significant. The results are consistent for the crisis period sample.

5.5 Test on the relative explanatory power of the three models

We adopt the likelihood ratio test to examine whether incorporating order imbalance in the framework improves the overall goodness-of-fit of the system. Table 6 shows the results for all three cross comparisons and for both periods. The large F-statistics allow us to reject the null hypothesis that the performance of the benchmark model and those that incorporate order imbalance are equal for both sample periods. This means that both the 4-regime and 3-regime models outperform the benchmark framework. There is no significant difference in performance of the two models that incorporate the market impact of order imbalance.

6. Conclusion

This paper examines the impact of order imbalance on the dynamics of index and index futures. Order imbalance leads the cash index returns by 3 minutes but the lead is reduced to 1 minute during the crisis period. Order imbalance and futures returns both lead and lag each other but the lead of futures over order imbalance strengthened during the crisis period. The benchmark model fails to provide a consistent explanation of the error correction dynamics especially of the cash index under the stressful market condition during the crisis period. The results show that incorporating the market impact of order imbalance provides consistent explanation of the error dynamic process particularly under volatile market conditions and when arbitrage and trading activities are intense. Moreover, factoring the potential impact of order imbalance significantly improves the explanatory power of the framework.

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Table 1
The lead-lag relationship between order imbalance and futures returns

Dependent variable: Futures Return

	Pre-crisis Period		Crisis Period	
	Parameter Estimate	t-value (p-value)	Parameter Estimate	t-value (p-value)
Intercept	– 0.00026562	– 0.57 (0.5693)	– 0.00013477	– 0.13 (0.8935)
$OIres_{-5}$	– 0.00004087	– 1.37 (0.1710)	0.00007151	1.46 (0.1445)
$OIres_{-4}$	0.00005152	1.71 (0.0867)	0.00000691	0.14 (0.8884)
$OIres_{-3}$	0.00009808	3.26 (0.0011)	0.00004752	0.97 (0.3315)
$OIres_{-2}$	0.00018273	6.09 (<.0001)	0.00005361	1.10 (0.2713
$OIres_{-1}$	0.00025442	8.52 (<.0001)	0.00047648	9.80 (<.0001)
OIres	0.00028638	9.59 (<.0001)	0.00099068	20.44 (<.0001)
$OIres_{1}$	0.00040973	13.72 (<.0001)	0.00128	26.29 (<.0001)
$OIres_{2}$	0.00029279	9.76 (<.0001)	0.00073243	14.96 (<.0001)
$OIres_{3}$	0.00023527	7.86 (<.0001)	0.00035124	7.19 (<.0001)
$OIres_{4}$	0.00013740	4.60 (<.0001)	0.00026036	5.35 (<.0001)
$OIres_{5}$	0.00008216	2.74 (0.0061)	0.00014219	2.92 (0.0035)
R^2	0.0612		0.1014	
No. of obs.	9223		14160	
F-value (p-value)	54.61 (<.0001)		145.11 (<.0001)	

$$\Delta f_t^{'} = a_1 + \sum_{i=-5}^{5} b_{1i} OI_{t+i}^{'} + e_{1t}$$

Table 2
The lead-lag relationship between order imbalance and index returns

Dependent variable: Stock Return

	Pre-crisis Period		Crisis Period	
	Parameter Estimate	t-value (p-value)	Parameter Estimate	t-value (p-value)
Intercept	– 0.00013572	– 0.45 (0.6519)	– 0.00022652	– 0.42 (0.6721)
$OIres_{-5}$	– 0.00000802	– 0.42 (0.6768)	– 0.00000607	– 0.23 (0.8157)
$OIres_{-4}$	0.00000368	0.19 (0.8493)	– 0.00006395	– 2.44 (0.0146)
$OIres_{-3}$	0.00007054	3.64 (<.0001)	– 0.00004647	– 1.79 (0.0740)
$OIres_{-2}$	0.00010929	5.65 (<.0001)	– 0.00000852	– 0.33 (0.7421)
$OIres_{-1}$	0.00031909	16.58 (<.0001)	0.00039015	15.10 (<.0001)
OIres	0.00074508	38.71 (<.0001)	0.00133	51.60 (<.0001)
$OIres_{1}$	0.00000322	0.17 (0.8672)	0.00016312	6.30 (<.0001)
$OIres_{2}$	0.00009046	4.67 (<.0001)	0.00027439	10.54 (<.0001)
$OIres_{3}$	0.00005897	3.05 (0.0023)	0.00020946	8.06 (<.0001)
$OIres_{4}$	0.00005990	3.11 (0.0019)	0.00011351	4.39 (<.0001)
$OIres_{5}$	0.00000999	0.52 (0.6049)	0.00010417	4.03 (<.0001)
R^2	0.1698		0.1855	
No. of obs.	9223		14160	
F-value (p-value)	171.34 (<.0001)		292.98 (<.0001)	

$$\Delta S_t^{'} = a_2 + \sum_{i=-5}^{5} b_{2i} OI_{t+i}^{'} + e_{2t}$$

Table 3
Summary of the estimation results of the benchmark STAR model

$$\Delta f_t = a_{10} + \sum_{n=1}^{p} a_{1(2n-1)} \Delta f_{t-1} + \sum_{n=1}^{p} a_{1(2n)} \Delta s_{t-1} + \left[\sum_{n=1}^{p} b_{1(2n-1)} \Delta f_{t-1} + \sum_{n=1}^{p} b_{1(2n)} \Delta s_{t-1} + (\alpha_{11} + \alpha_{12} D) z_{t-1} \right] * F(z_{t-1}; \gamma) + \eta_{1t}$$

$$\Delta s_t = a_{20} + \sum_{n=1}^{p} a_{2(2n-1)} \Delta f_{t-1} + \sum_{n=1}^{p} a_{2(2n)} \Delta s_{t-1} + \left[\sum_{n=1}^{p} b_{2(2n-1)} \Delta f_{t-1} + \sum_{n=1}^{p} b_{2(2n)} \Delta s_{t-1} + (\alpha_{21} + \alpha_{22} D) z_{t-1} \right] * F(z_{t-1}; \gamma) + \eta_{2t}$$

D=1 when $z_{t-1} < 0$; $= 0$. $F(z_{t-1}; \gamma_t) = 1 - \exp\left(-\gamma_i z_{t-1}^2 / \sigma_{z_{t-1}}^2\right)$.

	Pre-crisis Period (11/96 to 4/97)		Crisis period (8/97 to 1/98)	
	Futures *	Cash *	Futures	Cash
R^2	0.0435	0.1663	0.0823	0.3224
α_{i1}	-0.0011 (-0.39)	0.002685 (1.41)	-0.02043 (-1.12)	-0.00769 (-0.74)
α_{i2}	0.000303 (0.09)	-0.00171 (-0.73)	0.015351 (0.82)	0.0064 (0.60)
A_i	0.101741 (11.71)	0.089565 (7.67)	–	–
B_i	0.867286 (63.83)	0.833386 (30.89)	–	–
γ_i	1.011882 (3.18)	0.956565 (3.05)	0.174346 (1.91)	0.139064 (1.71)
u_t	0.6112	0.9039	0.0505	0.2478
u_t^2	0.0776	0.3143	–	–

Note: the following GARCH (1,1) process is adopted to account for the non-constant error variance in the index and futures equations: $\sigma_{it}^2 = \varpi_i + A_i a_{it-1}^2 + B_i \sigma_{it-1}^2$; i=1,2. a_{it-1}^2 is the lag 1 squared residuals and σ_{it-1}^2 the lag 1 residual variance of Δf_t and Δs_t. The system is estimated with full information maximum likelihood method. For all periods, outliers with absolute values of either one of df, ds, lagz and boi exceeding 7 standard deviation. are removed. Numbers corresponding to ut and ut2 are the p-values of Ljung-Box Q(24)-statistics residual diagnosis on the null hypothesis that the residuals are white noise. The Ljung-Box Q(24) statistics show that GARCH (1,1) is sufficient to capture the stochastic error variance during the pre-crisis sample period. However, the same process is found insufficient to fit the volatility structure. We adopt the robust Heteroscedastic Consistent Covariance Matrix Estimation (HCCME) to provide consistent estimates of the model parameters. We use 15 lag terms for each estimation.

Table 4
Summary of the estimation results of the STAR model with 4 order imbalance regimes

$$\Delta f_t = a_{10} + \sum_{n=1}^{p} a_{1(2n-1)}\Delta f_{t-1} + \sum_{n=1}^{p} a_{1(2n)}\Delta s_{t-1} + \left[\sum_{n=1}^{p} b_{1(2n-1)}\Delta f_{t-1} + \sum_{n=1}^{p} b_{1(2n)}\Delta s_{t-1} + (\beta_{11}D_1 + \beta_{12}D_2 + \beta_{13}D_3 + \beta_{14}D_4)z_{t-1}\right] * F(z_{t-1};\gamma_1) + \eta_{1t}$$

$$\Delta s_t = a_{20} + \sum_{n=1}^{p} a_{2(2n-1)}\Delta f_{t-1} + \sum_{n=1}^{p} a_{2(2n)}\Delta s_{t-1} + \left[\sum_{n=1}^{p} b_{2(2n-1)}\Delta f_{t-1} + \sum_{n=1}^{p} b_{2(2n)}\Delta s_{t-1} + (\beta_{21}D_1 + \beta_{22}D_2 + \beta_{23}D_3 + \beta_{24}D_4)z_{t-1}\right] * F(z_{t-1};\gamma_2) + \eta_{2t}$$

$$D_1 = \begin{cases} 1 \text{ when } z_{t-1} \geq 0 \,\&\, OI_t \geq 0 \\ 0 \text{ otherwise} \end{cases} \qquad D_2 = \begin{cases} 1 \text{ when } z_{t-1} \geq 0 \,\&\, OI_t < 0 \\ 0 \text{ otherwise} \end{cases}$$

$$D_3 = \begin{cases} 1 \text{ when } z_{t-1} < 0 \,\&\, OI_t \geq 0 \\ 0 \text{ otherwise} \end{cases} \qquad D_4 = \begin{cases} 1 \text{ when } z_{t-1} < 0 \,\&\, OI_t < 0 \\ 0 \text{ otherwise} \end{cases}$$

$$F(z_{t-1};\gamma_t) = 1 - \exp\left(-\gamma_i z_{t-1}^2 / \sigma_{z_{t-1}}^2\right).$$

	Pre-crisis Period (11/96 to 4/97)		Crisis period (8/97 to 1/98)	
	Futures	Cash	Futures	Cash
R^2	0.0454	0.1882	0.0863	0.3421
β_{i1}	0.007588 (1.84)	0.010054 (5.39)	0.004988 (0.64)	0.015415 (5.92)
β_{i2}	-0.01376 (-2.43)	-0.00698 (-3.35)	-0.02598 (-2.55)	-0.0175 (-8.01)
β_{i3}	-0.00378 (-1.98)	-0.01044 (-7.79)	-0.01841 (-5.43)	-0.01401 (-10.06)
β_{i4}	0.003339 (1.64)	0.013337 (9.36)	0.006331 (2.06)	0.009749 (6.37)
A_i	0.1118 (10.96)	0.11444 (8.20)	–	–
B_i	0.852678 (51.77)	0.78098 (22.71)	–	–
γ_i	0.897909 (1.81)	2.202972 (3.30)	0.941743 (2.53)	11.88921 (3.23)
u_t	0.5934	0.7815	0.0813	0.1060
u_t^2	0.0992	0.5507	–	–

Note: refer to the footnote in Table 6.

Table 5
Summary of the estimation results of the STAR model with 3 order imbalance regimes

$$\Delta f_t = a_{10} + \sum_{n=1}^{p} a_{1(2n-1)}\Delta f_{t-1} + \sum_{n=1}^{p} a_{1(2n)}\Delta s_{t-1} + \Big[\sum_{n=1}^{p} b_{1(2n-1)}\Delta f_{t-1} + \sum_{n=1}^{p} b_{1(2n)}\Delta s_{t-1} + (\lambda_{11}D_{11} + \lambda_{12}D_{12} + \lambda_{13}D_{13})z_{t-1}\Big]F(z_{t-1};\gamma_1) + \eta_{1t}$$

$$\Delta s_t = a_{20} + \sum_{n=1}^{p} a_{2(2n-1)}\Delta f_{t-1} + \sum_{n=1}^{p} a_{2(2n)}\Delta s_{t-1} + \Big[\sum_{n=1}^{p} b_{2(2n-1)}\Delta f_{t-1} + \sum_{n=1}^{p} b_{2(2n)}\Delta s_{t-1} + (\lambda_{21}D_{21} + \lambda_{22}D_{22} + \lambda_{23}D_{23})z_{t-1}\Big]F(z_{t-1};\gamma_1) + \eta_{2t}$$

For the futures equation:

$$D_{11} = \begin{cases} 1 \text{ when } z_{t-1} \geq 0 \,\&\, OI_t \geq 0; \\ \quad \text{and when } z_{t-1} < 0 \,\&\, OI_t < 0 \\ 0 \text{ otherwise} \end{cases}$$

$$D_{12} = \begin{cases} 1 \text{ when } z_{t-1} \geq 0 \,\&\, OI_t < 0 \\ 0 \text{ otherwise} \end{cases}$$

$$D_{13} = \begin{cases} 1 \text{ when } z_{t-1} < 0 \,\&\, OI_t \geq 0 \\ 0 \text{ otherwise} \end{cases}$$

For the index equation

$$D_{21} = \begin{cases} 1 \text{ when } z_{t-1} \geq 0 \,\&\, OI_t \geq 0 \\ 0 \text{ otherwise} \end{cases}$$

$$D_{22} = \begin{cases} 1 \text{ when } z_{t-1} \geq 0 \,\&\, OI_t < 0; \\ \quad \text{and when } z_{t-1} < 0 \,\&\, OI_t \geq 0 \\ 0 \text{ otherwise} \end{cases}$$

$$D_{23} = \begin{cases} 1 \text{ when } z_{t-1} < 0 \,\&\, OI_t < 0 \\ 0 \text{ otherwise} \end{cases}$$

$$F(z_{t-1};\gamma_t) = 1 - \exp\left(-\gamma_i z_{t-1}^2 / \sigma_{z_{t-1}}^2\right).$$

	Pre-crisis Period (11/96 to 4/97)		Crisis period (8/97 to 1/98)	
	Futures	Cash	Futures	Cash
R^2	0.0454	0.1880	0.0863	0.3419
λ_{i1}	0.004323 (2.51)	0.019296 (8.28)	0.006061 (2.10)	0.03103 (10.44)
λ_{i2}	-0.01783 (-3.10)	-0.00941 (-8.60)	-0.03185 (-2.93)	-0.01525 (-13.09)
λ_{i3}	-0.00781 (-3.18)	0.023108 (13.03)	-0.02453 (-5.66)	0.024756 (12.99)
A_i	0.112023 (10.72)	0.11371 (7.86)	–	–
B_i	0.852275 (50.33)	0.782588 (21.63)	–	–
γ_i	1.001553 (1.87)	2.032275 (3.22)	0.971266 (2.53)	11.60231 (3.22)
u_t	0.5906	0.7766	0.0804	0.0894
u_t^2	0.0976	0.5536	–	–

Note: refer to the footnote in Table 6.

Table 6
A comparison between the explanatory power of the benchmark model and the two models that incorporate the impact of order imbalance

					H0: 2-alpha model H1: 3-alpha model		H0: 2-alpha model H1: 3/4-alpha model		H0: 3-alpha model H1: 4-alpha model	
Pre-crisis period		N	7332							
		R-sq	k	q	F	p-value	F	p-value	F	p-value
Benchmark model	futures	0.0435	67							
	cash	0.1663	67							
3-regime model	futures	0.0454	68	1	14.45799288	0.0001445				
	cash	0.188	68	1	194.1241379	1.45E-43				
4-regime model	futures	0.0454	69	2			7.228001257	0.0007312	0	1
	cash	0.1882	69	2			97.9672949	1.04E-42	0.894678492	0.408784
Crisis period		N	14354							
		R-sq	k	q	F	p-value	F	p-value	F	p-value
Benchmark model	futures	0.0823	64							
	cash	0.3224	64							
3-regime model	futures	0.0863	65	1	62.55444894	2.781E-15				
	cash	0.3419	65	1	423.3938611	9.792E-93				
4-regime model	futures	0.0863	66	2			31.27503557	2.799E-14	0	1
	cash	0.3421	66	2			213.9182247	2.886E-92	2.171758626	0.114015