Vehicle Currency*

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Abstract

An important feature of the international financial system is that it overwhelmingly uses a major currency, such as the US dollar, as a vehicle in the exchange between other currencies. In this paper we construct an equilibrium model to study this feature. The model economy has a finite number of countries, and each country's goods are sold only for the country's own currency. Households obtain foreign currencies at trading posts. Each post involves one pair of currencies, and there is a fixed cost of operating the post. We study two types of equilibria. One is the symmetric trading equilibrium, in which there is an active post for every pair of currencies and so there is no vehicle currency. The other is the vehicle currency equilibrium, in which all countries exchange for a particular currency first and then use that currency to exchange for other currencies. We analyze how the use of a vehicle currency changes each country's consumption and welfare, relative to the symmetric equilibrium. We study the constraints on the inflation rate of the vehicle currency that must be satisfied in order for a currency to be a robust vehicle currency.

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1. Introduction

The international financial system is very far from the ideal symmetric mechanism that is often described in theoretical models. Countries differ greatly in market size, financial openness, and asset positions. One of the most obvious asymmetries in the financial system concerns the role of currencies. In recent history, the US dollar has occupied a central role in the international economy. The dollar has acted as a international unit of account, in that it operates as an invoicing currency for commodity and asset trade, a store of value, in that official reserves assets of central banks are predominantly held in US dollars, and an international means of exchange, in that global foreign exchange transactions are overwhelmingly conducted using the US dollar on one side of the transaction.

In this latter role, the dollar acts as a 'vehicle currency'[1]. In general bilateral markets for smaller country currencies are quite thin or even non-existent. To engage in currency trade between the currencies of small countries, generally the US dollar is used as a vehicle. We would expect this to generate efficiency gains if there are fixed costs to setting up trading technologies. By having trade from many disparate country/currencies all go through one large currency market, the average cost of setting up trading technologies may be greatly reduced. On the other hand, it would seem to give the US dollar and US monetary policy a predominance over the rest of the world in a way that it would not necessarily always be beneficial, particularly in light of the fact that US monetary policy is focused on domestic rather than international goals.

This paper develops a simple dynamic general equilibrium model of a vehicle currency. We build a multi-country monetary exchange economy model , where each country's money is required to finance purchases in that country, through a cash-in-advance constraint. But

[1]See Krugman (1979), Hartman (1990) and Rey (1990) for discussion on the nature of a vehicle currency.

the way in which agents acquire foreign currencies may differ. We model foreign exchange trade as a costly process that takes place through 'trading post' technologies. Trading posts have been modelled by Starr (2000) and Howitt (2005). They represent locations where agents can go in order to buy or sell one currency for another; that is, they facilitate bilateral trade in currencies. In a purely symmetric world, there would be one trading post for all possible bilateral pair of currencies. Trading possibilities would be the same for the holders of any currency, so that currencies and countries would be treated equally. But trading posts are costly to set up. In a world with a large number of currencies, having trading in all possible bilateral pairs of currencies would involve significant real resources used up in setting up trading posts.

An alternative equilibrium is where one country operates as a 'vehicle currency'. This offers significant efficiencies, since less resources are used up in trading. At the same time however, it confers benefits on the vehicle currency issuer. The main object of the paper is to explore this trade off.

Our model has $N > 3$ countries, labeled $1, 2..N$. In a Symmetric Trading Equilibrium, there are $N(N-1)/2$ bilateral foreign exchange trading posts, and agents from any country can use their currency directly to buy the currency of any other country. In a Vehicle Currency Equilibrium, country 1 acts as an intermediary. There are only $N-1$ trading posts, with currency 1 being on one side of all currency trades. Agents from any country $i > 1$ who wish to purchase currency $j \notin \{i, 1\}$ must first purchase currency 1 and then use currency 1 to purchase currency j.

The gains to a Vehicle Currency Equilibrium come from being able to facilitate all possible trades while reducing the number of trading posts by $(N/2-1)(N-1)$. For large N, the resources saved may be significant. But the Vehicle Currency Equilibrium treats countries asymmetrically. Residents of the issuing country have the same opportunity set as in a Symmetric Trading Equilibrium, since they can directly buy the currency of any other country. But residents of the peripheral countries (i.e. all countries $i > 1$) must visit two trading posts in order to complete an exchange with another peripheral country. We find that a Vehicle Currency Equilibrium always benefits residents of country 1. But residents of peripheral countries may lose or gain. They gain from reduced trading costs, but lose from lower terms of trade in international trade, delayed consumption, and exposure to country 1 inflation tax. We find that for a small number of countries, it is never desirable (for peripheral countries) to have a vehicle currency.

The paper is not only motivated by the analysis of how a vehicle currency system works. We are also interested in determining how such situations come about. Historically, different currencies have acted as the standard in the international economy during different epochs. The pound sterling represented the central international currency in the pre WWI period, and to a lesser extent in the interwar period. More contemporaneously, now the euro offers a viable alternative international standard, what forces would lead it to supplant the US dollar as a vehicle currency?

To address these issues, we explore the robustness of a vehicle currency equilibrium. Given that a vehicle currency is in use, what factors might lead to its abandonment? One aspect of the 'trading post' is that there are many Nash equilibria where agents will follow a particular trading pattern because they expect that all others will do so as well. But we can examine how robust these equilibria are to deviations, where a (large) group of agents choose a different trading pattern. We use this approach to ask how robust is a vehicle currency equilibrium to a deviation where all agents in two countries use their own currency to facilitate bilateral trade, but continue to use the vehicle currency for all other trade, and all other countries continue to the the vehicle currency for all trade. We find that the vehicle currency is robust to deviations if it has the lowest rate of inflation among all countries.

There is a relatively small literature on the nature of an international currency. Krugman (1980) defines a vehicle currency in the same way that is used here, within a partial equilibrium setting, and explores alternative trading patterns. Rey (2001) looks at how increasing returns to scale technologies in financial markets may give rise to an international currency. Hartmann (1998) looks at a model of a vehicle currency in financial markets and endogenizes a bid-ask spread. A different literature on search and money has explored the use of international currencies in an environment where agents can choose the currency they will hold to make purchases (e.g. Matsuyama et al. 1991, Trejos and Wright 2001). This differs from ours principally in that we assume the existence of a cash-in-advance constraint for all goods purchases, but look specifically at the nature of trade between currencies.[2]

The next section sets out the basic model. Section 3 constructs an equilibrium where bilateral markets in all currencies exist. Section 4 constructs the vehicle currency equilibrium and compares it to the symmetric trading equilibrium. Section 5 explores the robustness of the vehicle currency equilibrium. Section 6 examines the sensitivity of the results to the trading assumptions. Some conclusions follow.

2. The Model

2.1. Technology and Preferences

Time is discrete, indexed by $t = 0, 1....$ There are $N \geq 3$ countries, indexed by $i = 1, 2, ..N$, and every country has the same population which is normalized to 1. Within a country, all households are alike. Each country has a given endowment of its own good, so each household in a country i is endowed with y_{it} units of good i in period t. All goods are

[2]Head and Shi (2002) construct a search-based model of two countries in which goods trade for money, and moneys also trade for one another.

perishable within a period. A country i household's preferences are written;

$$U^i = \sum_{t=0}^{\infty} \beta^t \left[u(c_{iit}) + \theta \sum_{j \neq i} u(c_{ijt}) \right], \quad \theta > 0$$

where $\beta \in (0, 1)$ is the discount factor. The parameter θ may be used to capture home preference. When $\theta = 1$, consumption of home goods and all foreign goods are equally weighted. When $\theta < 1$, there is a home good preference. Throughout the analysis, $u(c) = \ln(c)$ is assumed.

Each country has its own currency, and residents of a country receive lump-sum transfers only from their own country's monetary authority. Let M_{it} be the total stock of currency i in period t and the gross rate of growth of currency i be $\gamma_{it} = M_{it}/M_{it-1}$. Each household in country i receives an amount, $(\gamma_{it} - 1)M_{it-1}$, of currency i at the beginning of period t. There are no fiscal transfers across countries. We normalize all nominal variables by the money stock of the relevant country. Lower case notation denotes normalized expressions. Thus the normalize price level of good i is $p_{it} = \frac{P_{it}}{M_{it}}$, and normalized holdings of currency j by a country i household are $m_{ijt} = \frac{M_{ijt}}{M_{jt}}$. Nominal exchange rates are normalized by the ratio of the stocks of two currencies involved.

2.2. Monetary Exchange at Trading Posts

We impose a cash in advance constraint at the national level. That is, purchases of country i's goods must use only currency i.[3] Therefore, in order to consume country j's good, a household in country i must obtain currency j. How currency trade takes place is the main focus of interest in the paper.

We assume that currency trade is organized in bilateral trading posts. That is, at a trading post, one currency is exchanged for another. There can be many agents on each

[3] One way to view this assumption is as a result of a legal restriction on settlement with domestic currency within a domestic market.

side of a trading post. We order the two currencies at a post in ascending order and refer to a trading post with currencies k and j as post kj, where $k < j$.

Anyone can set up a trading post, but doing so involves fixed costs. In order to set up trading post kj, the manager of a trading post must incur a fixed cost $y_{kt}\phi_k$ in good k and $y_{jt}\phi_j$ in good j. There is also a cash-in-advance constraint on trading posts - the fixed cost in each country's good needs to be paid in that country's money.

The managers of each trading post announce two prices for a pairwise trade, one for sale of a currency (ask) for another currency, and one for purchase (bid) of a currency for another currency. We assume that potential entry into a trading post leads each manager to follow a Bertrand pricing rule. In equilibrium the bid and ask prices announced by the manager of the trading post are just sufficient to cover the fixed costs of setting up the trading post, given the buyers and sellers of the currency pair in which the trading post operates. These prices then represent the equilibrium nominal exchange rates for each currency pair.[4]

Note that with N countries and trading posts for each pair of currencies, there are $N(N-1)/2$ possible trading posts. But with each trading post incurring fixed costs, in principle this can be improved upon by using one currency as an an intermediate, and trading twice. When one currency plays the role of a 'vehicle', then only $N-1$ trading posts need to exist in order to facilitate trade between all countries.

With fixed costs of setting up trading posts, there can be many Nash equilibria that differ from each other in the number of active posts. To see this, suppose that an agent believes that only a few other agents will go to a particular trading post. Then trading

[4] In reality of course, currency traders do not just trade one currency for another. But there are clear limits on the number of exchange possibilities that exist. Few commercial currency exchanges are willing to buy or sell much more than about a half dozen currencies. Moreover, bid ask spreads are typically higher for smaller currencies. The use of trading posts allows us a simplified way to handle the frictions inherent in currency trading.

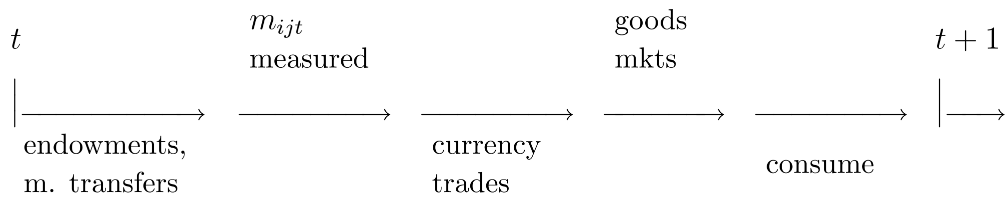
at that post will not be sufficient to cover the fixed cost, and so the agent will have no incentive to bring a currency to buy or sell at that trading post. In this case, the trading post will remain inactive.

We will focus on the equilibria that lie at the two ends of the spectrum. One is the Symmetric Trading Equilibrium (STE henceforth), in which there is a trading post for each pair of countries and there are $N(N-1)/2$ trading posts. The other is the Vehicle Currency Equilibrium (VCE henceforth), in which one currency is common to all trading posts, and there are only $N-1$ trading posts.

Of course it is unsatisfactory merely to focus on alternative equilibria, since using this criterion, it is hard to rule out any trading configuration, however inefficient. In light of this, we will investigate the robustness of the Vehicle Currency Equilibrium in a later section of the paper.

2.3. Timing of Events

The timing of events is as follows. At the beginning of a period, agents receive any unspent cash balances in each currency. They receive their income from last period sales of their endowment, in domestic currency, plus a domestic currency transfer from the monetary authorities. In sum this gives m_{ijt}. Agents then visit the trading posts of their choice in order to exchange currencies. For the present, we assume that the household can visit a given trading post only once within the period. After currency exchange at trading posts, they hold m'_{ijt} of each currency. After the currency trading is over, agents visit the goods market, with each household dividing into a shopper and a seller. At the end of the period, the households consumes all the goods purchased. The following illustration describes the timing:



In the following analysis, we will suppress the time subscript t whenever possible and use the subscript $\pm n$ to stand for $t \pm n$, where $n \geq 1$. Denote the (normalized) nominal exchange rate for a buyer of currency j at a post kj as $s_{kj}^{a} \equiv S_{kj}^{a} \frac{M_k}{M_j}$. This is the (normalized) amount of currency k required to purchase one unit of currency j at the post kj, or the 'ask' price of j in terms of k. For a seller of currency j, at trading post kj, the exchange rate is s_{kj}^{b}. This is the amount of k that can be obtained at the post, for one unit of j, or the bid price of currency j in terms of k. Clearly, $s_{kj}^{a} \geq s_{kj}^{b}$ is required for trading post kj to be viable.

Let f_{ik}^{kj} be the amount of currency k (normalized by the total stock of currency k, M_k) brought to the post kj by the representative country i household. We impose a trading restriction such that households cannot short on currencies at any post. That is, $f_{ik}^{kj} \geq 0$ for all i, k, j.[5]

3. Symmetric Trading Equilibrium

In this section, we describe an equilibrium where all bilateral currency posts are open, and residents of each country engage in currency trade with all other countries in order to obtain the currency required to purchase the country's good. In this equilibrium, the household begins the period holding all of its own country's cash balances, and engages in trade in foreign currency markets with all other currencies to finance its imports.

[5]The post kj is said to be active if at least one side of the post has a positive amount of currency, i.e., if $\left(\sum_{i=1}^{N} f_{ik}^{kj}\right) + \left(\sum_{i=1}^{N} f_{ij}^{kj}\right) > 0$.

3.1. Household Choices

Consider an arbitrary country i and let us examine the decision problem of a representative household in country i. For given money holdings, the household chooses a sequence $\{h_{it}\}_{t=0}^{\infty}$, where $h_i = \left((c_{ij})_{j=1}^{N}, (f_{ii}^{ij})_{i<j}, (f_{ii}^{ji})_{i>j}, (m'_{ij})_{j=1}^{N}, (m_{ij(+1)})_{j=1}^{N}\right)$, to maximize U^i subject to the following constraints:

$$m_{ii} = \frac{1}{\gamma_i}\left[m'_{ii(-1)} - p_{i(-1)}c_{ii(-1)} + p_{i(-1)}y_{i(-1)}\right] + \tau_i \tag{3.1}$$

$$m_{ij} = \frac{1}{\gamma_j}\left[m'_{ij(-1)} - p_{j(-1)}c_{ij(-1)}\right], \quad j \neq i, \tag{3.2}$$

$$m'_{ii} = m_{ii} - \sum_{j>i} f_{ii}^{ij} - \sum_{j<i} f_{ii}^{ji}, \tag{3.3}$$

$$m'_{ij} = m_{ij} + \frac{1}{s_{ij}^{a}} f_{ii}^{ij}, \quad i < j, \tag{3.4}$$

$$m'_{ij} = m_{ij} + s_{ji}^{b} f_{ii}^{ji}, \quad i > j, \tag{3.5}$$

$$m'_{ij} \geq p_j c_{ij}, \text{ all } j. \tag{3.6}$$

Equation (3.1) describes the dynamics of domestic cash balances and (3.2) the dynamics of the balances of foreign currencies. For the domestic currency, holdings at the beginning of the period must come from left-over currency in the last period, sales of goods in the last period, or monetary transfers. Money growth γ_i is applied to the money carried over from the last period because $(m'_{ii(-1)}, p_{i(-1)})$ are normalized by last period's money stock. For a foreign currency $j \neq i$, holdings at the beginning of the period consist of entirely the left-over currency in the last period, as described in (3.2).

The household then visits the $N-1$ currency trading posts ij (for $i<j$) and ji (for $i>j$), supplying respectively f_{ii}^{ij} and f_{ii}^{ji} to these posts, as described in (3.3). Recall that m is measured immediately before currency trades and m' is measured immediately after currency trades. At the ij trading post ($i < j$), the household pays the 'ask' price for currency j, and receives f_{ii}^{ij}/s_{ij}^{a} units of currency j in return. At the ji trading post ($i > j$), the household receives the 'bid' price for its sale of currency i, and gets $s_{ji}^{b}f_{ii}^{ji}$ units of currency j. These constraints are described in (3.4) and (3.5). In addition, the cash in advance constraint (3.6) must be satisfied for all goods.

We first examine the optimal choices of households, taking exchange rates as given, and then look at equilibrium exchange rates which ensure that trading posts are viable in a Symmetric Trading Equilibrium. To proceed, we assume for this section is that all cash-in-advance constraints are binding.[6] This means that, when households enter a period, they have no foreign currency left over and they hold the entire stock of domestic currency. That is, $m_{ij} = 0$ for all $j \neq i$ and so $m_{ii} = m_i = 1$. Because there is only one currency trading session per period, the households must visit all trading posts in order to ensure that they can consume all goods.

In Appendix A we show that optimal choices for household i give the conditions:

$$\text{for } j > i: \quad p_j c_{ij} = \frac{\theta}{s_{ij}^{a}} p_i c_{ii}, \tag{3.7}$$

$$\text{for } j < i: \quad p_j c_{ij} = \theta s_{ji}^{b} p_i c_{ii}. \tag{3.8}$$

Because the household holds no foreign currency entering the period, consumption of a foreign good j must be financed entirely by the amount of currency j that the household purchases in the current period. That is, $f_{ii}^{ij}/s_{ij}^{a} = p_j c_{ij}$ for $j > i$ and $f_{ii}^{ji} s_{ji}^{b} = p_j c_{ij}$ for $j < i$. Also, all purchases of foreign currencies in the period must come from holdings of domestic currency at the beginning of the period. Therefore,

$$1 = m_{ii} = p_i c_{ii} + \sum_{j>i} s_{ij}^{a} p_j c_{ij} + \sum_{j<i} \frac{1}{s_{ji}^{b}} p_j c_{ij}. \tag{3.9}$$

[6] Conditions under which this will be confirmed are given below.

Now, substituting the first-order conditions for consumption, we have:

$$p_i c_{ii} = \frac{1}{\theta\sigma}, \tag{3.10}$$

$$f_{ii}^{ij} = \frac{1}{\sigma},\ j > i; \quad f_{ii}^{ji} = \frac{1}{\sigma},\ j < i, \tag{3.11}$$

where $\sigma \equiv \frac{1}{\theta} + N - 1$. Thus, households bring a fixed fraction of their initial cash balances to each bilateral trading post.

3.2. Exchange Rates and Consumption

There is a firm at each trading post ij. The firm sets prices s_{ij}^a and s_{ij}^b so as to just break even, after it incurs the fixed cost $y_i\phi_i$ in good i and $y_j\phi_j$ in good j. The firm must pay these fixed costs with currency. Hence, the firm must hold currency i in the (normalized) amount $p_i y_i \phi_i$ and currency j in the amount $p_j y_j \phi_j$.

The implicit idea here is that if the firm were to make profits, then there is another firm in the background which would enter the ij trading post. So the firm engages in Bertrand pricing (see Howitt, 2005, for a formalization of this assumption).

Exchange rates in trading post ij are set by the firm so as to satisfy two conditions. The first condition, determining the ask price of currency j, is written as:

$$s_{ij}^a \left[f_{jj}^{ij} - p_j y_j \phi_j \right] = f_{ii}^{ij}, \tag{3.12}$$

This is explained as follows. In an STE, trading post ij receives total currency j payments of f_{jj}^{ij} (since only country j agents hold currency j in this equilibrium), and must hold currency $p_j y_j \phi_j$ to pay the good j fixed costs of setting up the trading post. It receives f_{ii}^{ij} deliveries of currency i from country i residents. It must set the asking price of currency j that country i residents will pay so that its holdings of currency j, in excess of its fixed costs, are all paid out to country i households. From this condition, s_{ij}^a exactly satisfies this property.

In a similar manner, to determine the bid price, s^b_{ij}, the trading post must satisfy the condition that deliveries of currency i made by country i households, less required currency holdings of $p_i y_i \phi_i$, must equal the deliveries of currency j by country j residents. This condition is:

$$s^b_{ij} f^{ij}_{jj} = f^{ij}_{ii} - p_i y_i \phi_i. \tag{3.13}$$

From the fact that all cash in advance constraints bind, in conjunction with market clearing, we have that $m_i = p_i y_i$, so that $p_i = 1/y_i$, for all i. Using this in (3.12) and (3.13), and substituting the solutions for the currency trades f^{ij}_{ii}, we get (for $i < j$):

$$s^{aSTE}_{ij} = \frac{1}{1 - \sigma\phi_j}; \quad s^{bSTE}_{ij} = 1 - \sigma\phi_i. \tag{3.14}$$

We impose the restriction $max_i\ \sigma\phi_i < 1$ so that these solutions are meaningful.[7] Hence, the bid-ask spread at trading post ij under STE is

$$\left(\frac{s^a_{ij}}{s^b_{ij}}\right)^{STE} = \frac{1}{(1 - \sigma\phi_i)(1 - \sigma\phi_j)} > 1.$$

The bid-ask spread reflects the presence of trading costs. It is increasing in the cost parameters ϕ_i and ϕ_j. If the trading posts could operate without costs, so that $\phi_i = 0$ for all i, then the terms of trade between any two countries would be $\frac{y_i}{y_j}$, and the normalized exchange rate would be unity among all pairs of currencies. But with fixed costs of setting up each trading post the ask price must exceed the bid price. To explain (3.14), note that agents in any country j will spend an amount $\frac{1}{\sigma}$ on each trading post that they visit, then the fixed cost relative to the cash deliveries is $\phi_j/(\frac{1}{\sigma}) = \sigma\phi_i$. To break even, the trading post must ensure that the bid price of currency j for each currency i is reduced by this amount. A similar argument holds for the ask price. Put together, the bid-ask spread reflects the cost of the trading post in terms of both countries' good.

[7]This restriction just requires that the average cost of setting up a trading post must be less than the total endowment of the respective commodities.

Recall that $\sigma = \frac{1}{\theta} + N - 1$. Thus, a rise in N (the number of countries) or a fall in θ (the preference weight on foreign goods) reduces the share of the budget devoted to any one trading post, hence increases the fixed cost per unit of currency trade in that post. As a result, these forces will increase ask exchange rates and reduce bid exchange rates. It is precisely the large trading costs in setting up bilateral foreign exchange trading posts with many countries that give rise to potential benefits of a vehicle currency.

We calculate the consumption levels of a country i household. Substituting the solutions for exchange rates together into (3.10), (3.7) and (3.8), we get:

$$c_{ii}^{STE} = \frac{y_i}{\theta\sigma}, \tag{3.15}$$

$$c_{ij}^{STE} = [1 - \sigma\phi_j] \frac{y_j}{\sigma}, \; j \neq i. \tag{3.16}$$

Each country i consumes a share $1/(\theta\sigma)$ of its own good, and $(1 - \sigma\phi_j)/\sigma$ of good $j \neq i$. The presence of trading costs in the currency market introduces an endogenous home bias in consumption, in addition to the home bias implied by preferences. Given the form of preferences and the trading cost technology, the STE has the property that the fixed costs of setting up the ij trading post are fully borne by households of country i and j. The fixed costs in terms of good j (i) are borne by country i (j).

Note that in the STE, consumption is independent of home or foreign country money growth. Money is completely neutral, and there are no international 'spillovers' of monetary policy.

Finally, we check that cash in advance constraints indeed bind. Using the first order conditions above, it is easy to establish that cash in advance constraints for each currency i will bind in a steady state if $\gamma_i > \beta$.

4. Currency 1 as a Vehicle

We now examine the VCE (Vehicle Currency Equilibrium) where currency 1 serves as the vehicle currency. That is, currency 1 has active trading posts with all other currencies, but there are no bilateral posts other than those with country 1. Instead of $N(N-1)/2$ trading posts as in the STE, now there are only $N-1$ posts. We call country 1 the VC country or the center country and other countries the peripheral countries. For now, we continue to maintain the assumption that there is only one round of trading in the currency markets in each time period.

4.1. Households' Decisions

When currency 1 is a vehicle currency, residents of all other countries $i > 1$ must engage in two foreign exchange transactions in order to consume goods other than their own or country 1's good. This means that, from the time of their decision to consume an additional unit of these goods, they must wait one period for consumption to take place. Because there is only one round of trading in the currency market in each period, to obtain other currencies $j \neq i, 1$, a household in a peripheral country i ($\neq 1$) must carry a positive amount of the vehicle currency between periods. That is, $m_{i1} > 0$ for all $i \neq 1$ (and so $m_{11} < 1$). This means that the cash in advance constraint on currency 1 does not bind for the peripheral countries. In contrast, for the VC country, the cash in advance constraint on currency 1 binds.

Moreover, the cash in advance constraints on all non-vehicle currencies bind for all countries, as in the previous section. Thus, $m_{ij} = 0$ for all $i \neq j$ and $j \neq 1$, and $m_{ii} = m_i = 1$ for all $i \neq 1$.

The decision problem facing country 1 is identical to that described above, because country 1 has active trading posts with all other countries. For country $i > 1$, the dynamics of money holdings are still given by (3.1) and (3.2), and the cash in advance constraints by (3.6). However, the other constraints are modified as follows:

$$m'_{ii} = m_{ii} - f_{ii}^{1i}, \tag{4.1}$$

$$m'_{ij} = m_{ij} + \frac{1}{s_{1j}^a} f_{i1}^{1j}, \quad j \notin \{i, 1\}, \tag{4.2}$$

$$m'_{i1} = m_{i1} - \sum_{j \notin \{i,1\}} f_{i1}^{1j} + s_{1i}^b f_{ii}^{1i}, \tag{4.3}$$

$$\sum_{j \neq i} f_{i1}^{1j} \leq m_{i1}. \tag{4.4}$$

Constraint (4.1) says that the only amount of domestic currency (i) that the household spends in the currency market is the one brought to the $1i$ post. The constraint (4.2) gives the household's holdings of other non-vehicle currency $j \notin \{1, i\}$ after the currency exchange. The household uses the vehicle currency to exchange for such a non-vehicle currency at the $1j$ post, and the amount of the vehicle currency that the household brings to the post is f_{i1}^{1j}. The total amount of the vehicle currency that the household brings into the posts is given by the sum in (4.3). Thus, (4.3) gives the household's holdings of the vehicle currency after the currency exchange. Finally, (4.4) requires that the total amount of the vehicle currency that the household brings into the posts should not exceed the amount that the household has when it enters the period. We may call this constraint the 'vehicle currency constraint'. It arises from the fact that the household cannot arbitrage between different posts. This constraint binds, provided $\gamma_1 > \beta$.

In Appendix A, we show that the optimal choices of a peripheral country i household yield the following conditions:

$$p_i c_{ii} = \frac{1}{\theta s_{1i}^b} p_1 c_{i1} \tag{4.5}$$

$$p_i c_{ii} = \frac{\gamma_{1(+1)} s_{1j(+1)}^a}{\theta \beta s_{1i}^b} p_{j(+1)} c_{ij(+1)}, \; j \notin \{i, 1\}. \tag{4.6}$$

The condition (4.5) characterizes the trade-off between good 1 consumption and that of the domestic good, which is the same as before. For each country $i > 1$, the relative price of good 1 is $p_1/(s_{1i}^b p_i)$. But the trade-off involved between consumption of the domestic good and that of a third country is quite different. Acquiring one dollar of the vehicle currency incurs a sacrifice of $1/s_{1i}$ in terms of domestic currency and $1/(s_{1i}^b p_1)$ in terms of the domestic good. Since the vehicle currency constraint is binding, this can only be converted into a country j's ($j \notin \{i, 1\}$) currency in next period's foreign exchange trading session. In the next period, each dollar of currency 1 can obtain $1/[\gamma_{1(+1)} s_{1j(+1)}^a p_{j(+1)}]$ units of good j. Equating the costs and benefits in utility terms, and discounting, gives condition (4.6).

Hence, there are three basic features of the vehicle currency environment that impact on the decisions of peripheral countries. First, in their consumption of third country goods, they must undertake two foreign exchange transactions, accepting the bid price of their own currency, and paying the ask price of currency 1 for the third country currency. Second, this involves a delay, which is costly because agents discount future utility. Finally, it also involves a cost due to country 1 money growth, as country 1 inflation will reduce the real value of their currency 1 money holdings over time.

As in the previous section, $m_{ii} = m_i = 1$ and $p_i = 1/y_i$ for all $i \neq 1$. Also, a country i's holdings of currency i are equal to the sum of the expenditures on goods. However, because the expenditures on other peripheral countries' goods occur with one period delay, as explained above, the condition (3.9) needs to be modified. In Appendix A, we provide this modification and show the following results for country i ($\neq 1$):

$$c_{ii} = \frac{y_i}{\theta[\sigma - (1-\beta)(N-2)]}, \tag{4.7}$$

$$f_{ii}^{1i} = 1 - p_i c_{ii} = \frac{1 + \beta(N-2)}{\sigma - (1-\beta)(N-2)}, \tag{4.8}$$

$$f_{i1}^{1j} = \frac{\beta}{\sigma - (1-\beta)(N-2)} \left(\frac{s_{1i(-1)}^b}{\gamma_1} \right), \; j \notin \{i, 1\}, \tag{4.9}$$

$$m_{i1} = \sum_{j \notin \{i,1\}} f_{i1}^{1j} = \frac{\beta(N-2)}{\sigma - (1-\beta)(N-2)} \left(\frac{s_{1i(-1)}^b}{\gamma_1} \right). \tag{4.10}$$

Recall that $\sigma = N - 1 + 1/\theta > N - 1$. Expression (4.7) shows that for $\beta < 1$, a peripheral country i will devote a greater share of its budget to the domestic good that under STE, since ceteris paribus, commodities $j \neq \{1, i\}$ become more expensive, as described below. Condition (4.8) says that whatever country $i \neq 1$ does not spend on its home good, it brings to the $1i$ trading post to obtain currency 1. Condition (4.9) gives the amount of currency 1 brought to the $1j$ trading post ($j \neq i$). This is determined by the price that country i received for its sales of currency i, in the previous period; i.e. $s_{1i(-1)}^b$, adjusted by country 1 money growth. The condition (4.10) gives the total amount of currency 1 that country i holds at the beginning of the period.

For country 1, optimal consumption is chosen in the same manner as under the STE:

$$p_j c_{1j} = \frac{\theta}{s_{1j}^a} p_1 c_{11}, \text{ all } j \neq 1. \tag{4.11}$$

Because other countries hold currency 1 between periods, however, it is no longer true that $m_{11} = m_1$ $(= 1)$. In fact, since $m_{11} + \sum_{i \neq 1} m_{i1} = 1$, it must be the case that:

$$m_{11} = 1 - \frac{\beta(N-2)}{\sigma - (1-\beta)(N-2)} \left(\frac{1}{\gamma_1} \right) \sum_{i \neq 1} s_{1i(-1)}^b \tag{4.12}$$

The level of consumption of good 1 by country 1 is given by:

$$c_{11} = \frac{m_{11}/p_1}{\theta\sigma}. \tag{4.13}$$

Consumption levels of other goods by country 1 can be calculated using (**??**). Likewise, the amount of currency 1 brought to the $1i$ post by country 1 is:

$$f_{11}^{1i} = \frac{m_{11}}{\sigma}. \tag{4.14}$$

To compute the price level, p_1, notice that the cash in advance constraint on currency 1 binds for country 1. Using this fact and the fact $\tau_1 = (\gamma_1 - 1)/\gamma_1$, we rewrite the constraint (3.1) for $i = 1$ as follows:

$$p_1 y_1 = 1 - \gamma_{1(+1)} \left[1 - m_{11(+1)}\right]. \tag{4.15}$$

Thus, country 1's normalized price level is influenced by the holdings of all other countries of currency 1.

The above analysis also implies that country 1's current account surplus is:

$$p_1 y_1 - p_1 c_{11} - \sum_{j \neq 1} s^a_{1j} p_j c_{1j} = p_1 y_1 - m_{11} = (1 - m_{11}) - \gamma_{1(+1)} \left[1 - m_{11(+1)}\right].$$

In a steady state, $m_{11(+1)} = m_{11}$. With a positive growth rate of money, country 1 can run a continual current account deficit by virtue of the fact that all other countries must maintain currency 1 balances.

4.2. Trading Posts with a Vehicle Currency

We determine exchange rates in trading post $1i$, $i > 1$, as follows. In a period, country i residents bring f^{1i}_{ii} to the $1i$ post. Currency 1 at the post $1i$ is supplied by country 1, in the amount f^{1i}_{11}, and by each of the other peripheral countries $j \notin \{i, 1\}$, in the amount f^{1i}_{j1}. The peripheral countries supply currency 1 to post $1i$ in order to purchase country i's good later in the period. To supply currency 1, they hold currency 1 from the last period. Then, the ask and bid prices of currency i are determined by:

$$s^a_{1i} \left[f^{1i}_{ii} - \phi_i\right] = f^{1i}_{11} + \sum_{j \notin \{i,1\}} f^{1i}_{j1}, \tag{4.16}$$

$$s^b_{1i} f^{1i}_{ii} = f^{1i}_{11} + \sum_{j \notin \{i,1\}} f^{1i}_{j1} - p_1 y_1 \phi_1. \tag{4.17}$$

We focus on a steady state where γ_i is constant over time for all i. Then, all real variables and all normalized nominal variables are constant over time. In the steady state, the above conditions in the currency market and the condition (4.12) yield the following results (see Appendix A):

$$m_{11} = 1 - \frac{\beta(N-2)(N-1)}{[\sigma-(1-\beta)(N-2)]\,\gamma_1} s^b_{1i}, \tag{4.18}$$

$$s^{bVCE}_{1i} = \frac{\gamma_1}{A}\left[\frac{1+\theta}{\beta(N-2)} + \theta\right][1-\sigma\phi_1], \tag{4.19}$$

$$s^{aVCE}_{1i} = \frac{\theta\gamma_1\left[\frac{1}{\beta(N-2)} + 1 - (N-1)\phi_1\right] - \phi_1}{A\left[\frac{1+\beta(N-2)}{\sigma-(1-\beta)(N-2)} - \phi_i\right]}, \tag{4.20}$$

where

$$A \equiv \gamma_1\theta\sigma\left[\frac{1}{\beta(N-2)} + 1 - (N-1)\phi_1\right] - 1.$$

Then, the bid-ask spread in the $1i$ market is:

$$\left(\frac{s^a_{1i}}{s^b_{1i}}\right)^{VCE} = \frac{1 - \frac{\beta(N-2)}{1+\beta(N-2)}\phi_1\left(N-1+\frac{1}{\theta\gamma_1}\right)}{(1-\sigma\phi_1)\left[1 - \frac{\sigma-(1-\beta)(N-2)}{1+\beta(N-2)}\phi_i\right]}. \tag{4.21}$$

From this, we may establish the following proposition (see Appendix B for a proof):

Proposition 4.1. *The bid-ask spread between currency 1 and all other currencies is lower when currency 1 is the vehicle currency than in an STE. Moreover, the bid-ask spread is increasing in* γ_1.

It is intuitive that the bid-ask spread is lower when all other countries use currency 1 for foreign exchange trading. In a VCE, each peripheral country needs currency 1 to exchange for all other non-vehicle foreign currencies. Thus, the deliveries of all other currencies to currency 1 trading posts must be larger than in an STE.[8] It is also true that the deliveries of currency 1 to each trading post is higher, because both country 1 and all other country residents will wish to exchange currency 1 to purchase other peripheral country goods.

[8]More precisely, the fraction of any currency $i > 1$ supplied to the $1i$ post is equal to $\frac{1+\beta(N-2)}{\sigma-(1-\beta)(N-2)}$, which is greater than that supplied in the STE, i.e. $1/\sigma$.

The high volumes on both sides of each post reduce the average cost of trading and hence reduce the bid-ask spread.

There is a second, more indirect factor reducing the bid-ask spread, however. Because some currency 1 is held by $j > 1$ countries, the normalized price level in country 1 is lower than it would be in the symmetric equilibrium. As a result, the fixed cost of setting up a post in terms of good 1, $p_1 y_1 \phi_1$, is less than ϕ_1. This raises the bid price of all other currencies against currency 1, reducing the bid-ask spread.

While the bid-ask spread is lower in the economy with a vehicle currency, it is raised by country 1 money growth. A rise in country 1 money growth leads to a fall in the fraction of country 1 money balances held by peripheral countries, in a steady state. This leads to a fall in both s_{1i}^a and s_{1i}^b, since country $i > 1$ has a higher marginal propensity to trade its holdings of currency 1 than does country 1. But inspection of conditions (4.16) and (4.17) makes it clear that for an equal rise in m_{11}, the bid exchange rate will fall by more than the ask exchange rate, so the bid-ask spread is widened by higher country 1 money growth.

4.3. Effects of the Vehicle Currency on Efficiency and Resource Allocations

The VCE enhances world efficiency in the sense that, with less resources used up in trading posts, there are more of all goods $i > 1$ available for consumption, and the same amount of good 1. For large N, this efficiency gain can be substantial. On the other hand, the presence of the vehicle currency introduces a fundamental asymmetry into the allocation of world resources. Country 1 occupies a special role as a provider of the vehicle currency. We now analyze these implications of the vehicle currency on efficiency and resource allocations.

Consider the VC country first. Using the expression for m_{11} in (4.18), we can solve for the price level of good 1 as:

$$p_1 = \frac{\gamma_1 \left[\frac{\theta\sigma}{\beta(N-2)} + 1\right] - 1}{A y_1}.$$

Then, from (4.7) and (4.13), we solve country 1's consumption levels as:

$$c_{11}^{VCE} = y_1 \frac{1 + (1 - \frac{1}{\gamma_1})\beta(N-2)\left[1 - (N-1)\phi_1\right]}{\theta\sigma + (1 - \frac{1}{\gamma_1})\beta(N-2)}, \tag{4.22}$$

$$c_{1i}^{VCE} = y_i \frac{1 + (1 - \frac{1}{\gamma_1})\beta(N-2)\left[1 - (N-1)\phi_1\right]}{1 + \beta(N-2)\left[1 - \phi_1(N - 1 + \frac{1}{\theta\gamma_1})\right]} \left[\frac{1 + \beta(N-2)}{\sigma - (1-\beta)(N-2)} - \phi_i\right]. \tag{4.23}$$

To compare these consumption levels with those in the STE, consider the special case where $\gamma_1 = 1$ and $\beta = 1$. In this case, country 1's consumption of good 1 is the same as in the STE. But consumption of good $i > 1$ exceeds that of the STE equilibrium because

$$c_{1i}^{VCE} = \frac{\left[1 - \frac{\sigma}{N-1}\phi_i\right] y_i}{\sigma\left[1 - \frac{N-2}{N-1}\sigma\phi_1\right]} > \frac{1 - \sigma\phi_i}{\sigma} y_i. \tag{4.24}$$

This exceeds consumption under the STE for two reasons. First, the denominator shows that the average costs of a $1i$ trading post is lower, since a larger fraction of currency i is traded at this post. Secondly, the numerator captures the fact that country 1 receives a higher price for its currency s_{1i}^a, than in the STE.

More generally, we establish the following proposition (see Appendix B for a proof):

Proposition 4.2. *Consumption levels of the VC country increase in γ_1. For all $\gamma_1 \geq 1$, welfare of residents of the VC country always exceed those under the STE.*

For the peripheral countries, we can use a similar procedure to calculate consumption. Hence, for $i \neq 1$:

$$c_{ii}^{VCE} = \frac{y_i}{\theta[\sigma - (1-\beta)(N-2)]}, \tag{4.25}$$

$$c_{ij}^{VCE} = \frac{\beta y_j}{\sigma - (1-\beta)(N-2)} \frac{\left(s_{1j}^b / s_{1j}^a\right)^{VCE}}{\gamma_1}, \; j \notin \{i, 1\}, \tag{4.26}$$

$$c_{i1}^{VCE} = \frac{y_1 \theta \left(1 - \sigma\phi_1\right)}{\theta\sigma + \beta(N-2)\left(1 - \frac{1}{\gamma_1}\right)}. \tag{4.27}$$

From (4.25), we find that consumption of the home good is *higher* in the VCE than in the STE, so long as $\beta < 1$. Because consumption of other peripheral country goods requires waiting one period, and there is discounting, consumers will substitute towards the home good.

Consumption of good 1 by country $i > 1$ is at most equal to that under STE, for all $\gamma_1 \geq 1$. If $\gamma_1 = 1$, then $c_{i1}^{VCE} = c_{i1}^{STE}$. But higher country 1 money growth reduces consumption in a vehicle currency equilibrium.

Consumption of other peripheral country's goods may be greater or less than that under STE. In the special case where $\gamma_1 = 1$ and $\beta = 1$, we may express c_{ij} as:

$$c_{ij}^{VCE} = \left(\frac{1 - \sigma\phi_1}{1 - \sigma\frac{N-2}{N-1}\phi_1} \right) \left(1 - \frac{\sigma}{N-1}\phi_j \right) \frac{1}{\sigma}, \quad j \notin \{i, 1\}. \tag{4.28}$$

The peripheral countries face a basic trade-off, evident in (4.28). In order for country i to consume country j's good (where $i \neq j$, $i, j > 1$), the country must first sell its own currency at the $1i$ trading post, and then, in the next period, purchase currency j at the $1j$ trading post. Hence, consumption of other peripheral country goods is subject to the bid-ask spread implied by both these trades. This is reflected in the first expression in (4.28), which is less than unity. On the other hand, relative to the STE, the reduction in the average cost of the $1j$ trading posts tends to increase c_{ij}. This is captured by the second expression in (4.28). The second factor becomes more important, the higher is N. If the cost of setting up a trading post is the same for all posts, then (4.28) implies that $c_{ij}^{VC} > (=)\ c_{ij}^{STE}$ as $N > (=)\ 3$.[9]

Finally, looking back at the general case, it is evident from (4.27) that c_{i1} is decreasing

[9] In the STE, each country bears the cost (in terms of reduced consumption) of setting up the bilateral trading post with all other countries. In the VCE, with $\beta = 1$, and $\gamma_1 = 1$, the same property applies to the consumption of good 1, by all peripheral countries (see (4.27)). But as a consequence of having to pay the full bid-ask spread in order to consume, country i (> 1) bears a disproportionate share of the cost of setting up the trading post $1j$, ($j > 1$), as seen in (4.28). It follows that country 1 bears a less than proportionate share of the cost.

in γ_1. Also, since the bid-ask spread increases in γ_1, then (4.26) implies that c_{ij} decreases in γ_1, where $j \notin \{1, i\}$.

We summarize this discussion in the following proposition:

Proposition 4.3. *(i) If $\beta = 1$, $\gamma_1 = 1$, and $\phi_i = \phi$ for all $i > 0$, then a peripheral country's welfare is higher (the same) in a VCE than in an STE if and only if $N > 3$ ($N = 3$). (ii) For general values of $\beta < 1$, $\gamma_1 < 1$, and ϕ_j, welfare for country $i > 1$ may be higher or lower in the VCE relative to the STE. (iii) A peripheral country's consumption of foreign goods decreases in the VC money growth rate.*

The vehicle currency gives the center country three distinct advantages. First, it receives a better return in trade than the peripheral countries, because it needs to trade only once in order to consume, and because there is more of all other currencies supplied for trade with country 1 than in the symmetric trading equilibrium. Secondly, relative to all other countries, country 1 consumers do not have to wait one period in order to obtain the currency required to consume the goods of other non-vehicle currency countries. Finally, residents of non-vehicle countries lose from country 1 money growth, since they must hold currency 1 across time. Conversely, country 1 residents gain from country 1 money growth.

For a peripheral country, the fundamental trade-off is between the gains from greater efficiency in foreign exchange trading, relative to the losses from reduced terms of trade, delayed consumption, and center country money growth. Note that a key aspect of the vehicle currency equilibrium is that money is not neutral. Center country money growth determines the distribution of gains of moving to the vehicle currency equilibrium.

Figure 1 shows the gains to peripheral countries (in terms of percentage of permanent consumption) as a function of the number of countries. In this Figure, we set $\beta = .99$, $\gamma_1 = 1$, $\phi_i = .01$ for all i, and $\theta = 1$. Hence there is a small degree of discounting in preferences, country 1 money growth is zero, the real cost of setting up a trading post is one percent of the endowment, and finally, there is no home bias in preferences.

In this case, for $N = 3$, peripheral countries are worse off than STE, barely. But the gains are increasing in N, because the efficiency gains from fewer trading posts increase in N.

A rise in γ_1 however, still shrinks these gains. Figure 2 shows that, for a center country money growth rate equal to 5 percent, peripheral countries will lose under VCE relative to STE, for N less than 10.

An interesting feature that arises in Figure 2 is that the effect of N on the gains from a vehicle currency may be non-monotonic. When $\gamma_1 > 0$, increasing N initially leads the peripheral country to lose, relative to STE. As N continues to increase, this effect is reversed, and welfare is higher under VCE relative to STE. Intuitively, as we increase the number of countries, in the case $\theta = 1$, each country becomes more open, since in this case preferences are assumed equally weighted towards all country's goods. Hence, each peripheral country is more exposed to country 1 money growth. Thus, the losses from adopting a vehicle currency tends to rise, as N increases. Offsetting this however, is the fundamental efficiency of a vehicle currency, leading to a greater welfare gains, the greater the number of bilateral trading posts that are closed down by its adoption. For small N, the first force tends to dominate, and increasing numbers tends to reduce the gains to a vehicle currency. For larger N, the second force is predominant, and the gains to a vehicle currency begin to rise and become positive.

An alternative parameterization is shown in Figure 3. There, we assume that $\theta = 1/(N-1)$, which ensures that the share of the budget spent on foreign goods remains at 0.5, whatever the number of countries. In this case, the non-monotonic characteristic of the gains from a vehicle currency is much less pronounced, since intuitively, increasing the number of countries does not increase the exposure to currency 1 inflation as much as the case where $\theta = 1$.

5. Robustness of the Vehicle Currency Equilibrium

In addition to the STE and the VCE analyzed in previous sections, there are many other equilibria in the model. For example, other currencies can also be the vehicle currency. Such multiplicity is inevitable when there are fixed costs of organizing the currency exchange. Moreover, much of the multiplicity is robust to the refinements of trembling hands by a small measure of agents or of evolutionary stability.[10] Such robustness illustrates the fact that, once a currency has established itself as the vehicle currency, a large disturbance is needed to dethrone it.

In this section we examine whether the VC equilibrium is robust to the following two deviations by a large number of agents. The first is a deviation by all households in two countries to trading their currencies directly. We call this deviation a bilateral deviation. The second is a deviation by all households in all peripheral countries to using a different currency as the vehicle currency. We show that, for a vehicle currency to survive these deviations, its inflation rate and its fixed trading cost cannot be too high.

Let us first consider a bilateral deviation by two countries, say, country 2 and country 3. Suppose that all households in the two countries deviate to trading the two currencies directly. Other countries do not participate in the 23 post. Moreover, countries 2 and 3 still supply their domestic currencies to trade for currency 1 and use currency 1 to get other peripheral currencies. However, country 2 does not use currency 1 to buy currency 3, and country 3 does not use currency 1 to buy currency 2.

[10] For example, if a small measure of agents from any two countries exchange their domestic currencies directly in the VCE constructed above, they will make a loss as the amount of currencies brought into that post will not be sufficient to cover the fixed trading cost. Similarly, if a small measure of agents deviate to using a different currency as the vehicle currency, they will make a loss.

Denote $I = \{1,2,3\}$. For a country $i \notin \{2,3\}$, the decision problem is the same as in the VCE characterized in the previous section, because all currency posts which the country participated before are still active after the above deviation. Since the decision problems of a household in country 2 and of a household in country 3 are similarly, we only formulate the problem for country 2.

With the deviation, a household in country 2 faces the following constraints involving currencies 1, 2 and 3:

$$m'_{22} = m_{22} - f^{12}_{22} - f^{23}_{22}, \quad m'_{23} = m_{23} + \frac{1}{s^a_{23}} f^{23}_{22},$$
$$m'_{2j} = m_{2j} + \frac{1}{s^a_{1j}} f^{1j}_{21},\ j \notin I, \quad m'_{21} = m_{21} - \sum_{j \notin I} f^{1j}_{21} + s^b_{12} f^{12}_{22},$$
$$\sum_{j \notin I} f^{1j}_{21} \leq m_{21}.$$

Other constraints that the household faces, such as the cash in advance constraints in the goods markets, are the same as those in the previous section.

Because country 2 still needs currency 1 to exchange for other currencies, the cash in advance constraint on currency 1 in the goods market does not bind for country 2, as in the previous section. All other cash in advance constraints bind. Then, the household's optimal choices yield:

$$\theta p_2 c_{22} = \frac{1}{s^b_{12}} p_1 c_{21} = s^a_{23} p_3 c_{23} = \frac{\gamma_{1(+1)} s^a_{1j(+1)}}{\beta s^b_{12}} p_{j(+1)} c_{2j(+1)},\ j \notin I.$$

As before, $m_{22} = m_2 = 1$, $m_{j2} = 0$ $(j \neq 2)$, and $p_2 = 1/y_2$. Adding up country 2's spending of currency 2 and substituting the first-order conditions for c yields:

$$c_{22} = \frac{y_2}{\theta[\sigma - (1-\beta)(N-3)]}.$$

The household's consumption levels of other goods can be calculated accordingly. Also, for $j \notin I$, the household's optimal decisions on the quantities of currency trade yield:

$$f^{1j}_{21} = \frac{\beta}{\sigma - (1-\beta)(N-3)} \left(\frac{s^b_{12(-1)}}{\gamma_1} \right), \tag{5.1}$$

$$m_{21} = \sum_{j \notin I} f_{21}^{1j} = \frac{\beta(N-3)}{\sigma - (1-\beta)(N-3)} \left(\frac{s_{12(-1)}^b}{\gamma_1} \right). \tag{5.2}$$

$$f_{22}^{23} = \frac{1}{\sigma - (1-\beta)(N-3)}, \quad f_{22}^{12} = \frac{1+\beta(N-3)}{\sigma - (1-\beta)(N-3)}.$$

At the 23 post, bid/ask prices satisfy $f_{22}^{23}/s_{23}^a = f_{33}^{23} - \phi_3$ and $s_{23}^b f_{33}^{23} = f_{22}^{23} - \phi_2$. The solutions are:

$$s_{23}^b = 1 - [\sigma - (1-\beta)(N-3)]\, \phi_2, \tag{5.3}$$

$$s_{23}^a = \frac{1}{1 - [\sigma - (1-\beta)(N-3)]\, \phi_3}. \tag{5.4}$$

The bid-ask spread at the 23 post is smaller than that in the STE, provided $N > 3$. This is because, when $\beta < 1$, countries 2 and 3 will assign a higher fraction of their budget to each other's good that they will to other periperal country goods, given that the consumption of those other goods requires a one-period delay in consumption.

In the analysis below, $j \notin I$ unless it is specified otherwise. To compute exchange rates at the 12 post and the 13 post after the deviation by countries 2 and 3, we count the total amount of currency 1 that is held by the peripheral countries at the beginning of a period as follows:

$$1 - m_{11} = m_{21} + m_{31} + \sum_{j \notin I} m_{j1} = 2m_{21} + (N-3)m_{j1}.$$

The second equality comes from the fact that $m_{31} = m_{21}$ and that m_{j1} is the same for all $j \notin I$. Substituting (5.2) for m_{21} and (4.10) for m_{j1} yields:

$$1 - m_{11} = \frac{2\beta(N-3)s_{12}^b/\gamma_1}{\sigma - (1-\beta)(N-3)} + \frac{\beta(N-2)(N-3)s_{1j}^b/\gamma_1}{\sigma - (1-\beta)(N-2)}. \tag{5.5}$$

With modification of m_{11}, a country 1 household's optimal choices of consumption are still given by (4.11) and (4.13), the amount of currency trade by (4.14), and the price level of good 1 by (4.15).

At the 12 post, bid/ask prices satisfy the following conditions:

$$\frac{1}{s^a_{12}}\left(f^{12}_{11}+\sum_{j\notin I} f^{12}_{j1}\right)=f^{12}_{22}-\phi_2 \tag{5.6}$$

$$s^b_{12}f^{12}_{22}=f^{12}_{11}+\sum_{j\notin I} f^{12}_{j1}-p_1y_1\phi_1. \tag{5.7}$$

At the 13 post, the conditions are similar. At the $1j$ post ($j\notin I$), the conditions are:

$$\frac{1}{s^a_{1j}}\left(f^{1j}_{11}+2f^{1j}_{21}+\sum_{j\notin I\cup\{j\}} f^{1j}_{i1}\right)=f^{1j}_{jj}-\phi_j \tag{5.8}$$

$$s^b_{1j}f^{1j}_{jj}=f^{1j}_{11}+2f^{1j}_{21}+\sum_{j\notin I\cup\{j\}} f^{1j}_{i1}-p_1y_1\phi_1. \tag{5.9}$$

Here we used the fact that $f^{1j}_{21}=f^{1j}_{31}$. These equations solve for the exchange rate at each post involving currency 1 and the solutions are provided in Appendix C.

To see whether the deviation is profitable to countries 2 and 3, let us compare the direct exchange of currency 2 for currency 3 and the indirect exchange through the vehicle currency. With the direct exchange, a household in country 2 gets $1/s^a_{23}$ units of currency 3 for each unit of currency 2. With the indirect exchange, one unit of currency 2 returns s^b_{12} units of currency 1 in the current period, which the household can use to exchange for $s^b_{12}/s^a_{13(+1)}$ next period. Taking into account time discounting and the inflation in currency 1 between the two periods, the effective number of units of currency 3 that one unit of currency 2 can exchange for in the VCE is $\beta s^b_{12}/[\gamma_1 s^a_{13(+1)}]$. In the steady state, $s^a_{13(+1)}=s^a_{13}$, and so the indirect exchange through the vehicle currency gives a higher payoff to a household in country 2 than the direct exchange if and only if $\beta s^b_{12}/[\gamma_1 s^a_{13}]>1/s^a_{23}$, that is, if and only if $\gamma_1/\beta<s^a_{23}s^b_{12}/s^a_{13}$.

Again, let's look at the special case where γ_1 and β approach one. In this case, (4.21) and (5.4) imply:

$$s^a_{23}\frac{s^b_{12}}{s^a_{13}}=\left(\frac{1-\sigma\phi_1}{1-\sigma\phi_3}\right)\left[\frac{N-1-\sigma\phi_3}{N-1-(N-2)\sigma\phi_1}\right]. \tag{5.10}$$

The indirect exchange is better for country 2 than the direct exchange iff $s^a_{23}s^b_{12}/s^a_{13} > 1$. We can express this condition as follows:

$$\phi_1 < \frac{(N-2)\phi_3}{(N-3)\sigma\phi_3 + 1}. \tag{5.11}$$

Under this condition, the deviation described above makes country 2 households worse off.[11] Similarly, the deviation makes country 3 worse off if (5.11), with ϕ_3 being replaced with ϕ_2, is satisfied.

The following proposition extends the above analysis to general values of γ_1 (see Appendix C for a proof):

Proposition 5.1. *Assume $\phi_1 < \bar{\phi}$, where $\bar{\phi}$ is defined as*

$$\bar{\phi} = min_{i \neq 1} \left\{ \frac{(N-2)\phi_i}{(N-3)\sigma\phi_i + 1} \right\}. \tag{5.12}$$

Also assume that β is sufficiently close to 1. Then there exists $\bar{\gamma}_1 \in (1, \infty)$ such that, iff $\gamma_1 < \bar{\gamma}_1$, the VC equilibrium is robust to joint deviations by all households in any two countries to a direct exchange of the two countries' currencies.

This proposition is intuitive. High growth rates of the vehicle currency reduce the peripheral countries' consumption. When this money growth rate passes a critical level, the loss to the peripheral countries exceeds the gain from the economy of scale of using a vehicle currency in the exchange market. In this case, the peripheral countries can be better off by trading currencies bilaterally. Because the center country will lose when its currency ceases to be a vehicle currency, the potential deviation by the peripheral countries puts an upper bound on the money growth of the vehicle currency.

[11] In fact, we can show that, in the special case where β and γ_1 approach 1, the deviation does not change country 2's consumption level of any good except good 3. That is, $c_{2j} = (c_{2j})_{VC}$ for all $j \neq 3$, where the subscripts VC indicate the levels in the VCE before the deviation. However, the deviation does change country 2's consumption level of good 3. It can be verified that the ratio of consumption of good 3 by a country 2 household in the VCE relative to the level after the deviation, $\frac{(c_{23})_{VC}}{c_{23}}$, is equal to $s^a_{23}\frac{s^b_{12}}{s^a_{13}}$ given by (5.10). Thus, under (5.11), the deviation makes country 2 worse off.

The condition $\phi_1 < \bar{\phi}$ is not very stringent, under the maintained assumption $\sigma\phi_i < 1$ for all i. For example, if $\phi_i = \phi$ for all $i \neq 1$, then $\bar{\phi} = \phi$ if $N = 3$ and $\bar{\phi} > \phi$ if $N > 3$. Thus, for all $N > 3$, the condition $\phi_1 < \bar{\phi}$ is satisfied if all currencies have the same trading cost. The condition can be satisfied even if the vehicle currency has a higher cost than other currencies.

Figure 4 describes the utility gains from a deviation by country 2 (or country 3) using the same calibration as in Figures 1-2, as a function of the country 1 rate (gross) inflation rate γ_1. With $\gamma_1 = 1$, there is no gain to deviating, as we have shown in Proposition 5.1. But as inflation rises, the utility gain to a deviation by any two countries increases. As should be clear from the previous section, the gains to deviating are inversely related to the number of countries N. With $N = 5$, a deviation is beneficial even for very very small rates of inflation. But with $N = 20$, there is no incentive to deviate unless inflation rates exceed 10 percent.

Now consider the deviation by which all peripheral countries choose any currency $k \neq 1$ as the new vehicle currency. After this deviation, consumption levels for a country $i \notin \{k, 1\}$ are given by the same formula as those in the original VCE, with the modification that (ϕ_1, γ_1) are replaced with (ϕ_k, γ_k). Because these consumption levels are decreasing functions of the money growth rate of the vehicle currency, then in the case $\phi_j = \phi_1$ for all j, the deviation reduces country i's utility if and only if $\gamma_k > \gamma_1$. This result can be extended as the following proposition:

Proposition 5.2. *Assume $\phi_1 \leq min_k\{\phi_k\}$. If $\gamma_1 \leq min_k\{\gamma_k\}$, then currency 1 is a robust vehicle currency with respect to the deviation by which all $(N-1)$ peripheral countries switch to use another currency as the new vehicle currency.*

Propositions 5.1 and 5.2 have shown that a currency can serve as a more robust vehicle currency if its inflation rate is lower and if its trading cost is lower.

6. A Vehicle Currency with Two Rounds of Currency Trading per Period

In the Vehicle Currency Equilibrium of section 4, peripheral countries must carry amounts of the vehicle currency from one period to the next, exposing them to losses from vehicle currency inflation, as well as from time discounting. We now allow for two rounds of currency trading in every period. This means that peripheral countries can avoid carrying over the vehicle currency across periods, eliminating both of these welfare losses. Nevertheless, they still cannot avoid the need to 'trade twice' in order to consume other peripheral country goods.

Now let the currency trading session within each period be broken into two rounds; A and B. There are a number of ways to allow currency trading within each round, and for each country. In any configuration, country $i > 1$ must sell currency i in the first round of trading. But it may wish to sell more of its own currency in the second round, in order to finance purchases of good 1. Its choices will depend on equilibrium exchange rates. Country 1 has more options. It needs to buy currencies $j > 1$, but it could do this in the first round or the second round. Country 1 is not obliged to participate in both rounds. Again its choices will depend on the exchange rate.

In equilibrium, the choices of country 1 and the peripheral countries must be consistent with one another. Hence, it is clear that in any configuration of equilibrium, country 1 must sell currency 1 in the first round, if each country $i > 1$ is to have money to finance consumption of goods $j \neq i, 1$ in the second round. Similarly either country 1 or peripheral countries must offer all currencies $j \neq 1$ for sale in round B, so that other peripheral countries can use currency 1 to purchase these currencies. This still allows for a number of possible trading equilibria. In Appendix D, we discuss alternative possibilities. Here however, we focus on an equilibrium trading configuration which has the following characteristics; a) in Round A, each peripheral country i ($\neq 1$) trades currency i for currency 1 (i.e. the vehicle currency), and b) in round B, country i sells currency 1 for currency j ($\neq i$), and again sells currency i for currency 1, and c) country 1 purchases all currencies $i \neq 1$ in round A, and sells some of these currencies in round B. In fact, this trading pattern offers the best chance for the peripheral countries to gain from the vehicle currency, relative to other configurations of two-round trading (see Appendix D).

The changes in the constraints facing a household in country 1, relative to the one-round trading environment, are as follows: (Note that I deleted four constraints here because three repeat the ones in the one-round trading and the other one does not bind.)

$$m'_{11} \leq m_{11} - \sum_{j \neq 1} f_{11}^{1jA} + \sum_{j \neq 1} s_{1j}^{bB} f_{1j}^{1jB}, \tag{6.1}$$

$$m'_{1j} \leq m_{1j} + \frac{1}{s_{1j}^{aA}} f_{11}^{1jA} - f_{1j}^{1jB}, \text{ all } j \neq 1. \tag{6.2}$$

The superscripts A and B indicate the round of trade. The constraint (6.1) replaces (3.3) with $i = 1$. The new element here is the the last term in the constraint, which is the sum of currency 1 that the household obtains in the second round by selling peripheral currencies obtained in the first round. Similarly, (6.2) replaces (3.4) and (3.5) with $i = 1$. The household continues to face the constraints (3.1), (3.2) and (3.6), with $i = 1$.

In the two rounds described above, a household in country 1 delivers f_{11}^{1jA} to the $1j$ trading post in round A, and then returns f_{1j}^{1jB} back to the $1j$ post in round B. For this to be rational, it must be that $s_{1j}^{aA} \leq s_{1j}^{bB}$. Otherwise, country 1 residents would never hold currency j in excess of their consumption needs (i.e. $f_{1j}^{1jB} = 0$ would hold). If $s_{1j}^{aA} < s_{1j}^{bB}$, then country 1 residents would like to purchase as much currency j as possible in the first round of trading, constrained only by their initial money balances. However, it can be shown that this cannot be an equilibrium in the model, because if country 1 sold all its initial holdings of currency 1 in the first round, then $s_{1j}^{aA} > s_{1j}^{bB}$ would obtain. Hence, we focus attention on the case where $s_{1j}^{aA} = s_{1j}^{bB}$. In this case, country 1 is indifferent between entering the currency market in Round B and not entering. Then f_{1j}^{1jB} is determined residually from the equilibrium equations.

In this equilibrium, country 1's optimality conditions satisfy:

$$m_{11} = 1, \quad f_{1j}^{1jB} = \frac{1}{s_{1j}^{bB}} \left(f_{11}^{1jA} - \frac{1}{\sigma} \right), \tag{6.3}$$

$$p_1 c_{11} = \frac{s_{1j}^{aA}}{\theta} p_j c_{1j} = \frac{1}{\theta\sigma}, \quad j > 1. \tag{6.4}$$

Here, as before, all cash in advance constraints in the goods market facing a country 1's household bind. This means that $m_{1j} = 0$, so that, by (6.2), f_{1j}^{1jB} is determined residually by the currency j that is not used for purchases of good j. In trading off consumption between goods 1 and good j, country 1 faces the round A ask price for currency j.

For country $i \neq 1$, currency i must be traded for currency 1 in the first Round, so as to ensure that consumption of other peripheral goods is positive. But the country may or may not bring currency i to the $1i$ post in Round B. If $s_{1i}^{bA} < s_{1i}^{bB}$, then country i will be better off participating in Round B, selling currency i in return for currency 1 that can finance the purchase of country 1 goods. We may illustrate the constraints on country $i \neq 1$ as follows:

$$m'_{ii} \leq m_{ii} - f_{ii}^{1iA} - f_{ii}^{1iB}, \tag{6.5}$$

$$m'_{ij} \leq m_{ij} + \frac{1}{s_{1j}^{aB}} f_{i1}^{1jB}, \text{ all } j \neq i, 1, \tag{6.6}$$

$$m'_{i1} \leq \left(m_{i1} + s_{1i}^{bA} f_{ii}^{1iA} - \sum_{j \neq 1, i} f_{i1}^{1jB} \right) + s_{1i}^{bB} f_{ii}^{1iB}, \tag{6.7}$$

$$0 \leq m_{i1} + s_{1i}^{bA} f_{ii}^{1iA} - \sum_{j \neq 1,i} f_{i1}^{1jB}. \tag{6.8}$$

These conditions are analogous to (4.1)-(4.4), except allowing for two rounds of trade. The condition (6.8) states that currency 1 spent in Round B by a country i cannot exceed the initial holdings of currency 1 plus the amount of currency 1 that the country acquired in Round A. If $s_{1i}^{bA} < s_{1i}^{bB}$, this constraint binds.

Because a household in a peripheral country i can now obtain other peripheral currencies in one period by going through the two rounds of trade, it is not necessary for the household to hold the vehicle currency from one period to the next. In fact, it is not optimal to do so, provided that (gross) rates of growth of these currencies exceed the discount factor. This implies that a household always spends all foreign currencies on goods before a period ends. That is, all cash in advance constraints in the goods market now bind including the one involving the vehicle currency.

Assume $s_{1i}^{bA} < s_{1i}^{bB}$, which will be shown to hold. The optimality choices of a country $i \neq 1$ household generate the following results:

$$\frac{1}{\theta s_{1i}^{bB}} p_1 c_{i1} = \frac{s_{1j}^{aB}}{\theta s_{1i}^{bA}} p_j c_{ij} = p_i c_{ii} = \frac{1}{\theta\sigma},\ j \neq 1, i, \tag{6.9}$$

$$f_{ii}^{1iB} = \frac{1}{\sigma}, \qquad f_{i1}^{1jB} = \frac{s_{1i}^{bA}}{\sigma},\ j \neq 1, i, \tag{6.10}$$

$$f_{ii}^{1iA} = \frac{1}{s_{1i}^{bA}} \sum_{j \neq 1,i} f_{i1}^{1jB} = \frac{N-2}{\sigma}. \tag{6.11}$$

As in the VCE, $m_{ii} = 1$. Also, country i ($\neq 1$) must face the bid price at trading post $1i$ and the ask price at post $1j$ in order to consume good j, ($j \neq i, 1$). But now both trades take place within the period.

Trading posts are active across two rounds. In round A of post $1j$, s_{1j}^{aA} and s_{1j}^{bA} are determined by:

$$s_{1j}^{aA}\left(f_{jj}^{1jA} - \phi_j\right) = f_{11}^{1jA}, \quad s_{1j}^{bA} f_{jj}^{1jA} = f_{11}^{1jA} - \phi_1. \tag{6.12}$$

In Round B, the $1j$ trading post yields:

$$s_{1j}^{aB}\left(f_{jj}^{1jB}+f_{1j}^{1jB}-\phi_j\right)=\sum_{i\neq 1,j} f_{i1}^{1jB}, \tag{6.13}$$

$$s_{1j}^{bB}\left(f_{jj}^{1jB}+f_{1j}^{1jB}\right)=\sum_{i\neq 1,j} f_{i1}^{1jB}-\phi_1. \tag{6.14}$$

In Appendix D, we solve the exchange rates as follows:

$$s_{1j}^{bA}=(1-2\sigma\phi_1)\left(1-\frac{\sigma}{N-2}\phi_j\right)-\frac{\sigma}{N-2}\phi_1, \tag{6.15}$$

$$s_{1j}^{aA}=s_{1j}^{bB}=1-2\sigma\phi_1, \tag{6.16}$$

$$s_{1j}^{aB}=\frac{(1-2\sigma\phi_1)(\frac{N-2}{\sigma}-\phi_j)-\phi_1}{\frac{N-1}{\sigma}-\frac{1}{\sigma(1-2\sigma\phi_1)}-2\phi_j}. \tag{6.17}$$

These solutions are enough to determine consumption rates for all countries. In particular, because all cash in advance constraints bind, consumption of the local good is identical to that under the STE:

$$c_{ii}=\frac{y_i}{\theta\sigma}.$$

For country 1, consumption of periphery goods is determined by s_{1j}^{bB} $(=s_{1j}^{aA})$:

$$c_{1j}=\frac{y_i}{s_{1j}^{bB}\sigma}=\left(1+\frac{2\sigma\phi_1}{1-2\sigma\phi_1}\right)\frac{y_j}{\sigma},\quad j>1. \tag{6.18}$$

For the peripheral countries, consumption of good 1 is determined simply by s_{1j}^{bB}:

$$c_{i1}=\frac{s_{1i}^{bB}}{\sigma}y_1=(1-2\sigma\phi_1)\frac{y_1}{\sigma},\quad i>1. \tag{6.19}$$

Consumption of good j $(\neq i,\, 1)$, however, is constrained by the bid-ask spread. As before, assume that $\phi_i=\phi_j$, $i,j>1$. Then,

$$c_{ij}=\frac{s_{1i}^{bA}}{s_{1j}^{aB}}\frac{y_j}{\sigma}=\left(1-\frac{2\sigma\phi_1}{(N-2)(1-2\sigma\phi_1)}-\frac{2\sigma\phi_j}{N-2}\right)\frac{y_j}{\sigma}. \tag{6.20}$$

We may explain these expressions as follows. The peripheral countries consumption of good 1 is determined by the bid price of currency $i > 1$. Because currency 1 is no longer held across periods, there is no inflation or discounting effects in (6.19). As in the STE, country i bears the full good 1 cost of setting up the $1i$ trading post. Because there are *two* rounds of currency trading, the costs of trading are higher, and c_{i1} is actually lower here than under the STE, and hence lower than under the one-round vehicle currency equilibrium of the last section, in the absence of discounting and money growth.

Consumption of other periphery goods is affected in two ways. First, given reduced average costs of trading posts (the third term inside parenthesis in (6.20)), the trading cost borne in consumption is now only $\frac{2\sigma\phi_j}{N-2}$ rather than $2\sigma\phi_j$, leading to an increase in c_{ij}. But because it incurs the bid-ask spread across the $1i$ and $1j$ trading posts, country i bears additional costs relative to the STE (the second term inside parentheses in (6.20)). Note that, in the two-round vehicle currency case, these costs represent an effective transfer to country 1, as seen by a comparison on (6.18) and (6.20).

How does the two round of trading case compare in welfare terms to the STE? As in the one round VCE case, country 1 is better off in the two round VCE, relative to STE. Again, as in the one round case with no discounting and zero money growth, it consumes an equal amount of its own good, but more of all other goods, relative to STE. But the peripheral countries again are faced with a trade off. The reduced average costs of trading posts offers a welfare gain, but there is a loss from having to trade twice in the currency trading posts. In addition there is an additional cost from having more rounds of trade within a period. We may establish the following proposition (see Appendix D for a proof)

Proposition 6.1. *With two rounds of currency trade in each period, peripheral countries are better of in a VCE relative to STE only if $N \geq 7$.*

In the equilibrium with two rounds of currency trade, it requires at least 7 countries to ensure gains from a vehicle currency, for peripheral countries. With two rounds of trade, peripheral countries face a less advantageous relative price of currency, because relative to the VCE of the previous section, in round A, currency 1 is now supplied only by country 1. This leads to a much lower round A bid price for currency $j > 1$, reducing the bid-ask spread facing these countries.

Note that the equilibrium under two rounds of trade is independent of country 1 money growth and the subjective discount factor. Peripheral countries are not exposed to country 1 inflation, since they no longer hold currency 1 across periods. Nor do they bear a welfare cost of waiting. But they suffer in a different way; from much reduced terms of trade (as well as costs of more trading posts). In general, they tend to be worse off in a steady state with two rounds of trading relative to the VCE of the previous section.

7. Conclusions

This paper has developed a model in which a globally acceptable currency can arise endogenously as a medium of exchange among countries, facilitating international trade, and economizing on resources when trading currencies requires costly transactions technologies. By eliminating the need to set up bilateral currency trading posts among all possible countries, a vehicle country reduces the average cost of currency trade. But the cost savings are distributed unevenly, with the center country gaining disproportionally. With a small number of countries, peripheral countries will be worse off with a vehicle currency relative to a symmetric trading equilibrium. But the gains from a vehicle currency may be substantial when there are a large number of countries and currencies. Even with many countries, however, these gains are eroded by higher rates of inflation in the center country. If inflation in the center country goes to high, then our robustness analysis suggests that the use of the vehicle currency will collapse.

The model could be extended in a number of ways. We could allow for uncertainty in money growth and output levels. In this case, the risk-hedging properties of a vehicle currency would be important, in addition to its exchange use. We could do a more explicit welfare analysis of monetary policy, assuming a social planner that weights each countries utility and can make monetary transfers across countries. We leave these issues for future research.

8. Appendix

A. Derivations for Sections 3 and 4

First, we derive (3.7) and (3.8). Let the current-value Lagrangian multiplier be λ_{ii} for (3.1), λ_{ij} for (3.2), η_{ii} for (3.3), η_{ij} for (3.4) and (3.5), and ψ_{ij} for (3.6). Define $\theta_{ij} = \theta$ if $j \neq i$ and $\theta_{ij} = 1$ if $j = i$. With the logarithmic utility function, the first-order conditions for c_{ij} and m'_{ij} yield the following result for all i and j:

$$\frac{\theta_{ij}}{p_j c_{ij}} = \frac{\beta}{\gamma_{j(+1)}} \lambda_{ij(+1)} + \psi_{ij} = \eta_{ij}, \tag{A.1}$$

where the subscript +1 indicates the next period. The first-order conditions for f_{ii}^{ij} $(i < j)$ and f_{ii}^{ji} $(i > j)$ yield:

$$\eta_{ii} = \frac{1}{s_{ij}^a} \eta_{ij} \ (i < j); \quad \eta_{ii} = s_{ji}^b \eta_{ij}. \tag{A.2}$$

Dividing (A.1) for $j \neq i$ by the condition for $j = i$, and using (A.2), we obtain (3.7) and (3.8).

Second, we derive (4.5) and (4.6). Let the current-value Lagrangian multiplier be η_{ii} for (4.1), η_{ij} for (4.2), η_{i1} for (4.3), and μ_{i1} for (4.4). As in the STE, the multiplier is λ_{ii} for (3.1), λ_{ij} for (3.2), and ψ_{ij} for (3.6). It is easy to verify that the first-order conditions for c_{ij} and m'_{ij} yield the same result, (A.1), as in the STE. The first-order conditions for f_{ii}^{1i} and f_{i1}^{1j} are as follows:

$$\eta_{ii} = s_{1i}^b \eta_{i1}, \quad i \neq 1, \tag{A.3}$$

$$\eta_{i1} + \mu_{i1} = \eta_{ij} / s_{1j}^a, \quad j \neq i, 1. \tag{A.4}$$

The envelope conditions for m_{ij} are:

$$\lambda_{ij} = \eta_{ij} \ (j \neq 1); \qquad \lambda_{i1} = \eta_{i1} + \mu_{i1}. \tag{A.5}$$

Substituting the last condition into (A.4) yields $\eta_{ij} = s_{1j}^a \lambda_{i1}$ for all $j \neq i, 1$. Dividing (A.1) for $j = 1$ by (A.1) for $j = 1$, and using (A.3), we obtain (4.5).

To establish (4.6), we show that $\psi_{i1} = 0$ for all $i \neq 1$. Suppose, to the contrary, that $\psi_{i1} > 0$. Then, $m'_{i1} = p_1 c_{i1}$, and so $m_{i1(+1)} = 0$ by (3.2). With (4.4), this further implies $f_{i1(+1)}^{1j} = 0$ for all $j \neq i$. That is, the household will have no foreign currency in the next period. As a result, consumption of foreign goods will be zero. This is not optimal since the marginal utility of such consumption is infinite when consumption is zero.

Since $\psi_{i1} = 0$, (A.1) implies $\lambda_{i1(+1)} = \eta_{i1} \gamma_{1(+1)} / \beta$. Then, for all $j \neq i, 1$, we have:

$$\eta_{ij(+1)} = s_{1j(+1)}^a \lambda_{i1(+1)} = \frac{\gamma_{1(+1)}}{\beta} s_{1j(+1)}^a \eta_{i1} = \frac{\gamma_{1(+1)}}{\beta} \left(\frac{s_{1j(+1)}^a}{s_{1i}^b} \right).$$

The first equality comes from a result derived above, the second equality is obvious, and the last equality comes from (A.3). Now, dividing (A.1) for $j \neq i, 1$ in the next period by (A.1) for $j = i$ in the current period, and using the above condition, we get (4.6).

Third, we derive the results (4.7) – (4.10). To do so, consider a household in a country $i \neq 1$. Notice that the household spends the domestic currency in the current period to acquire currency 1 and to purchase domestic good. Part of currency 1 that the household acquires today is spent on good 1. The rest will be spent in *the next period* to purchase other peripheral currencies which, in turn, will be spent on goods of these peripheral countries. Thus, the household's holdings of domestic currency at the beginning of the period are equal to the sum of current expenditures on good 1 and domestic good and expenditures in the next period on goods of other peripheral countries. This constraint is as follows:

$$1 = m_{ii} = p_i c_{ii} + \frac{1}{s^b_{1i}} p_1 c_{i1} + \frac{\gamma_{1(+1)}}{s^b_{1i}} \sum_{j \notin \{i,1\}} s^a_{1j(+1)} p_{j(+1)} c_{ij(+1)}. \tag{A.6}$$

Substituting 4.5) and (4.6), we obtain (4.7). The result (4.8) comes from the fact that the household spends all domestic currency on domestic goods and on acquiring the vehicle currency. The result (4.9) comes from the constraint $f^{1j}_{i1} = s^a_{1j} m'_{ij} = s^a_{1j} p_j c_{ij}$ for $j \neq i, 1$. The result (4.10) comes from (4.4).

Finally, we derive (4.18), (4.19) and (4.20). Substituting f and p_1 from the (4.8), (4.9), (4.10), and (4.15) into (4.16) and (4.17), we get:

$$s^a_{1i} \left[\frac{1 + \beta(N-2)}{\sigma - (1-\beta)(N-2)} - \phi_i \right] = \frac{m_{11}}{\sigma} + \frac{1 - m_{11}}{N-1},$$

$$s^b_{1i} \left[\frac{1 + \beta(N-2)}{\sigma - (1-\beta)(N-2)} \right] = \frac{m_{11}}{\sigma} + \frac{1 - m_{11}}{N-1} - \left[1 - \gamma_{1(+1)}(1 - m_{11(+1)}) \right] \phi_1.$$

Notice that the second condition implies that s^b_{1i} is independent of i. Thus, (4.12) can be simplified to (4.18). Substituting m_{11} from (4.18) into the above two equations and focusing on the steady state, we obtain (4.19) and (4.20).

B. Proofs of Propositions 4.1 and 4.2

For Proposition 4.1, note first that the ratio s^a_{1i}/s^b_{1i}, given by (4.21), is increasing in γ_1. Because $N - 2 > 0$, then the numerator in (4.21) is less than one and

$$\frac{\sigma - (1-\beta)(N-2)}{1 + \beta(N-2)} < \sigma.$$

Thus, $s^a_{1i}/s^b_{1i} < [(1 - \sigma\phi_1)(1 - \sigma\phi_i)]^{-1}$, the latter of which is the bid-ask spread in the STE.

For Proposition 4.2, it is straightforward to show from (4.22) and (4.23) that c_{1i} is strictly increasing in γ_1 for all i under the maintained assumption that $max_i \sigma\phi_i < 1$.

Thus, a sufficient condition for the VC country's consumption levels to be higher in the VCE than in the STE is that they are so in the case $\gamma_1 = 1$. When $\gamma_1 = 1$, we have $c_{11} = y_1/(\theta\sigma)$, which is equal to the level in the STE. However, when $\gamma_1 = 1$, c_{1i} $(i \neq 1)$ is given as follows:

$$c_{1i} = \frac{\left[1 - \frac{\sigma-(1-\beta)(N-2)}{1+\beta(N-2)}\phi_i\right] y_i}{[\sigma - (1-\beta)(N-2)]\left[1 - \frac{\beta(N-2)}{1+\beta(N-2)}\sigma\phi_1\right]}.$$

Since the numerator is greater than $(1 - \sigma\phi_i)y_i$ and the denominator is less than σ, the above consumption level is greater than that in the STE. Therefore, the VC country's welfare is higher in the VCE than in the STE.

C. Proofs for Section 5

We derive bid/ask prices at each post that involves currency 1. Denote:

$$\xi = \frac{1 + \beta(N-2) + \beta/\gamma_1}{1 + \beta(N-3) + 2\beta/\gamma_1},$$

$$\widehat{s}_{12}^z = \frac{\sigma s_{12}^z}{\sigma - (1-\beta)(N-3)}, \quad \widehat{s}_{1j}^z = \frac{\sigma s_{1j}^z}{\sigma - (1-\beta)(N-2)},$$

where $z \in \{a, b\}$. Also, $j \notin \{1, 2, 3\}$ unless indicated otherwise. Substituting the quantities of currencies brought to the posts involving currency 1 from (4.8), (4.9), (4.14) and (5.1) into (5.6) – (5.9), we solve exchange rates as follows:

$$\widehat{s}_{12}^b = \xi \widehat{s}_{1j}^b,$$

$$\widehat{s}_{1j}^b = \frac{1 - \sigma\phi_1}{[1 + \beta(N-3)]\,\xi - \frac{\beta(N-3)}{\gamma_1} + \beta(N-3)(2\xi + N - 2)\left(\frac{1}{\sigma\gamma_1} - \phi_1\right)},$$

$$\widehat{s}_{12}^a = \frac{1 + \widehat{s}_{1j}^b \frac{2\beta(N-3)}{\sigma\gamma_1}\left(\frac{1}{\theta} + 1 - 2\xi\right)}{1 + \beta(N-3) - [\sigma - (1-\beta)(N-3)]\,\phi_2},$$

$$\widehat{s}_{1j}^a = \frac{1 + \widehat{s}_{1j}^b \frac{\beta}{\sigma\theta\gamma_1}\left[2\xi(1+2\theta) + N - 4 - 2\theta\right]}{1 + \beta(N-2) - [\sigma - (1-\beta)(N-2)]\,\phi_j}.$$

In addition,

$$p_1 = \frac{1}{y_1}\left[1 - \frac{1}{\sigma}\beta(N-3)(2\xi + N - 2)\widehat{s}_{1j}^b\right].$$

Proof of Proposition 5.1.

To prove Proposition, we compute the consumption levels in the VCE relative to the ones after the deviation. Use the subscript VC to indicate the levels in the VCE. Then,

$$\frac{(c_{22})_{VC}}{c_{22}} = \frac{\sigma - (1-\beta)(N-3)}{\sigma - (1-\beta)(N-2)},$$

$$\frac{(c_{2j})_{VC}}{c_{2j}} = \left(\frac{\widehat{s}^b_{1j}}{\widehat{s}^a_{1j}}\right)_{VC} \Big/ \left(\frac{\xi\widehat{s}^b_{1j}}{\widehat{s}^a_{1j}}\right),$$

$$\frac{(c_{21})_{VC}}{c_{21}} = \left(\frac{\widehat{s}^b_{1j}}{p_1}\right)_{VC} \Big/ \left(\frac{\xi\widehat{s}^b_{1j}}{p_1}\right),$$

$$\frac{(c_{23})_{VC}}{c_{23}} = \frac{\frac{\beta/\gamma_1}{\sigma-(1-\beta)(N-2)}\left(\widehat{s}^b_{1j}/\widehat{s}^a_{1j}\right)_{VC}}{\frac{1}{\sigma-(1-\beta)(N-3)} - \phi_3}.$$

We examine the special case $\beta = 1$ first. When $\beta = 1$, we have $c_{22} = (c_{22})_{VC}$. Also,

$$\frac{\xi\widehat{s}^b_{1j}}{\widehat{s}^a_{1j}} = \frac{(1-\sigma\phi_1)(N-1-\sigma\phi_j)}{N-2+\frac{2-1/\xi}{\gamma_1} - \phi_1\left[(N-3)\left(2+\frac{N-2}{\xi}\right) + \frac{1}{\theta\gamma_1}\left(2(1+2\theta)+\frac{N-4-2\theta}{\xi}\right)\right]}.$$

When ϕ_1 is small, this expression is an increasing function of γ_1. Recall that $\left(s^b_{1j}/s^a_{1j}\right)_{VC}$ is a decreasing function of γ_1. Thus, $(c_{2j})_{VC}/c_{2j}$ is a decreasing function of γ_1. Similarly, $(c_{23})_{VC}/c_{23}$ is a decreasing function of γ_1. Also, when $\beta = 1$, we have:

$$\frac{(c_{21})_{VC}}{c_{21}} = \frac{\sigma\left[(N-2)\gamma_1 - \frac{N-3}{\xi}\right] + (1-\gamma_1)(N-3)\left(2+\frac{N-2}{\xi}\right)}{\left[\gamma_1(N-1)(1+\theta)-(N-2)\right]/\theta}.$$

This is a decreasing function of γ_1.

Now we compare the utility level of country 2 before and after the deviation. Denote $\Delta(\gamma_1) = (U^2)_{VC} - U^2$. With logarithmic utility functions, the above properties of the consumption ratios imply that $\Delta(\gamma_1)$ decreases in γ_1. It can be verified that $\Delta(\infty) = -\infty$, and so the deviation is profitable to countries 2 and 3 if γ_1 is sufficiently large. On the other hand, when $\gamma_1 = 1$, we can verify that $c_{2j} = (c_{2j})_{VC}$ and $c_{21} = (c_{21})_{VC}$. In addition,

$$\left(\frac{(c_{23})_{VC}}{c_{23}}\right)_{\gamma_1=1} = RHS(5.10).$$

This ratio is greater than one, and hence $\Delta(1) > 0$, iff (5.11) is satisfied. Because $\Delta(\gamma_1)$ decreases in γ_1, then there exists a critical level of γ_1 such that the deviation makes countries 2 and 3 worse off if and only if γ_1 is below this critical level.

Extending the above analysis to allow for a deviation by any arbitrary pair of countries, we establish Proposition 5.1.

D. Proofs for Section 6

We derive (6.15) – (6.17). Substituting f_{jj}^{1jA} from (6.11) into (6.12), we obtain:

$$s_{1j}^{bA} = \frac{\sigma}{N-2}\left(f_{11}^{1jA} - \phi_1\right), \qquad s_{1j}^{aA} = \frac{f_{11}^{1jA}}{\frac{N-2}{\sigma} - \phi_j}. \tag{D.1}$$

where f_{11}^{1jA} is still to be determined. Substitute f_{jj}^{1jB} from (6.10) and f_{1j}^{1jB} from (6.3) into (**??**). Furthermore, using the above result to substitute for s_{1j}^{bA}, we get the solution for s_{1j}^{bB} as in (6.16). As explained in the main text, the equality $s_{1j}^{aA} = s_{1j}^{bB}$ is necessary for households in country 1 to trade in both rounds of the currency exchange. This equality and the expression for s_{1j}^{aA} in (D.1) imply:

$$f_{11}^{1jA} = (1 - 2\sigma\phi_1)(\frac{N-2}{\sigma} - \phi_j).$$

Then, (D.1) gives s_{1j}^{bA} as in (6.15). To solve for s_{1j}^{aB}, substitute f_{jj}^{1jB}, f_{1j}^{1jB} and f_{i1}^{1jB} from (6.3), (6.10) and (6.11) into (6.13). Re-arranging the result yields (6.17).

Proof of Proposition 6.1
to be written.

Other configurations of two rounds of trade per period
to be written.

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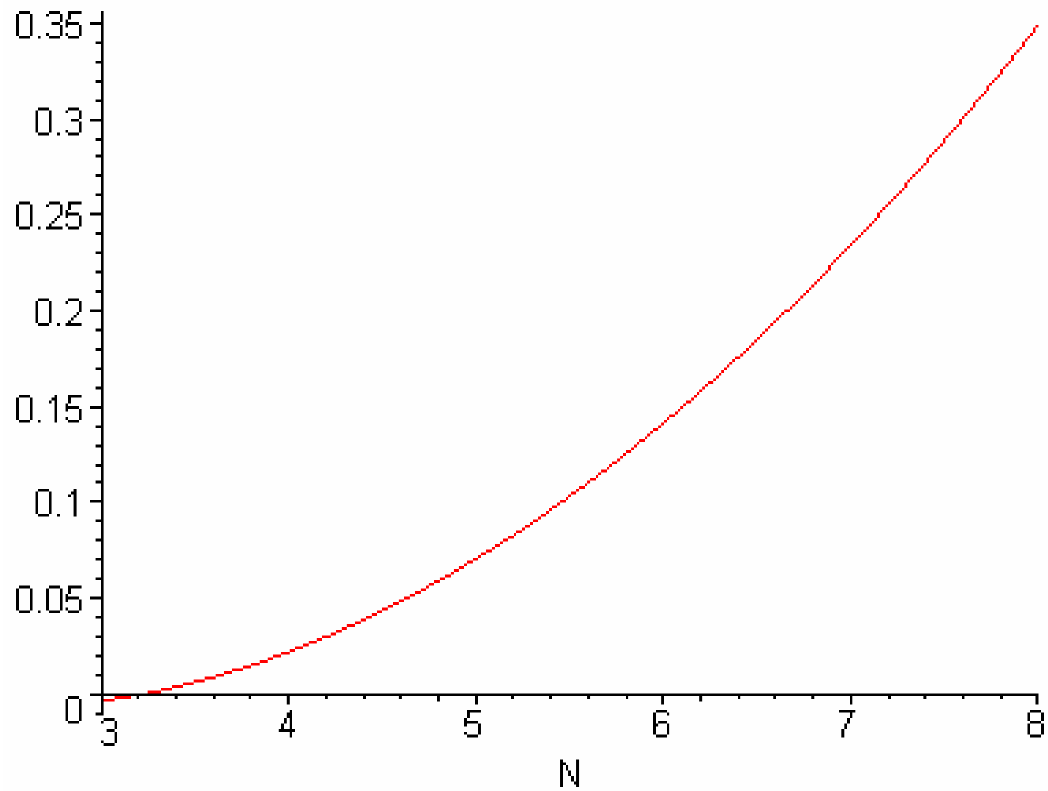

Figure 1

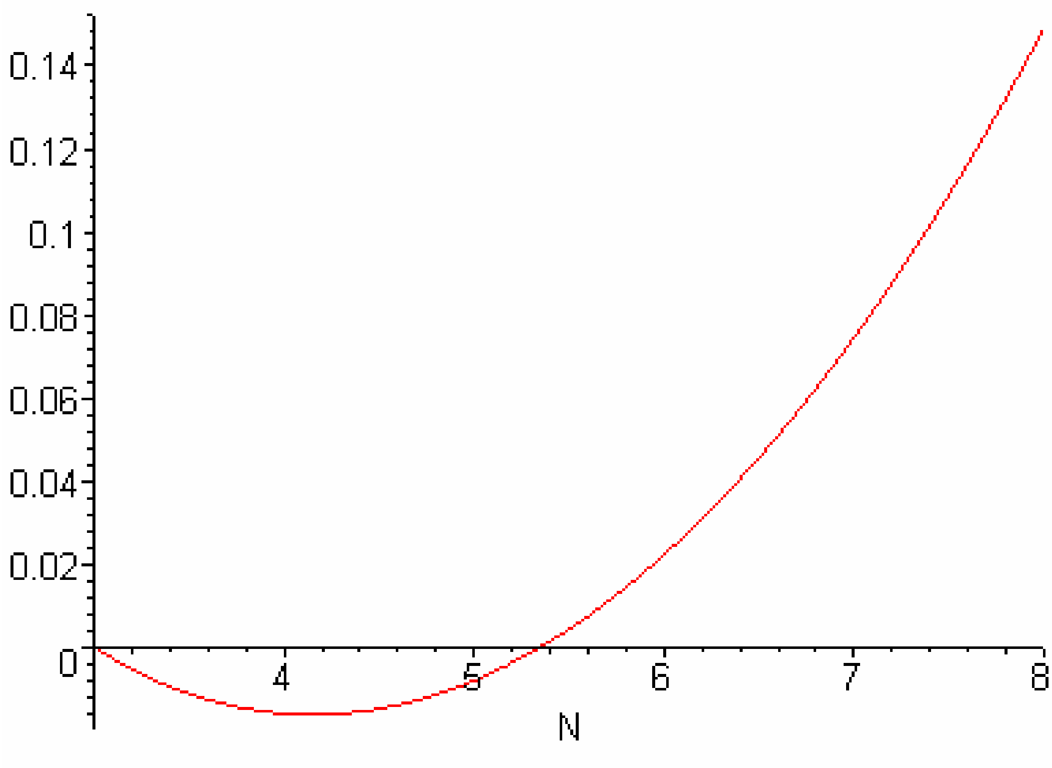

Figure 2

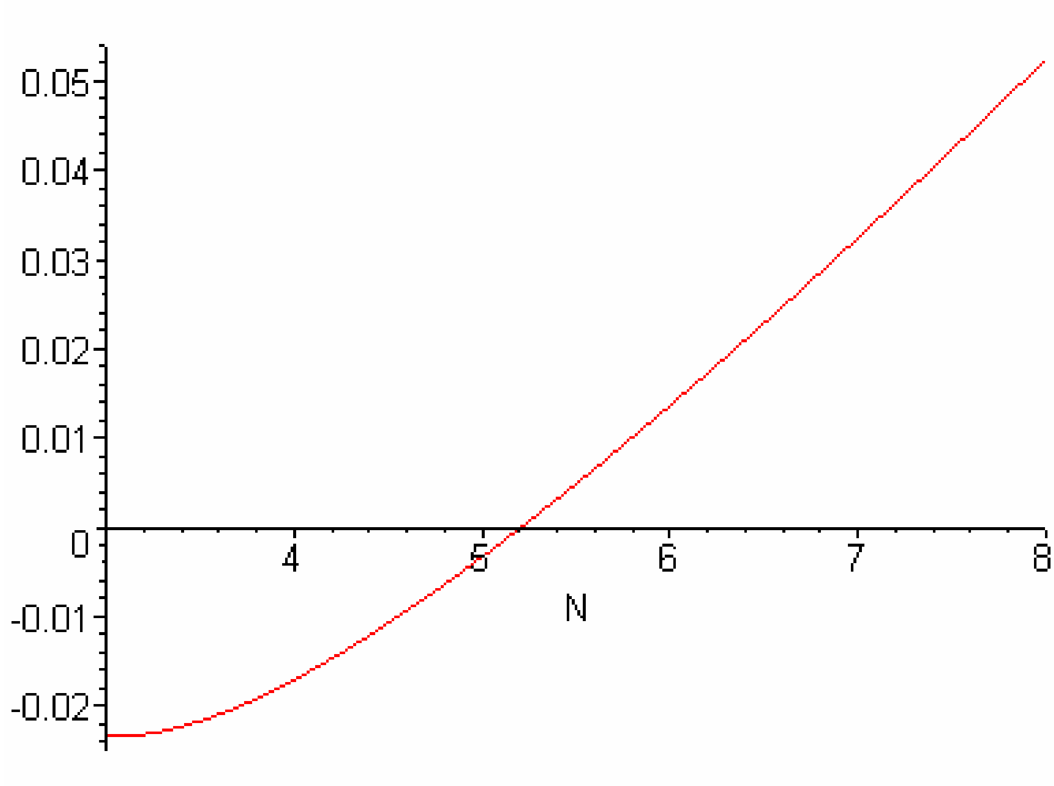

Figure 3

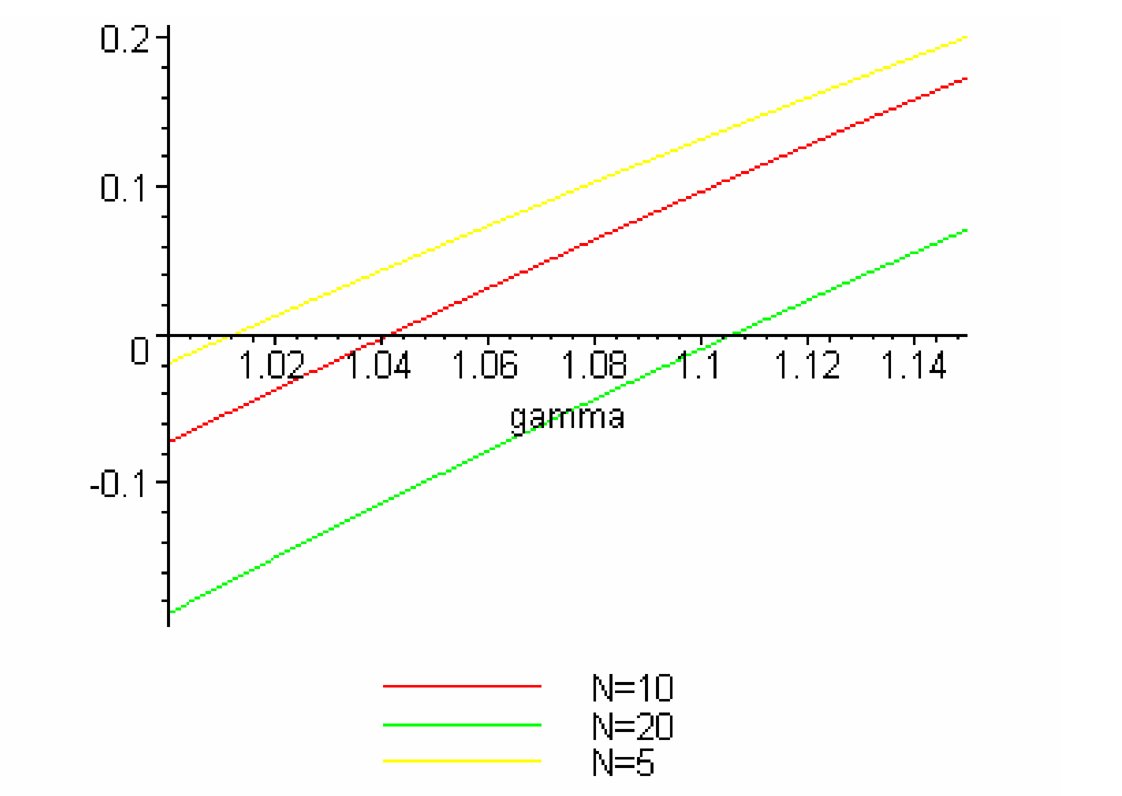

Figure 4