On Creditors and Debtors, and their Rates of Interest and Exchange

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1. INTRODUCTION

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The main focus of this paper is on real and nominal exchange rates in the long run, and the relationships they bear to real interest rates, inflation, output, capital and other assets. We seek to solve for equilibrium values of real and nominal exchange rates, along with other key endogenous variables to which they are linked – stocks of capital, currency and debt, flows of output and rates of interest. The point of departure from standard models is that residents of different countries are assumed to discount utility *at different rates*. What keeps cross-border claims finite in the steady state in the model proposed here is an assumption that agents care, in addition to regular goods, about their *net wealth*, with an increasing aversion to financial instruments denominated in an alien currency[2].

Here are a number of questions on which this paper aims to throw light:

(i) Do free international capital movements mean that real interest differentials across currencies have to vanish in the absence of the usual risk premia? Why might they not vanish?

(ii) How can we avoid Lucas's disagreeable result (Lucas, 1992) that, when agents (or countries) differ in rates of time preference, the most far sighted own everything in the steady state?

(iii) What determines the currency mix in which foreign lending is conducted?

(iv) Does greater indebtedness imply a lower or a higher real exchange rate in the long run? What else governs the equilibrium real exchange rate, and why?

(v) How and why might inflation affect output and debt levels at home and abroad?

(vi) What explains and determines long run levels of inter-country claims?

(vii) What else, besides relative nominal money supplies, do equilibrium nominal exchange rates depend upon?

(viii) How does removal of restrictions on international lending affect welfare in lending and borrowing countries?

This paper explores the effects of (and limits to) international lending with bonds. There are no equity trades across borders, nor direct foreign investment. The focus is on steady states, and the simplest functional forms for preferences and technology are employed to ease exposition and pinpoint results. The physical capital that each country accumulates is assumed to be constructed directly from its own exportable product: but the sheer presence of capital as an explicit factor of production is a point worthy of note, since capital is unfortunately an absentee in many contemporary models regarded as canonical (such as Obstfeld and Rogoff (1995)). And a central feature of the argument of this paper is that domestic and foreign residents discount utilities at *different rates*.

2. THE MODEL: PRELIMINARIES

This paper presents a long run, two-country model of floating exchange rates, where agents' decisions are driven by optimization conditions. One main feature that needs to be incorporated is the idea that residents of one country prefer not to have claims, or obligations, expressed in the currency of another. There is a home currency bias in this respect. Exchange rate uncertainty and transactions costs will be prime considerations underlying this. The model below captures the idea by introducing a subjective marginal cost of holding assets or debts denominated in foreign (as opposed to domestic) currency, which increases linearly with their size.

A debtor who has issued claims in his own currency – which we are assuming to be unindexed - has an incentive to generate surprise devaluation: that would reduce the real burden of servicing them. The size of this incentive increases with the scale of his obligations – hence the notion that the creditor will be increasingly reluctant to hold them as that scale goes up. Similarly a creditor holding unindexed claims against foreigners denominated in the creditor's currency has an incentive to generate a revaluation, unanticipated when they were issued, and this temptation increases with the magnitude of those claims. What follows will depict an equilibrium where such time inconsistent behaviour does not in fact occur. The power of each to act that way is counterbalanced by the other's. But the thought that surprises of this kind could occur is what limits the scale of loans of both kinds.

Net claims in home or foreign currency, which could be positive or negative, along with domestic capital and real balances of home currency, are all assets for which individuals will presumably choose optimal accumulation paths. Their choices will also extend to labour, and to purchases of domestic and foreign goods.

For a variety of reasons, residents of different countries may exhibit different rates of time preference. To the extent that discount rates may reflect probabilities of death, life expectations could differ, for example. So might attitudes to heirs. Family links may be not as durable in one country as another (a greater prospect of divorce may lead couples to save less (or more) than they might have chosen otherwise, as a result of a "tragedy of the commons" or other effects). Countries might differ in their perceived probabilities of financial crime and property theft, or in the insurance premia needed to cover such risks. Corporate and political institutions may take key decisions with greater myopia in one polity than another. And finally people in different countries might simply wish to have differently shaped time profiles of consumption.

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[2] There are other ways, too, of "closing" the current account in an intemporal optimizing setup, discussed, for example, by Schmitt-Grohe and Uribe (2003).

If discount rates do differ across countries, some feature needs to be added if one is to avoid the depressing conclusion that the most patient will end up owning everything. The important findings of Lane and Milesi-Ferretti (2001) illustrate the fact that this is certainly not the case at present. Virtually all countries studied have claims on (and obligations to) each other. The device explored in this paper is the notion that utility depends upon net wealth as well as the traditional goods of leisure and consumption. But within the definition of net wealth, pride of place, with special emphasis, goes to real balances of local currency. This captures the idea that titles to physical capital and other non-monetary claims are nothing like as good as cash for smoothing consumption in the face of shocks, but may still be able to perform that function to some degree. There is a well-established tradition of allowing utility to include real money holdings, and, in the open economy context, this usually means domestic currency. So one way of thinking of the modest innovation to agents' utility functions to be shown below is that non-cash assets are added in here, at a relative discount. Another is to treat the new term as one in net wealth, essentially a portfolio, which displays an additional weight on real local currency holdings.

The exogenous variables in the system are parameters governing preferences and technology, and nominal money supply paths assumed to be set by governments or central banks (in this respect the model is rather traditional: central banks do not explicitly, as in Woodford (2003), set nominal interest rates). But the results can be reinterpreted to cover cases where the monetary authorities are in fact nominal interest rate setters, assuming that they determine them, in the long run, at the (neutral) point of Fisherian equilibrium where they cover the sum of the inflation and real interest rates exactly. The focus will be on steady states with full information, price flexibility and perfect competition. The simplest, Cobb Douglas functional forms for preferences and technology are adopted, to aid the identification and interpretation of concrete solutions.

Although there are some new features of this model, which yield interesting conclusions, it must be admitted at the outset that there are others that limit its scope. The main focus will be on the steady state, and much less upon how it is reached. Partly for this reason, short-run considerations such as nominal inertia in prices are set aside. So, too, is imperfect competition. Keynesian features are therefore very limited. But there is one, until relaxed later in the paper, which has powerful (if controversial) implications. This is the Tobinesque notion that inflation aids capital formation.

3.1: ANALYSIS OF THE HOME COUNTRY

Consider the following optimization problem for each of the residents of the home country, all of whom are assumed similar in preferences and endowments, and to display unit mass: maximize Φ which is given by:

$$\Phi = \int_0^\infty \{\exp(-\beta t)[\eta \ell n c_h(t) + (1-\eta)\ell n c_f(t) + \theta \ell n(1-h(t))$$
$$+ \zeta \ell n\{k(t) + [m(t)(1+\sigma) + D^*(t)S(t)(1-\xi(D^*(t)S(t)/2P_h) + D(t)]/P_h(t)$$
$$-\zeta(1-\eta)\ell n X(t)]\} + \lambda(t)\{P_h((f(k(t),h(t)) - \dot{k}(t) - c_h(t) - c_f(t)X(t)) - \dot{m}(t) + \Omega(t)$$
$$+ D^*S(t)r^* + D(t)r - \dot{D}^*(t)S(t) - \dot{D}(t)\}\}dt \qquad (1)$$

The decision date is t=0. Here c_h, c_f denote consumption of home produced and foreign goods; *h* is labour supply, and *k* capital; *m* is the nominal domestic money supply, and D^*, D are claims in foreign and domestic currency respectively. Nominal interest rates are *r* and *r**. *S* and *X* are the nominal price of foreign currency and real exchange rate ($X = P_f S / P_h$), where P_h, P_f are local-currency nominal prices of home and foreign goods. An overdot denotes a time derivative. $\Omega(t)$ is nominal seignorage, returned to the individual as a lump sum. The budget constraint in (1) is couched in nominal terms from the standpoint of a home currency resident.

The non-standard element in utility is the fourth "good", the term introduced by the parameter ζ. This may be thought of as "amplified" real money. It is written as real money, multiplied by a coefficient σ, plus real wealth. All the terms in this expression are deflated by the nominal price of home goods. But since the Cobb Douglas tastes give the implicit price index as $P_h^\eta (P_f S)^{1-\eta} = P_h X^{1-\eta}$, $(1-\eta)\ell n X$ is subtracted from direct utility to capture the adverse effect of a terms of trade deterioration. If σ is very large, the fourth good is overwhelmingly dominated by money holdings, and other forms of net wealth become relatively unimportant. There are several reasons for handling wealth and money in this way.

Part of the justification for putting money into the utility function is because of its outstanding convenience as a resource-saving transactions medium. But it is not unthinkable that title deeds to other assets, if, as assumed here, perfectly divisible, could also perform this function, though much less well of course, if real money were for some reason unavailable. Second, to the extent that money is held as a precautionary fund to smooth consumption in the face of shocks, non-money assets can be collateralized to do this too. Then there is the idea that the possession of both money and (to a lesser extent) non-money assets confers on the holder the freedom and flexibility to take timely advantage of investment or purchasing opportunities, for example. The analysis that follows abstracts from these issues, and does not model them specifically.

Another feature of the wealth-amplified money term is that it reflects an increasing marginal distaste for acquiring claims denominated in foreign currency[3]. The strength of this bias towards home-currency denominated claims is captured by the parameter ξ. One reason underlying this is that debtors can surprise (and rob) creditors holding claims in debtor currency by unexpectedly devaluing their external and internal value. The larger the stock of such claims, the more the debtor country would stand to gain by doing this. But the debtor will have a symmetric concern that the creditor can unexpectedly revalue credits denominated in creditor currency: hence the debtor's decreasing willingness to issue them, modelled in the section that follows. In the steady state, it is assumed that these fears are unrealized. Yet that does not mean that they are irrational. One could imagine that the monetary authorities of each country are deterred from such time-inconsistent actions by the power of the other's to retaliate.

First order conditions for consumption of the two goods and labour imply the following intratemporal equalities

$$\begin{aligned}&\exp(-\beta t)/\lambda(t)P_h(t)\\&=c_h(t)/\eta\\&=c_f(t)X(t)/(1-\eta)\\&=(1-h(t))f_2/\theta.\end{aligned} \tag{2}$$

Here, $f_2=\partial f(k(t),h(t))/\partial h(t)$, the marginal product of labour. The proportionate rate of decline of λ may therefore be written as $\beta+\partial \ell n(P_h c_h)/\partial t$, which reduces to $\beta+\pi$ in the steady state with π the home currency rate of inflation for domestic goods.

For each of the assets a_i, we have a first order Euler condition for an interior intertemporal equilibrium is $\partial\Phi/\partial a_i(t)=\partial\{\partial\Phi/\partial \dot{a}_i(t)\}/\partial t$. Applying these conditions on *k(t), D(t), D*(t),* and *m(t)* establishes that, in the steady state (when *X* is stationary):

$$\begin{aligned}&\zeta c_h/\eta\Lambda=\\&\beta-r+\pi=\\&\beta-f_1=\\&(\beta-r^*+\pi^*)/(1-\xi D^*S/P_h\}\\&=(\beta+\pi)/(1+\sigma)\end{aligned} \tag{3}$$

where $\Lambda\equiv[k+[(1+\sigma)m+D^*S(1-\xi D^*S/2P_h)+D]/P_h]$, our term in wealth-amplified money.

One of the implications of (3), which follows from the Euler conditions on capital and home currency claims *D*, is that the marginal product of capital, f_1, will equal the real own-rate of interest on home goods. Another (revealed by combining the Euler conditions on *D* and money), is that this real rate, *R* let us call it, will equal

$$R=r-\pi=(\beta\sigma-\pi)/(1+\sigma). \tag{4}$$

This domestic real interest rate tends towards the time preference rate, β, as σ becomes very large; but with finite σ, it falls below β provided that the nominal price of domestic goods trends upwards, or if downwards, at a rate smaller than β. Equation (4) bears a second interpretation: the monetary authority may set the nominal rate of interest on its currency, and thus, implicitly in the steady state, its rate of inflation (we may rewrite (4) as $\pi=-\beta+r(1+\sigma^{-1})$).

The generalization of this story would be to modify (1) to

[3] Bordo, Meissner and Redish (2003) report how difficult it has proved even for some advanced countries to secure acceptance of their local-currency-denominated obligations by overseas residents.

Max $\phi = \int_0^{\infty} [e^{-\beta t}U(t) + \lambda(t)B(t)]dt$ (1a)

subject to given initial levels of $k(0), m(0)$ and any net claims on agents abroad, where

$U(t) = U(c_1(t), c_2(t), 1 - h(t), I(t)/P(t), m(t)/P(t), w(t))$,

$w(t) = k(t) + [D(t) + D^*(t)S(t)(1 - \xi D^*(t)S(t)/2P(t)]/P(t)$

and

$$B(t) = P(t)\{f(k(t), h(t)) - \dot{k}(t) - c_1(t) - c_2(t)X(t)\} - \dot{m}(t) + \Omega(t)$$
$$- \dot{D}(t) - \dot{D}^*(t)S(t) + r(t)D(t) + R^*(t)D^*(t)S(t) = 0.$$

The intratemporal first order conditions for this problem link consumption of the two goods and labour. They imply

$$e^{\beta t}\lambda(t)P(t) = U_1(t) = U_2(t)/X(t) = U_3(t)/f_2(k(t), h(t))$$

and, therefore, that the proportionate rate of decline of the marginal utility of nominal income is given by $-\hat{\lambda}(t) = \beta + \pi(t) - \hat{U}_1(t)$.

The intertemporal conditions are extensions of those considered above, with qualitatively broadly similar properties.

3.2: ANALYSIS OF THE FOREIGN ECONOMY

The next stage is to model the foreign economy. We shall assume for the moment that foreign residents, like domestic residents, have unit mass. Agents there will be presumed to differ in preferences from their home counterparts in five respects. First, they are less patient: their rate of discount, β^*, is larger than at home. Second, their intratemporal utility from home and foreign goods, call these c_h^*, and c_f^*, respectively, will be $\{1 - \theta - \zeta\}[\eta^* \ell n c_h^* + (1 - \eta^*)\ell n c_f^*)]$. There is home bias in preferences if and only if $\eta > \eta^*$. Third, for them, D^* and D represent obligations, not claims, so their sign in the wealth-amplified money term is reversed. Fourth, home currency money is replaced by foreign currency money, m^*, since it is this that they will use in purchases. Finally, foreigners have an increasing marginal dislike of obligations denominated in the home currency, D.

At each instant foreigners must respect a nominal budget constraint

$$P_f[f^*(k^*(t), h^*(t)) - \dot{k}^*(t) - c_h^*(t)/X(t) - c_f^*(t)] - \dot{m}^*(t) + \Omega^*(t)$$
$$+ \dot{D}^*(t) + \dot{D}(t)/S(t) - D(t)^* r^* - D(t)r/S(t) = 0 \quad (5)$$

and maximize Φ^* subject to them, where:

$$\Phi^* = \int_0^{\infty} \{\exp(-\beta^* t)[(\eta^* \ell n c_h(t)^* + (1 - \eta^*)\ell n c_f^*(t) + \theta \ell n(1 - h^*(t))$$
$$+ \zeta \ell n[k^*(t)X(t) + \{m^*(t)S(t) - D^*(t)S(t) - (D(t))(1 + \xi D(t)/2P_h(t))\}/P_h(t)]$$
$$- \zeta(1 - \eta^*)\ell n X(t)]\}dt \quad (6)$$

If $\mu(t)$ are the Lagrange multipliers for this problem, intratemporal first order conditions imply

$$e^{-\beta^* t}/P_f(t)\mu(t)$$
$$= c_h^*(t)/\eta^* X(t)$$
$$= c_f(t)/(1 - \eta^*)$$
$$= (1 - h^*(t)f_2^*/\theta \quad (7)$$

and from these conditions it follows that μ must, in the steady state, decline (proportionately) at the rate $\beta^* + \partial \ell n(P_f c_f^*)/\partial t \to \beta^* + \pi^*$.

The Euler conditions for the problem, meanwhile, entail for the steady state:

$$\beta * - f_1 *$$
$$= \beta * - R *$$
$$= (\beta * - R)/(1 + \xi D / P_h)$$
$$= (\beta * + \pi *)/(1 + \sigma)$$
$$= \zeta c_h * / \eta * \Lambda * \quad (8)$$

where $R*$ is the own real rate of interest on goods made abroad, and $\Lambda* = k * X + (m * S(1+\sigma) + D * S + D(1 - \xi D / 2P_h) / P_h)$. Equation (8) reveals that this will equal the marginal product of capital abroad. Furthermore, we may express $R*$ as

$$r*\text{-}\pi *= \; R* = (\beta * \sigma - \pi *)/(1+\sigma). \quad (9)$$

Another implication of (8) is that it expresses the steady state level of foreigners' home currency debt, *D*, in terms of *R* and *R**. But equation (3) did the same for home residents' claims in foreign currency. Combining (3) and (8) we obtain

$$D*S/D = (\beta * - R*)/(\beta - R)\xi. \quad (10)$$

What limits claims in home currency is foreigners' increasing reluctance to issue them; what restricts claims in foreign currency is home citizens' increasing unwillingness to hold them. Together, these restrictions imply a determinate long run value for the debt charges that foreigners will pay to home residents, in units of home goods:

$$\Gamma \equiv (D * SR * + DR)/P_h = (R * - R)[R */(\beta - R) + R/(\beta * - R*)]/\xi \quad (11)$$

These charges depend on local-currency rates of inflation for the two goods (taken to be policy parameters) and four preference parameters. These last capture aversion to financial instruments in alien currency (ξ), the degree of primacy of money in wealth (σ), and rates of impatience at home and abroad. Scrutiny reveals that Γ increases with $\beta *$, and decreases with β , given that $\beta* > \beta$. The greater the difference between the two countries' time preference rates, the larger these claims will be.

A further feature of (11) that merits emphasis is the fact that real interest rates *do not equalize*. This carries the implication that, in the steady state, the nominal interest differential does not match the trend in the nominal exchange rate. So uncovered interest parity fails to hold. The nominal interest rate on financial instruments denominated in the debtor's currency exceeds what the UIP condition would predict. The size of this gap is proportional to real debt charges. This is not really a "peso problem" of a conceivable future devaluation; rather, it reflects the consequences of assuming that claimants and debtors both display an increasing reluctance to hold claims or debts in what is, to them, an alien currency, and that the balance of temptation and possible punishment constrains both monetary authorities in equilibrium.

If real rates of interest in the two currencies were equal, there would be a zero level of debt charges and a zero level of claims by home residents on those abroad. With inflation rates similar, that could only happen if the two countries' impatience rates were equal. But a glance at equations (4) and (9) shows that inflation differences matter too. Real interest rates for one currency are negatively related to the trend in the nominal price of the goods produced by the country that issues it. When, as seems all too likely, real balances dominate non-money assets strongly, so that σ is large, the size of this influence will be very modest. So cross country impatience differences are not the only potential cause of enduring cross country claims: persistent inflation differences could also generate them. And the chance of a country being a long term debtor increases when its long term inflation rate *falls.*

This is a curious finding. Why does it arise? The mechanism is this. Faster inflation at home reduces the demand for real local currency, entirely as one would expect. Circulation velocity increases. Lower real money balances increase the demand for local capital. They also strengthen the demand for overseas claims – like capital, a partial substitute for the now reduced stock of real money. If the country was a marginal net debtor before the rise in inflation, the consequent fall in real money holdings could well tilt into repaying its external obligations and acquiring net overseas claims. That is the mechanism at work here.

Put another way, a debtor reduces its real overseas debt charges by raising its rate of inflation. But this is not the consequence of an unexpected once-only jump in the domestic price level: surprises do not happen in the steady state. The idea rather is that faster domestic inflation, operating through the mechanism just described, exerts a small negative effect on its real rate of interest. So the gap between the debtor's and the creditor's real rate of interest shrinks. And it is because real overseas debts are proportional to this gap, that real debt charges will then have to go down.

4: THE REAL EXCHANGE RATE

Now that we have solved for the steady state flow of real debt charges across the exchanges, we can begin to address the issue of the steady state real exchange rate, *X*. The balance of payments accounts will be stationary in units of home product when the value of domestic imports equals domestic exports plus Γ. So we may write:

$$c_f X = c_h * + \Gamma \tag{12}$$

Furthermore, in the steady state, investment will have vanished, and nominal borrowing will match any erosion of real debt due to inflation. Our assumptions on preferences imply that $c_f X = (1-\eta)(f+\Gamma)$ and that $c_h * = \eta * [f * X - \Gamma]$. The long run value of the real exchange rate is therefore given by

$$X = [(1-\eta) f + (\eta * - \eta)\Gamma] / \eta * f *. \tag{13}$$

(13) tells us that the real exchange rate will be independent of international debts if and only if foreign and domestic consumption preferences are the same ($\eta = \eta *$). A home bias in preferences will imply a boost to the lender's terms of trade; an anti-home bias implies a dent. These results follow, at least, for given levels of home and foreign output.

The intuition here is straightforward. Persistent impatience differences imply that foreigners must eventually keep transferring to domestic residents, as a consequence of their relatively high consumption in the past, and the debts that resulted from this. A transfer benefits the transferor, by improving his terms of trade, when the sum of the two countries' marginal import propensities exceeds unity (the "Keynes-Samuelson" condition). This happens in our case when $\eta * > \eta$, and not otherwise. A home bias in preferences implies the opposite – the transferor suffers the secondary burden of a deterioration in its terms of trade.

Our remarks about the real exchange rate are premature however. The levels of output at home and abroad have been taken as given. But capital and labour, upon which output depends, are both endogenous. So a proper solution for the real exchange rate requires further analysis.

Each country's supply of local goods is assumed to be Cobb-Douglas, differing only in a Hicks-neutral technology coefficient:

$$f(k(t), h(t)) = Tk(t)^{\gamma} h(t)^{1-\gamma},$$

$$f * (k * (t), h * (t)) = T * k * (t)^{\gamma} h * (t)^{1-\gamma}. \tag{14}$$

There is perfect competition, so that $f = kR/\gamma = hf_2/(1-\gamma)$, with similar conditions holding abroad (where, due to (8), capital's marginal product is *R**). As returns to scale are constant, the capital labour ratio is a function of the local real interest rate: in country i, $(k/h)_i = [\gamma T / R_i]^{1/(1-\gamma)}$. So output and capital can both be expressed as ratios to labour supply, h.

The next stage is to employ the labour supply-consumption and wealth – consumption trade-offs in the two countries, contained within (2), (3), (7) and (8). They enable us to solve for domestic and foreign output, and the nominal and real exchange rates. We shall do this in stages. First, use the production functions, the long run budget constraints, and the first order conditions on labour and consumption. These allow one to relate each country's output to its real wage rate (which is negatively related to its local real interest rate) and to the flow of real servicing costs on external claims between them. We may therefore write:

$$f = \{ f_2 - \theta\Gamma \} / \Xi \tag{15}$$

$$f * = \{ f *_2 + \theta\Gamma / X \} / \Xi \tag{16}$$

where $\Xi \equiv 1 - \gamma + \theta$. It is interesting to observe that the real debt charges paid by foreigners to home residents reduce output at home and raise it abroad. The reason here is that leisure is a normal good. The exogenous income that home residents receive dulls the incentive to work. Abroad, debt charges are a spur to labour.

Equation (15) is in fact a solution for home output (the marginal product of labour depends negatively on *R*, and *R*, *R** and debt charges have all been solved in terms of exogenous variables). On the other hand, (16) is not really a solution for the foreign country's output, because of the presence of the real exchange rate, *X*. (It is a solution, however, if one redefines debt charges in units of foreign output). But an equation for the real exchange rate has already been found, in terms of home output, foreign output and debt charges. This is (13). So we may combine (13), (15) and (16), to yield

$$X = [\{T/T*\}\{R*/R\}^{\gamma}]^{1/(1-\gamma)} (1-\eta)/\eta *$$
$$- \Gamma[\theta + (1-\gamma)(\eta - \eta *)](R*/\gamma)^{\gamma/(1-\gamma)} / \eta * (1-\gamma) T *^{1/(1-\gamma)} \tag{17}$$

It is worth exploring (17) in some detail. The real exchange rate may be thought of as the home country's terms of trade – and a rise means a deterioration. So we see an increase in the domestic technology coefficient, *T*, relative to its foreign counterpart, T^*, leading to a worsening in the home terms of trade: if home products are more plentiful, they must get cheaper. Then there are the demand parameters. An increased preference for home goods on the part of domestic residents means a rise in η, and in η^* on the part of foreigners, and in both cases (17) reveals that the home terms of trade must improve. The role of R^*/R, apparent in the first term on the right hand side of (17), arises because an increase in the relative cost of capital abroad will make inhibit foreign production relative to home, and foreign goods will get scarcer.

Next, there is the role played by real debt charges. As (17) reveals, these are highly likely to act as a drag on the real exchange rate, improving the terms of trade at home. Inspect the square bracketed term in the coefficient on Γ. This is very likely to be positive. Positive it must be, if commodity preferences are similar, or tilted towards local goods (home bias). This accords with the view expounded in several recent papers, notably Lane and Milesi-Ferretti (2002).

The key reason for this in our case is that larger debt charges must stimulate labour abroad and inhibit it at home, as we have already seen in the context of (15) and (16). Underlying this is the idea that the country where residents are more patient (home) will have spent time sacrificing not just consumption of the two goods, but also leisure, in the past, in anticipation of deferred gratification later on. Less patient foreign residents will have done the opposite, and are having to spend their steady state "paying" for the pleasure of previous leisure.

Yet there is just a possibility, however, that the borrower's terms of trade improve with the size of his borrowing, and that is what will in fact happen when preferences are so strongly biased against local products that these labour supply effects are offset. The strength of this bias must be very high, and the chance of its occurrence absolutely minimal. It is, however, a theoretical possibility in the circumstances of our model.

5. THE NOMINAL EXCHANGE RATE

To obtain the equilibrium (steady state) nominal exchange rate at a moment *t*, we need to explore (2), (3), (7) and (8) in further detail. The expressions for wealth-amplified money (or money-amplified wealth) have to be brought into play. They are not, unfortunately, particularly simple. Let us assume that each country's monetary authority controls both the level and the growth rate of the nominal stock of its local-currency money. In the steady state, with no trend in population or technology, inflation will equal the rate of nominal money growth in each currency. Furthermore, the real exchange rate will be trendless, implying no drift in relative prices, so that the rate of increase in the nominal, local-currency price of locally produced goods equals the general rate of inflation perceived by consumers in that country.

It helps to begin by recalling the definitions of "money amplified wealth", in units of home country product, given in (3) and (8): in the home country, we have $\Lambda \equiv k + [m(1+\sigma) + D^*S(1-\xi D^*S/2P_h) + D]/P_h$ and in the other. $\Lambda^* \equiv k^*X + [m^*S(1+\sigma) - D^*S - D(1+\xi D/2P_h)]/P_h$.
Meanwhile, (3) also gave us $\Lambda = \zeta c_h/\eta(\beta - R)$, and (8), $\Lambda^* = \zeta c_h^*/\eta^*(\beta^* - R^*)$.

If σ were allowed to be infinite, the nominal exchange rate (on the steady state path) at any date *t* would be given by

$$S = mr[f_2(1-\eta) - \Gamma(\eta(1-\gamma)+\theta]/m^*r^*[f_2\eta^* + \Gamma(\Xi - \theta\eta^*)] \qquad (18)$$

Here the nominal exchange rate would be proportional to relative nominal money supplies (at that date), and also to the ratio of nominal interest rates, *r/r**. A higher nominal interest rate at home would betoken faster domestic inflation and lower real home money demand. There would also be a positive influence on *S* from the domestic real wage rate, which is negatively related to *R* and positively to domestic output, signifying a weaker terms of trade for the home country; and debt charges would exert an unambiguously negative effect.

Equation (18) would hold when money dominated wealth in the utility function so strongly that the latter effectively vanished from the scene. So it would be a very special case. It is also a case – one of two as we shall see in the following section – where the model effectively collapses. With σ finite, on the other hand, we are in much safer (if rather more complex) territory. In these circumstances, *S* is given by

$$S = \frac{rm[f_2(\sigma\zeta - \gamma r/R) + \Gamma J]}{r^*m^*[f_2(1-\eta)(\sigma\zeta - \gamma r^*/R^*)/\eta^* + \Gamma J^*]}. \qquad (19)$$

Here we define $J \equiv \sigma\zeta(1-\gamma) + \theta r\gamma/R - \aleph(1 + \frac{r}{r^*} - \frac{\sigma}{2r}(R^* - R))$, $\aleph \equiv \frac{\sigma\Xi}{R^* + Rr/r^*}$,

$J^* \equiv \aleph(1 + \frac{r^*}{r} + \frac{\sigma}{2r}(R^* - R)) - \sigma\zeta(\theta + \eta(1-\gamma))/\eta^* + \Psi$, and lastly

$$\Psi \equiv \gamma r^*(\theta(1-\eta^*)+(\eta-\eta^*)(1-\gamma))/\eta^* R^*.$$

Nonetheless, (19) shares some features with (18), notably the proportionality of the nominal exchange rate in the ratios of home to foreign nominal money stocks, and home to foreign nominal interest rates. If the term in $\aleph$ dominates the coefficients on Γ in the numerator and denominator of (19), as appears highly likely, the impact of debt charges on the equilibrium nominal exchange rate will be qualitatively similar to what we observed when σ was infinite.

6. TWO PATHOLOGICAL CASES WHERE THE MODEL COLLAPSES

There are two cases where our model disintegrates. One arises when money dominates other components of wealth in the utility function so strongly that the latter effectively disappear. This arises, as we saw in the previous section, when the parameter σ is infinite. The problem here is that the two sets of Euler conditions (equation (3) for the home residents, and (8) for the foreigners) get infected with a bug that breaks the equalities. To be specific, Λ and Λ^* become infinite. The direct marginal utility of a stock of non-money assets, whether physical capital or net external claims, will vanish. The local-currency bond/capital/money tradeoffs still hold. But the real interest rate now equals the local residents' rate of impatience, and there is nothing to tie down relationships between real interest rates in the two economies, nor any way of determining the mix of currencies in which any foreign lending or borrowing is conducted. Worse still, the steady state has the property that the most patient owns everything, the feature that this paper's model was primarily built to avoid.

The same problem can also occur when the value of σ is finite. It arises in one special case, when countries aim for a "full liquidity" equilibrium by setting their rates of nominal money growth (and hence inflation) equal to minus the local real interest rate. That will drive R to β, and R^* to β^*, with just the same consequences. Real currency stocks will be infinite, and non-money wealth will cease to play a role. So it is crucial that the two countries' central banks disobey the Friedman rule, if the bridge between the two countries that the wealth-amplified-money functions have allowed us to construct.

7. REAL INTEREST RATES, AND THE LONG RUN EFFECTS OF ALLOWING CROSS BORDER LENDING

Several papers address the important questions about international capital movements. Examples include: why they often flow from poor to rich countries (Lucas (1990), Alfaro, Kalemi-Ozcan and Volosoyvych (2003)); whether they are harmful or beneficial (Obstfeld (1998)); and what is likely to happen if a country removes restrictions on them (Huynh and Sinclair (2005)). Our model can help to throw light on these issues.

It is a key feature of this model that real interest rates in each of the two countries depend on only three parameters: the rate of impatience shared by its citizens (β), the special additional emphasis on real money holdings in the wealth term in their utilities (σ), and the country's rate of inflation (π). The first two are positive influences, and the third is negative.

One direct implication of this is that international borrowing and lending have no effect on countries' real interest rates, which remain, in the steady state, at the long run values they would have displayed had cross-border lending not occurred. A second, already noted, is that this model makes money non-superneutral. Faster local inflation, if maintained, reduces the local long run real interest rate. The mechanism is that faster inflation squeezes real money balances, and this raises the marginal utility of wealth, in response to which more capital is accumulated, bringing its marginal product down. The consequence of this will be an increase in the country's output. Later, in section 9, we investigate a variant of the model that displays monetary superneutrality (and a further modification which makes inflation a damaging influence on real output).

Although opening up the countries to cross border lending leaves their real interest rates unchanged, it does bring other effects of note. The borrower now has debts to service. Real debt servicing costs raise the debtor's labour supply, and reduce the creditor's, since leisure is a normal good. This tends to raise the debtor's output, and reduce the creditor's. This tendency is reinforced by changes in long run stocks of capital in the two countries. For the debtor, net wealth has fallen; its enhanced marginal utility triggers accumulation. The opposite occurs in the creditor country. All these developments tend to improve the creditor's terms of trade, since his product is now scarcer in world markets. The debtor's real exchange rate depreciates. This must happen if goods preferences are similar or home-biased.

8. CHANGES IN CERTAIN PARAMETERS

Suppose everyone becomes less reluctant to hold assets or debts denominated in foreign currency. This might come about for a variety of reasons. Access to forward markets, where hedging can occur, might become easier or cheaper. Changes in central banks' policy, or a reduced incidence of serious shocks, might lower anticipated volatility in nominal exchange rates. Or there could develop, from at least the creditor's standpoint, the closer correspondence between the preference for foreign goods in consumption, and preference for foreign assets in the portfolio, that theory would lead us to predict – with the possible implication that claims expressed in foreign currency became less unpalatable.

What any of these changes imply is a fall in the parameter ξ. Since overseas debt charges are inversely proportional to ξ, a reduction would imply a greater long run level of real inter-country claims, and, as a consequence, a very likely (but, as we have seen, not quite certain) deterioration in the debtor's terms of trade.

Exactly the same consequences could be expected from an increased gap between the debtor's and the creditor's real rate of interest, R^*-R. This would normally happen if the difference between the two countries' impatience rates increased, or if the creditor's rate of inflation increased relative to the debtor's: in each case, any positive difference between R^* and R would tend to rise.

But the long run effects of a permanent change in one country's rate of inflation do not operate just through their impact on real overseas debt servicing flows and from them to the real exchange rate. There are direct output effects, too. If country 1 raises its rate of nominal money growth in perpetuity, its inflation rises, and its real rate of interest slips back a little. That means that capital is a cheaper factor of production, and, all else equal, country 1's steady state output level will rise. But higher output can only imply a worsened terms of trade.

Up to now, both population sizes have been normalized to one. Increasing the home country's population relative to the foreign country's will have the following effects. The worldwide supply of domestic output rises, so its relative price ($1/X$) must fall. As far as external claims are concerned, the enlarged population in the home country increases the ratio of home-currency denominated claims, relative to those expressed in foreign currency. Debt per head in the foreign country rises, while external claims per head in the home country will decline.

9. SUPERNEUTRALITY

The model embeds a Tobinesque view of the world (Tobin (1965) sees a rise in inflation as a portfolio-reshaping phenomenon that increases capital in the long run). Portfolios matter, and substitution between money and other assets occurs. Higher local inflation squeezes real money holdings, reducing wealth. The increased marginal utility of wealth then raises the attractiveness of capital, and net credits. An enlarged capital stock then tends to imply a higher level of domestic output. Some will object that money should be superneutral for capital and output, or that supernonneutrality might go the other way. It is the purpose of this section to show how the model can easily be amended to allow for either of these possibilities.

Superneutrality will arise if the utility from non-money net wealth is additively separable from the utility of real balances. Let us therefore amend the home and foreign agents' optimization problem from the maximization of (1) and (5) to

$$\phi_i = \int_0^\infty \{e^{-\beta_i t}[\eta_i \ell n c_{i1}(t) + (1-\eta_i)\ell n c_{i2}(t) + \theta \ell n(1-h_i(t)) + \mu_i \ell n(m_i(t)/P_i(t)) + \zeta \ell n \Lambda_i^*(t)]$$
$$+ \lambda_i(t) B_i\}dt \qquad (20)$$

with

$$B \equiv [P_i(t)\{f_i(k_i(t),h_i(t)) - \dot{k}_i(t) - X(t)^{-I}\{c_{i1}(t)+c_{i2}(t)X(t)\}\} - \dot{m}_i(t) + \Omega_i(t)$$
$$+\{(2I-1)(\dot{D}_1(t) + \dot{D}_2(t)S(t) - r_1 D_1(t) - r_2 D_2(t)S(t))S(t)^{I-1}\}],$$
$$\Lambda_i \equiv k_i(t)X^{1-I} + [m_i(t)(1+\sigma) + \{2I-1\}\{D_1(t) + D_2(t)S(t)\}]/P_1(t)$$
$$-\xi[ID_2(t)S(t) + (1-I)D_1(t)]^2/2P_1^2,$$

and I is an indicator variable that is unitary for the "home" country, 1, and zero for 2. Country i's representative resident consumes c_{i1} units of country 1's product and c_{i2} units of 2's. The variables r_1, r_2, D_1 and D_2 denote, respectively, the nominal interest rates ruling in each country, and the stock of country 1's claims against 2 denominated in 1's and 2's currency. Previously, these were r, r^*, D and D^*.

The intratemporal conditions for optimization remain as in the previous model. So, too, do the expressions for Γ, the tradeoffs in both countries between physical capital and each of the two debt instruments, and the link between the real exchange rate and the two countries' output levels. The big change is the Euler conditions for money, which now imply that the equilibrium nominal exchange rate, at an instant t in the steady state, will equal

$$S=\frac{\mu_2 m_1(t)(f_{21}(1-\eta)-\Gamma(\theta+\eta(1-\gamma)))(\beta_1+\pi_1)}{\mu_1 m_2(t)\eta^*(\beta_2+\pi_2)} \tag{21}$$

All else equal, then, the larger the real debt service flow, the cheaper the debtor's currency will be in terms of the creditor's. Thus the nominal exchange rate solution is much simpler than in the previous model.

Another change is that the two countries' real interest rates are now endogenous variables. They are no longer locked at values fixed by the two countries' discount and inflation rates and the parameter σ (which has now departed from the scene), and independent of the size of international claims. In particular, their values are now affected by the scale of inter-country lending. In financial autarky (denote this by an additional subscript A) either country's steady state real interest rate depends on three parameters. These are the profit share of income, γ, the preference parameter on (now non-monetary) wealth, ζ, and the discount rate on utility, β_i: $R_i=\beta_i[1+\zeta/\gamma]^{-1}$. Cross-border lending narrows the gap between the two countries' real interest rates. But it does not eliminate them. How far the process goes depends on each country's residents marginal distaste for claims or debts in the other's currency, the parameter ξ. The size of the remaining gap between the two real interest rates is proportional to ξ.

This new model consists of six key equations. Four of them are unchanged (except for notation: the starred country is subscripted 2, and the unstarred home country 1):

- (1) the relation between the real exchange rate, the real debt service flow, and the two countries' output levels (13);
- (2) the links between each country's output, its real wage rate, and the debt service flow ((15) and (16)) ;
- (3) the solution for real debt service flow (11).

The new ones express each country's optimization condition for capital, and may be written:

$$\zeta(f^1+\Gamma)=(\beta_1-R_1)[k+\{D_1+D_2S[1-\xi D_2S/2P_1]\}/P_1\}] \tag{22}$$

$$\zeta(f^2X-\Gamma)=(\beta_2-R_2)[kX-\{D_2S+D[1+\xi D_1/2P_1]\}/P_1] \tag{23}$$

where f^1 and f^2 denote output in the two countries.

Substitution between these six equations reduces to a pair of equations in the ratio of the real wage rate in country 1 to the real debt service flow, each of which depends on the two real interest rates alone. These are implicit functions; solution is easiest when $\gamma=0.5=\eta=\eta^*$. It becomes clear that real interest rates are drawn together somewhat, but not equalized (unless ξ were to vanish).

The fact that the steady state real interest rate in each country is now independent of its rate of inflation reflects the superneutrality of money that has been imposed in (20), the moment real balances have been split from the rest of the portfolio. A higher rate of sustained inflation in one country would bring just three effects, all local and purely nominal: real balances fall, and with it utility, and, from (21), the external nominal value of its currency (for a given level of the nominal money supply). There is just one qualification here: the higher rate of inflation considered here must be fully foreseen by the parties to the cross-border loans.

A second variant on the original model presented in this paper could make sustained inflation *reduce* output, rather than raise it or leave it unchanged. The simplest way of doing that would be to delete real balances as an explicit argument of utility, and, instead, model money as a device for saving time or resources. So leisure might be redefined as what remains from a time endowment of *1* once both labour time (*h*) and "shopping time" (*s*, say) had been subtracted. With *s* decreasing and convex in real balances, agents optimize by setting the local nominal interest rate equal to $-s'$, multiplied by the real wage rate. That equality expresses balance between the marginal benefit from holding money (time saved, valued at the wage) and its marginal (opportunity) cost. In such a case, the effect of faster inflation in squeezing real balances would raise shopping time; with Cobb Douglas utility, both leisure and labour supply would fall. In the steady state, capital would be reduced equiproportionately, so local output would come *down*, and not rise as in the original model. In most other respects, the analysis is very similar – or identical - to the first variant upon our original model.

10. ANSWERS TO OUR INITIAL QUESTIONS

Real interest rates diverge, even when, as in the steady state, real exchange rates are trendless. So uncovered interest parity condition does not hold. In the main model of our paper, cross border lending has no impact on countries' real interest rates. In the variant considered in the previous section, it brings them closer, but does not equalize them.

The currency mix for cross border loans can be inferred directly from our model: the ratio of debtor-currency loans to creditor-currency loans (expressed in either currency) equals the ratio of the debtor's discount rate-real interest differential to the creditor's. In the main model of this paper, this reduces to the ratio of their nominal interest rates. This follows from assuming that both countries' residents have a common marginal distaste for instruments denominated in alien currency. Had this preference parameter differed between the countries, the country with a greater willingness to digest instruments denominated in the other's currency would see a rise in foreign exchange-denominated instruments, relative to home-currency-denominated ones.

It is a general feature of the main model, and its variants, that the debtor's output rises, and that the creditor's falls, following the adoption of free capital movements. Wealth (or real interest) and labour supply effects both work in the same direction here. Such steady state output effects imply that the creditor's terms of trade should improve. So it will experience a real exchange rate appreciation.

On question (iv), relating to inflation, results differed sharply between the main model and its variants. In the former, inflation favoured output by reducing the real interest rate. So higher inflation for a creditor would tend to increase its lending, while faster inflation in a debtor country would reduce the steady state stock of its debt. The real exchange rate effects of inflation are governed chiefly by their output effects (the terms of trade deteriorating for a country whose output has risen) although there are also impacts from real debt servicing flows when commodity preferences differ.

The equilibrium level of inter-country loans is governed primarily by the real interest differential between borrower and lender, and the parameter reflecting each country's residents' increasing dislike of credits or debits in the other's currency. The ratio of the real interest differential to that parameter is approximately half the equilibrium debt service flow. In variant models, however, the picture is complicated by the fact that the real interest rate differential is endogenous – shrinking as the size of loans rises.

The equilibrium nominal exchange rate is trended by the two countries' inflation differential, which, in the steady state, matches the difference in their nominal money growth rates. The level at an instant is proportional to local-currency nominal money supplies then, but also varies in the usual fashion with relative incomes and (at least approximately) relative nominal interest differentials.

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