Measuring the Systemic Importance of Interconnected Banks*

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Abstract

We propose a method for measuring the systemic importance of interconnected banks. In order to capture contributions to system-wide risk, our measure accounts fully for the extent to which a bank propagates shocks across the system and is vulnerable to propagated shocks. An empirical implementation of this measure and a popular alternative reveals that interconnectedness is a key driver of systemic importance. That said, since the two measures incorporate the impact of interbank borrowing and lending on system-wide risk differently, they can disagree substantially about the systemic importance of individual banks.

Keywords: Systemic risk, Shapley values, Interbank market

JEL Classification: C15, G20, G28, L14.

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1 Introduction

It is commonly thought that interconnectedness is a key driver of systemic importance. Yet, the literature has produced few concrete insights to support this view. Who is more systemically important - the lender or the borrower in an interbank market transaction? How should the two counterparties share the rise in system-wide risk associated with the interbank link? These fundamental questions remain unanswered despite recent policy initiatives to strengthen the financial system by tightening the regulatory requirements for interconnected banks.[1] To assess whether interconnectedness is indeed a key driver of systemic importance and how it affects different interbank market participants, we propose a measurement methodology and implement it empirically.

We measure each bank's systemic importance as its share in the overall level of system-wide risk. And we explore two approaches, which decompose the *same* quantum of system-wide risk but allocate it differently across individual institutions. Adopting the perspective of a macroprudential regulator, we think of system-wide risk as the expected credit losses that the banking system as a whole may impose on non-banks in systemic events. These events, in turn, are characterised by aggregate losses exceeding a critical level.

The first approach measures systemic importance as the expected losses that a bank imposes on *its own* non-bank creditors in systemic events. This approach equates systemic importance with the expected *participation* of individual banks in systemic events. Thus, we label it the participation approach (PA).

Importantly, a bank's participation in systemic events is conceptually different from its contribution to system-wide risk. Consider, for example, a bank that is small in the sense that it can impose only small losses on its non-bank creditors. As this bank can *participate* little in systemic events, PA assigns only limited systemic importance to it. The same bank, however, might be highly

[1] See Basel Committee on Banking Supervision (2011).

interconnected in the interbank market and *contribute* materially to system-wide risk by transmitting distress from one bank to another. As PA is not designed to capture such transmission mechanisms, we consider an alternative: a contribution approach.

The key methodological innovation of our paper is to generalise an existing contribution approach – based on Shapley values – to the case of interconnected institutions. We call this the generalised contribution approach (GCA). It considers different groups of banks (or subsystems) in isolation and measures how much each bank adds – i.e. contributes – to the hypothetical risk of each subsystem it joins. The systemic importance of a bank is then simply the average of its risk contributions to all subsystems. When considering a subsystem within a system of interconnected banks, it is important to: (i) preserve the interbank network structure in that subsystem; and (ii) remove *all* the risk associated – directly or indirectly – with banks outside that subsystem. Only then do we fully capture the risk contribution of an interconnected bank to a particular subsystem. With GCA, we achieve this by modelling banks outside a particular subsystem as entities that impose no risk either on other banks or on non-banks.

The first main conclusion of our empirical analysis is to confirm the intuition that interconnectedness is a key driver of banks' systemic importance. Analyzing stylised banking systems and a system of 20 large globally active banks we find that interbank linkages raise materially the measured level of system-wide risk and, by extension, the portions of this risk allocated to individual banks. Thus, systemic importance rises in the presence of an interbank market, with the rise being greater for banks with greater interbank market activity.

Our second key finding is that the choice of a particular approach to measuring systemic importance matters not only from a conceptual but also from an empirical point of view. Quantitative differences between the two approaches are particularly pronounced in the presence of interbank linkages, as PA consistently assigns a higher (lower) degree of systemic importance to an interbank lender (borrower) than GCA.

To understand the main difference between the two approaches, consider an interbank transaction, which the lender funds by non-bank deposits and the borrower uses to buy assets. Assume also that this interbank link leads to contagion from the borrower to the lender in some systemic events. Thus, the link raises the expected participation of the lending bank in systemic events but leaves the participation of the borrowing bank unchanged. And since participation in systemic events is all that matters to PA, this approach attributes the entire risk associated with this interbank link to the interbank lender. By contrast, GCA splits this risk equally between the two counterparties. In this way, GCA captures the idea that an interconnected bank can contribute to system-wide risk through two channels: by directly imposing losses on its own non-bank creditors; and by indirectly imposing losses on the non-bank creditors of banks, from which it has borrowed.

The paper is organised as follows. In Section 2, we review the related literature. We then outline the analytic setup in Section 3, where we first present our definition and measure of system-wide risk and then define PA and GCA. In Section 4, we describe the empirical implementation. In Section, 5 we analyse stylised banking systems and, in Section 6, we measure systemic importance in a system of 20 large banks. We conclude with Section 7.

2. Literature review

The literature has recently proposed and used a number of alternative measures of the systemic importance of individual institutions. PA, for example, has been implemented, albeit under different names, by Huang et al (2010), Acharya et al (2009) and Brownlees and Engle (2010).[2] These papers compute the system-wide loss distribution, define a set of systemic events, and set the systemic importance of a particular bank equal to the expected losses it generates in these events. Thus, the

[2] Another strand of the literature gauges systemic importance by the impact that the failure of (or distress at) one bank has on the rest of the system. An often cited measure from this literature is CoVaR, which has been popularised by Adrian and Brunnermeier (2008). In this paper, we abstract from this and related measures as they do not attempt to allocate system-wide risk to individual institutions and, thus, are not additive across institutions. That said, as illustrated by Drehmann and Tarashev (2011), such measures can be easily implemented in the empirical framework we develop below.

papers measure systemic importance as the expected *participation* of individual institutions in systemic events. The measure corresponding to this approach can be interpreted as the actuarially fair premium that each institution should pay to a (hypothetical) provider of insurance covering system-wide losses in systemic events.

Tarashev et al (2010) propose a measure that captures the *contribution* of institutions to system-wide risk. This measure is based on Shapley values, a game theoretical concept developed by Shapley (1953) for allocating the value created in cooperative games across individual players. When the methodology is applied in the context of system-wide risk, a bank's Shapley value is a measure of its systemic importance.

Applications of the Shapley value methodology lead to important insights. For example, Tarashev et al (2010) highlight that bank size, institution-specific probabilities of default and exposures to common risk factors interact in a non-linear fashion to determine the contribution of financial institutions to system-wide risk. In a related paper, Staum (2010) uses Shapley values to design a deposit insurance scheme in the presence of fire-sale externalities and mergers. The insights of these papers are however incomplete because, just like articles adopting the participation approach, they do not consider explicitly the interbank *network* structure as a driver of systemic importance.

There is a large literature – surveyed recently by Upper (2011) and Allen and Babus (2009) – on how linkages in an interbank network influence system-wide risk. A number of the theoretical and empirical results of this literature, as well as some of the methodological challenges faced by it, re-emerge in our analysis below. An example is the finding, first reported by Allen and Gale (2000), that the interbank network determines the extent of contagion in the system and, thus, has a first-order impact on system-wide risk.

The large size of the interbank network literature notwithstanding, we are aware of only two papers that measure the systemic importance of interconnected institutions. In the first one, Gauthier et al (2009) use the approach of Tarashev et al (2010) in a system of five Canadian banks. As we show in Section 3.2.3 and Annex 1, however, this approach measures the hypothetical risk in a subsystem without removing fully the risk associated with banks outside that subsystem. As a result, the approach does not capture correctly the extent to which interconnected banks contribute to system-wide risk by propagating shocks across the network. In the second paper, Liu and Staum (2010) do tackle challenges related to the measurement of systemic importance in the presence of an interbank network. The approach they propose is different from the one we adopt below, requires complex linear programming techniques and is applied only in extremely stylized settings.

3. System-wide risk and systemic importance

In this section, we lay out the analytic framework. First, we present the system and the measure of risk we use throughout the paper. We then outline the Shapley value methodology as a general tool for attributing system-wide risk to individual institutions. Finally, we specify PA and GCA as special cases of the Shapley value methodology.

3.1 Measuring risk in the system and any subsystem

Let the system be a set N comprised of n banks, indexed by $i \in \{1,2,\cdots,n\}$. On the asset side of these banks' stylised balance sheets, there are claims on non-banks and other banks in the system. On the liability side, there are debt securities held by other banks or non-banks, as well as equity held by non-banks.

The fundamental source of distress in the system are exogenous random shocks to the value of each bank's *non-bank* assets. If an adverse realisation of these shocks drives the value of a bank's total assets below its non-equity liabilities, it experiences a *fundamental* default. In this case, the value of the bank's liabilities is written down, implying that its non-bank and interbank creditors incur losses. And if the latter losses push an otherwise solvent interbank creditor into default, there is a *contagion* default.

Even though a defaulting bank imposes losses on all its creditors – irrespective of whether it is a fundamental or contagion default – we consider *only* the stochastic losses of *non-bank* creditors for our measure of system-wide risk. Thus, we adopt the perspective of a macro-prudential authority who is only concerned with credit losses that the system as a whole could impose on the rest of the economy. Of course, we also take into account risk faced by interbank creditors and banks' equity holders but only *indirectly*, to the extent that it translates into risk for non-bank creditors. We motivate our choice as follows.

Losses on interbank exposures *should not* enter directly *any* measure of risk that takes a system-wide perspective. Since the interbank liabilities of one bank are the interbank assets of another, losses to the interbank creditors of one bank are ultimately incurred by the equity holders or non-bank creditors of one or more other banks in the system. As a result, including interbank losses in our measure of risk would involve double counting at a system-wide level.

In turn, by not *directly* accounting for losses to equity holders, we treat equity as fully loss-absorbing. In other words, a positive equity value, no matter how small, ensures the smooth functioning of a bank and does not imply any systemic repercussions. Clearly, shocks to equity would be important from a bank's *own* risk management perspective, which is beyond the scope of this paper.

In concrete terms, we measure risk by expected shortfall (ES). At an intuitive level, ES is the expected value of non-bank creditors' aggregate losses, given that these losses exceed a certain level. ES thus gauges the risk of large losses that lead to severe disruption in the financial sector and in this sense affect detrimentally the real economy.

We measure risk for the whole system N as well as for different subsystems $N^{sub} \subseteq N$. As the risk that a bank imposes on its creditors and investors can change across subsystems, we apply the superscript N^{sub} to indicate bank's risk characteristics in the particular subsystem. We further motivate this approach in the context of the Shapley value methodology (Section 3.2 below) and then demonstrate how it affects measured risk.

When applied to any subsystem, $N^{sub} \subseteq N$, ES can be written as follows:

$$ES\left(N^{sub}\right) = E\left(\sum_{i \in N^{sub}} L_i^{N^{sub}} \mid \sum_{i \in N^{sub}} L_i^{N^{sub}} \geq q^{N^{sub}}\right) \equiv E\left(\sum_{i \in N^{sub}} L_i^{N^{sub}} \mid e\left(N^{sub}\right)\right) \qquad (1)$$

where $L_i^{N^{sub}}$ are losses to the non-bank creditors of bank *i* and $q^{N^{sub}}$ is some, typically high, quantile of the probability distribution of the aggregate losses in N^{sub}.[3] In turn, $e\left(N^{sub}\right)$ is the set of loss configurations that deliver aggregate losses equal to or greater than $q^{N^{sub}}$. When the focus is on the whole system, $e(N)$ denotes *systemic events*.

3.2 Shapley values

We measure a bank's systemic importance by its Shapley value. More concretely, Shapley values are portions of system-wide risk that we attribute to individual institutions. In the next subsection, we outline the Shapley value methodology in its general form, which can be applied to a wide variety of settings and under very weak conditions. In the following subsections, we specify the participation and generalized contribution approaches as specific applications of the Shapley value methodology for measuring systemic importance.

3.2.1 General specification[4]

At the heart of Shapley values is a so-called characteristic function ϑ. In the context of a banking system, ϑ maps any subsystem $N^{sub} \subseteq N$ into a measure of risk and needs to satisfy two weak conditions. First, it should be defined on each of the 2^n possible subsystems of banks.[5] Second, when it is applied to the entire system, it should coincide with the chosen measure of system-wide risk. Thus, given equation (1), $\vartheta(N) = ES(N)$.

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[3] Even though it specifies ES in intuitive terms, expression (1) is not precise enough to be applied to general settings. For our numerical analysis, we use the more cumbersome but precise definition of Gordy (2003).

[4] This subsection draws heavily on Mas-Colell et al (1995). Shapley values were first introduced by Shapley (1953).

[5] These subsystems are: *Ø, {1}, {2}, {3}, …, {n}, {1,2}, {1,3}, …, {n-1,n}, …, {1,2,3,…,n}*.

For a given characteristic function ϑ, the Shapley value of bank *i* is a weighted average of the increments of risk that this bank generates as it joins any possible subsystem comprised of other banks. Denoting the risk of subsystem $N^{sub} \subseteq N$ by $\vartheta(N^{sub})$ and the risk in that subsystem without bank *i* by $\vartheta(N^{sub} - i)$, the Shapley value of bank *i* in system *N* is:

$$SbV_i(N) = \frac{1}{n}\sum_{n_S=1}^{n}\frac{1}{c(n_s)}\sum_{\substack{N^{sub}\supset i\\ |N^{sub}|=n_s}}\left(\vartheta(N^{sub}) - \vartheta(N^{sub} - i)\right) \quad \text{for all } i \in N \tag{2}$$

In this expression, $N^{sub} \supset i$ are all the subsystems $N^{sub} \subseteq N$ that contain bank *i*, $|N^{sub}|$ stands for the number of banks in subsystem N^{sub}, and $c(n_s) = (n-1)!/(n-n_s)!(n_s-1)!$ is the number of subsystems that contain bank *i* and are comprised of n_s banks. In addition, the empty set carries no risk: $\vartheta(\varnothing) = 0$.

For a given characteristic function ϑ, the Shapley values form the *unique* set of measures of systemic importance that satisfy a number of appealing and important properties. We mention two, while the rest are discussed at length in Tarashev et al (2010). First, the *additivity property* ensures that the sum of Shapley values equals exactly the aggregate measure of system-wide risk. Indeed, equation (2) implies that $\sum_{i=1}^{n} SbV_i(N) = \vartheta(N) = ES(N)$.

Second, the *fairness property* implies that the increment of the Shapley value of bank *i* that is due to the presence of bank *k* in the system equals the increment of the Shapley value of bank *k* that is due to the presence of bank *i*. Formally:

$$SbV_i(N) - SbV_i(N-k) = SbV_k(N) - SbV_k(N-i) \quad \text{for all } i, k \in N \tag{3}$$

While the fairness property in expression (3) holds for any well-defined characteristic function ϑ, the *size* of the increments on each side of the equality sign does change with ϑ. Thus, the choice of a characteristic function is crucial in determining the extent to which – because of interbank links, for example – the measured systemic importance of bank *i* depends on the risk generated by bank *k*, and vice-versa. Since the fairness property plays a key role in explaining our empirical results, we will concretise it further once we have introduced two characteristic functions that define two specific applications of the Shapley value methodology. These applications are the participation approach and the generalised contribution approach to measuring systemic importance.

3.2.2 Participation approach (PA)

PA can and has been defined without any reference to Shapley values. As discussed in Section 2, a number of recent papers have proposed to measure a bank's systemic importance by the expected losses the bank generates in systemic events. In the notation of equation (1), systemic events, *e(N)*, are states of the world in which aggregate losses generated by the banks in the *whole* system exceed a particular threshold: $\sum_{i\in N} L_i^N > q^N$. Thus, PA measures systemic importance as: $E\left(L_i^N \mid \sum_{i\in N} L_i^N > q^N \right) = E\left(L_i^N \mid e(N)\right)$. This is the expected *participation* of bank *i* in systemic events, through the losses it imposes directly on its own non-bank creditors. Note that $E\left(L_i^N \mid e(N)\right)$ is exactly the actuarially fair premium that bank *i* would have to pay to a scheme insuring non-bank creditors against losses in systemic events.

Interestingly, PA is a special application of the Shapley value methodology. To see this, let us define the characteristic function ϑ^{PA} as

$$\vartheta^{PA}\left(N^{sub}\right) \equiv E\left(\sum_{i\in N^{sub}} L_i^N \mid e(N) \right) \text{ for any } N^{sub} \subseteq N \tag{4}$$

Note that ϑ^{PA} qualifies as a characteristic function as it is defined on each subsystem N^{sub} and, when applied to the whole system *N*, coincides with the measure of system-wide risk: $\vartheta^{PA}(N) = ES(N)$.

To understand PA in the Shapley values context, it is useful to observe that two of the arguments of ϑ^{PA} remain constant across subsystems. First, irrespective of the subsystem, N^{sub}, ϑ^{PA} conditions on events determined at the level of the whole system: *e(N)*. Second, again irrespective of the subsystem in focus, ϑ^{PA} incorporates losses L_i^N that materialise when the whole system *N* is in place. This is despite the fact that the risk of bank *i* depends on its interbank linkages and these change typically with the subsystem. Thus, PA lets the banks in subsystem $N^{sub} \subset N$ be exposed to risks stemming from banks that are not part of this subsystem.

In deriving PA Shapley values, we use equation (4). Thus, irrespective of which subsystem a bank joins, it always changes the measured risk by the same increment: $\vartheta^{PA}\left(N^{sub}\right) - \vartheta^{PA}\left(N^{sub} - i\right) = E\left(L_i^N \mid e(N)\right)$. Since a Shapley value is a weighted average of such increments, we obtain the same PA measure of systemic importance as above:

$$ShV_i\left(N; \vartheta^{PA}\right) = ShV_i\left(N^{sub}; \vartheta^{PA}\right) = E\left(L_i^N \mid e(N)\right) \qquad \text{for } \forall i \in N^{sub} \text{ and } \forall N^{sub} \subseteq N \tag{5}$$

3.2.3 Generalised contribution approach (GCA)

The second approach to measuring systemic importance makes full use of the original idea behind the Shapley value methodology. This approach captures the risk that a bank generates on its own as well as this bank's contribution to the risk in each subsystem of other banks. And in gauging this contribution, the approach equates the exclusion of a bank from a subsystem with the removal of the *entire* risk the banks generates in that subsystem.

We define the generalized contribution approach with the following characteristic function:

$$\vartheta^{GCA}\left(N^{sub}\right) \equiv E\left(\sum_{i \in N^{sub}} L_i^{N^{sub}} \mid e\left(N^{sub}\right)\right) \text{ for any } N^{sub} \subseteq N \tag{6}$$

We note in passing that $\vartheta^{GCA}(N)$ qualifies as a characteristic function. This is because it is defined on each subsystem of banks and, when applied to the entire system *N*, coincides with the measure of system-wide risk: $\vartheta^{GCA}(N) = ES(N)$.

Even though the PA and GCA characteristic functions are consistent with the same measure of system-wide risk – i.e. $\vartheta^{GCA}(N) = \vartheta^{PA}(N) = ES(N)$ – the allocation of this risk across individual banks differs between the two approaches for two important reasons. First, unlike ϑ^{PA}, ϑ^{GCA} allows the stochastic losses $L_i^{N^{sub}}$ incurred by the non-bank creditors of bank *i* to depend on the subsystem considered. Of course, such dependence is redundant if there are no interbank links and a bank fails only because of shocks coming from outside the system. This is the setting in which Tarashev et al (2010) propose their contribution approach.

By contrast, when there is an interbank network, the absence of some banks from a particular subsystem can imply the absence of some interbank links and, thus, the absence of contagion risk for the banks remaining in the subsystem. This implies that bank-specific risks do depend on which other banks are in the subsystem, as reflected in ϑ^{GCA}. Gauthier et al (2010) abstract from this issue and thus fail to equate the removal of a bank from a (sub)system with the removal of the entire risk that this bank generates. As a result they misgauge the contribution of interconnected banks to system-wide risk. In Annex 1, we formulate their approach with our notation and show that it produces measures of systemic importance that, albeit roughly in line with PA, are in substantial disagreement with GCA.

To capture the dependence of bank-level losses on the subsystem, we implement GCA as follows. First, we assume that banks inside any given subsystem replace their exposures to banks outside this subsystem with a *risk-free* asset. Second, we make a similar assumption on the liability side: banks inside any given subsystem replace their liabilities to banks outside this subsystem by borrowing from outside the overall system. In this sense, excluding a bank from a (sub)system removes the entire risk that the bank generates in this (sub)system. The effect would be the same if the bank in question remained in the (sub)system but were fully funded with equity and, thus, could not impose credit losses on non-banks or other banks.

The second difference between ϑ^{GCA} and ϑ^{PA} is due to the fact that ϑ^{GCA} incorporates conditioning events, $e(N^{sub})$, that change with the subsystem. Thus, in contrast to ϑ^{PA}, ϑ^{GCA} measures risk as the expected shortfall in each subsystem: $\vartheta^{GCA}\left(N^{sub}\right)= ES(N^{sub})$. This leads to the following special case of the general Shapley value formula in (2):

$$ShV_i\left(N,\vartheta^{GCA}\right)=\frac{1}{n}\sum_{n_S=1}^{n}\frac{1}{c\left(n_s\right)}\sum_{\substack{N^{sub}\supset i\\ |N^{sub}|=n_s}}\left(ES\left(N^{sub}\right)-ES\left(N^{sub}-i\right)\right) \quad \text{for all } i\in N \qquad (7)$$

3.2.4 The fairness property under PA and GCA

The fairness property of Shapley values helps underscore key differences between PA and GCA. Since ϑ^{PA} does not allow the Shapley value of a bank to change with the subsystem considered (see equation (5)), it gives rise to an uninformative fairness property. Namely, for each $i,k \in N$:

$$ShV_i\left(N;\vartheta^{PA}\right)-ShV_i\left(N-k;\vartheta^{PA}\right)=0=ShV_k\left(N;\vartheta^{PA}\right)-ShV_k\left(N-i;\vartheta^{PA}\right) \tag{8}$$

By contrast, the GCA characteristic function leads to an insightful fairness property of the implied Shapley values. This comes to the fore if the simultaneous presence of two banks $i,k \in N$ raises system-wide risk (which occurs as soon as these banks have strictly positive sizes and PDs and positively correlated assets). ϑ^{GCA} then assigns a higher Shapley value to either bank in the entire system than in a system excluding the other bank. Or formally:

$$ShV_i\left(N;\vartheta^{GCA}\right)-ShV_i\left(N-k;\vartheta^{GCA}\right)=ShV_k\left(N;\vartheta^{GCA}\right)-ShV_k\left(N-i;\vartheta^{GCA}\right)>0 \tag{9}$$

To see what the fairness property of GCA implies concretely, consider an interbank transaction, which the lender (bank *i)* funds with non-bank deposits and the borrower (bank *k)* uses to buy assets. Assume also that this interbank link creates the risk of contagion from bank *k* to bank *i*. Equation (9) implies that ϑ^{GCA} splits this risk equally between the interbank lender and the interbank borrower. In this way, GCA captures the idea that an interconnected bank can contribute to system-wide risk through two channels: by directly imposing losses on its own non-bank creditors (e.g. the non-bank creditors of bank *i*); and by indirectly imposing losses on the non-bank creditors of banks, from which it (e.g. bank *k*) has borrowed.

At the same time, PA paints a different picture. To see why, note that the interbank link considered raises the expected participation of the interbank lender (bank *i*) in systemic events but leaves that of the interbank borrower (bank *k*) unchanged. And since participation in systemic events is all that matters to PA (recall Section 3.2.2), ϑ^{PA} attributes the *entire* risk associated with the interbank link to the interbank lender.

4 Empirical implementation

The methodology outlined so far can be applied to any probability distribution of losses, as long as it is well-defined for each subsystem. To specify a particular distribution, we start by defining the stochastic losses incurred by the non-bank creditors of bank *i* when subsystem N^{sub} is in place as:

$$L_i^{N^{sub}} = S_i \cdot LGD_i^{N^{sub}} \cdot I_i^{N^{sub}} \qquad \text{for all } i \in \{1,2,\cdots,n\} \tag{10}$$

In this expression S_i is the size of bank *i*. In the light of the discussion in Section 3.1, we set S_i to equal the debt liabilities of bank *i* to non-banks. In turn, $LGD_i^{N^{sub}}$, or loss given default, is the share of non-banks' credit exposure to bank *i* that is lost if this bank defaults. Finally, $I_i^{N^{sub}} = 1$ if bank *i* defaults and $I_i^{N^{sub}} = 0$ otherwise.

We denote by X_i the exogenous shocks that affect banks' non-bank assets and can lead to fundamental defaults. These shocks are driven by common and idiosyncratic factors, which we assume to be mutually independent normal variables, denoted respectively by M_j and Z_i:

$$X_i = \rho_i \cdot M + \sqrt{1-\rho_i^2} Z_i \qquad \text{for all } i \in \{1,2,\cdots,n\} \tag{11}$$

We refer to the probability that X_i pushes bank *i* into default as this bank's *fundamental PD*, This PD is independent of the subsystem considered. Higher common factor loadings, $\rho_i \in [0,1]$, lead to a higher probability of *joint* fundamental defaults.

The probability that a bank is pushed into default by the default of one (or more) of its bank obligors, even though it has survived the shocks to its non-bank assets, is this bank's *contagion* PD. By definition, the overall PD of a bank equals its fundamental PD plus its contagion PD.[6]

We employ the following simulation procedure. In line with equation (11), we draw a set of correlated exogenous shocks – one shock for each bank – which determines which banks experience

[6] Since the bank obligors of a bank can change with the subsystem, contagion PDs can change with the subsystem as well. This is captured by GCA but not by PA.

fundamental defaults. If there is a fundamental default, we derive the ensuing contagion defaults, if any, via the "clearing algorithm" of Eisenberg and Noe (2001). When any bank defaults, we assume that positions are not netted. A defaulting bank imposes losses on its creditors via two channels. First, the value of such a bank's liabilities are written down to reflect the drop in its assets. We assume that non-bank creditors are senior to bank creditors, which means that liabilities to non-banks are written down only when the value of the liabilities to other banks is written down to zero.[7] Second, there are bankruptcy costs, which reduce further the value of a defaulting bank's debt liabilities vis-à-vis both non-banks and other banks by a fraction a. In all simulations, a = 20%.[8] In this way, we obtain one realisation of aggregate losses for the initial set of exogenous shocks. To construct a probability distribution of these losses, we draw one million sets of exogenous shocks. Finally, for the calculation of ES, we set $q^{N^{sub}}$ to the 99th percentile of this distribution.

The discussion in Section 3.2 indicates that the computational burden could differ substantially between PA and GCA. Namely, we need to apply the above simulation procedure only once in the case of PA: to the entire system. By contrast, we need to apply this procedure 2^n times in the case of GCA: once to each subsystem in a system of n banks.

5 Hypothetical Networks

In this section, we build intuition for the proposed measures of systemic importance and their dependence on interbank networks by analyzing four stylized 9-bank systems that are easy to compare to each other. The differences among the systems stem entirely from the underlying interbank networks: we consider one system without interbank connections, as well as three different

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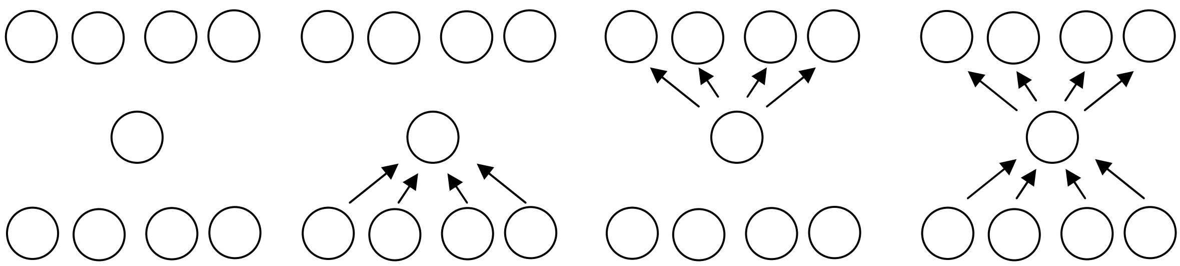
7 As reported by Upper (2011), non-bank depositors are for example senior to interbank creditors in Germany. Relaxing the assumption about the seniority of non-bank depositors would weaken the impact of the interbank network on system-wide risk but would not alter qualitatively the results we obtain below.

8 For a sample of failed US banks, James (1990) estimates that losses on bank assets amount to around 30% on average. This contains direct losses as well as loss of charter value and at least 10 percentage points of administrative and legal expenses. Background calculations reveal that the insights of our paper are preserved for any a between 10 and 30%, even if a is allowed to differ between bank and non-bank creditors.

structures of interbank networks (Graph 1). In each of the three setups with an interbank network there is one bank with a central role, henceforth the centre bank. In the first two of these networks, four periphery banks either borrow from or lend to the centre bank, while the remaining four banks do not participate in the interbank market. The last network captures in a stylised fashion the real-life phenomenon in which the centre bank intermediates between periphery banks, four of which borrow and four of which lend to it.[9]

Graph 1: Hypothetical interbank networks

No interconnections | Centre bank borrows | Centre bank lends | Centre bank intermediates



Balance sheets in the different systems are shown in Table 1. In all cases, banks have 5 units of equity and borrow 87 units from non-banks. This means that they are of the same size (recall Section 3.1). Periphery banks have 8 units of interbank liabilities (if they borrow from the centre bank) or 8 units of interbank assets (if they lend), which fully determines the interbank positions of

Table 1: Balance sheet of hypothetical banks

			No interbank connections			**Centre bank borrows**			**Centre bank lends**			**Centre bank intermediates**		
	EQ	NBL	IBL	IBA	TA	IBL	IBA	TA	IBL	IBA	TA	IBL	IBA	TA
No IB connection	5	87	0	0	92	0	0	92	0	0	92			
PB lender	5	87				0	8	92	0	0	92	0	8	92
PB borrower	5	87				0	0	92	8	0	100	8	0	100
CB	5	87				32	0	124	0	32	92	32	32	124

Note: IB: interbank market; PB: periphery bank; CB: centre bank; EQ: equity; NBL: non-bank liabilities (=size); IBL: interbank liabilities; IBA: interbank assets; TA: total assets.

[9] For theoretical and empirical investigation of such "tiered" network structures, see Bech and Atalay (2010), Craig and von Peter (2010) and Kahn and Roberts (2009).

the centre bank. The resulting share of interbank positions in a periphery (centre) bank's balance sheet is close to the mean (maximum) of the corresponding shares in our sample of 20 large banks (see Section 6 below).

When simulating the probability distribution of system-wide losses, we draw exogenous shocks to non-bank assets in line with the simulation procedure outlined in Section 4. The correlation of these shocks across banks is governed by a common factor loading ρ, which is the same across all banks and matches the average common-factor loading of 0.67 in our data on 20 large banks (see below). In addition, the exogenous shocks are calibrated so that the fundamental PD of each bank in the system with no interbank linkages is 0.42%, halfway between the median and mean PD estimates for the same 20 banks. We maintain the same fundamental PD for each bank in each of the other three stylised systems.

5.1 Systemic importance under different network structures

A priori, the network structure should affect both the absolute and relative levels of systemic importance of individual banks. If all banks are ex ante the same, the network that is more vulnerable to contagion defaults should lead to higher levels of system-wide ES and thus to higher uniform Shapley values. In a system of heterogeneous banks, however, it seems intuitive that the most interconnected bank – in the stylised examples, the centre bank – should have the highest Shapley value as it propagates shocks through the system and is subject to propagated shocks. Table 2 reports results related to the stylised systems, all expressed relative to total system size for the sake of comparability.

As expected, system-wide risk increases with the potential for contagion in the system. By design, the system without an interbank network does not experience contagion defaults and, as a result, has the lowest ES. The level of ES is higher in the two systems where the centre bank either borrows from or lends to the periphery, thus making contagion defaults possible. ES is highest when the centre bank intermediates between periphery banks. In this case, there is an additional channel of shock propagation, as the default of one (or several) borrowers in the periphery can be transmitted, via the default of the centre bank, to lenders in the periphery.

Table 2: Results for the hypothetical banking system[1]

				Shapley Values	
	ES	f.PD[2]	c.PD[2]	GCA	PA
No interconnections	4.01				
All (9 banks)		0.42	0	0.45	0.45
Centre bank borrows	4.95				
Centre bank		0.42	0	0.90	0.64
PB lender (4 banks)		0.42	0.10	0.56	0.66
No IB connection (4 banks)		0.42	0	0.45	0.42
Centre bank lends	5.16				
Centre bank		0.42	0.51	1.06	1.63
PB borrower (4 banks)		0.42	0	0.57	0.48
No IB connection (4 banks)		0.42	0	0.45	0.40
Centre bank intermediates	7.73				
Centre bank		0.42	0.51	2.06	1.78
PB lender (4 banks)		0.42	0.29	0.71	1.00
PB borrower (4 banks)		0.42	0	0.71	0.49

Note: [1] All values are in per cent. ES and Shapley values are expressed per unit of system size. ES pertains to the system as a whole. All other values pertain to a bank in the particular group. [2] f.PD and c.PD are fundamental and contagion PDs, respectively.

More interestingly, Table 2 also shows how the network structure and banks' position in it affect their systemic importance. For concreteness, we focus only on GCA Shapley values in this subsection (second to last column in the table) and compare GCA and PA Shapley values in the next. Not surprisingly, all banks feature their lowest Shapley values in the system without an interbank network, when system-wide ES is at its lowest.

Across the three setups with interbank linkages, the Shapley value of the centre bank is always higher than that of any periphery bank. While this is intuitive, it is not obvious when looking purely at contagion PDs. The system in which the centre bank only borrows from the periphery is a case in point. Here, the centre bank is a source of risk for periphery banks but is never pushed into default by another bank, i.e. has a contagion PD of zero. Nonetheless, the GCA Shapley value of the centre bank is more than 60% larger than the Shapley value of each interbank lender.

This result is driven by the fairness property of Shapley values. As discussed in Section 3.2.4, this property implies that, when an interbank link raises system-wide risk, GCA splits the rise equally between the borrower and the lender. This can be easily seen in the stylised systems, where drivers of system-wide risk unrelated to the interbank network – i.e. fundamental PDs, correlations of exogenous shocks and size – are held constant across banks. For example, switching from the system without a network to the system in which the centre bank only borrows from the periphery raises the system-wide ES by 0.94 percentage points (from 4.01% to 4.95%). To attribute this rise equally to borrowers and lenders is to attribute 0.5*0.94% = 0.47% to the centre bank and (0.5*0.94%)/4 = 0.1175% to each of the four lending periphery banks. This matches almost perfectly the actual increases in Shapley values relative to the system without an interbank network. These are 0.45% (= 0.9%-0.45%) for the centre bank and 0.11% (= 0.56%-0.45%) for the periphery banks.[10] The same intuition holds if the centre bank acts only as an interbank lender or if it intermediates.

When the centre bank intermediates between periphery banks (Table 2, bottom panel), its Shapley value (2.06%) is larger than the *sum* of its Shapley values when it only borrows or lends (0.90% + 1.06%). The reason is that an intermediating centre bank creates indirect links between periphery banks. Because of these links, an adverse shock to an interbank borrower in one part of the system can cause the default of an interbank lender in another part. This highlights that a correct measure of the contribution of an institution to system-wide risk is one that adopts a holistic perspective and takes into account both direct and indirect linkages in the system.

5.2 Comparing GCA and PA: Borrowers vs. lenders

PA and GCA can generate materially different Shapley values in the presence of an interbank network. To a large extent, this is due to the two approaches treating the counterparties in interbank links differently. As explained in Section 3.2.2 above, PA attributes the risk generated by an interbank

[10] The match is not exact because we compare two different systems, underpinned by different interbank networks, whereas the fairness property in equation (9) holds exactly within a given system of banks.

link to the lender. By contrast, GCA treats the two counterparties of an interbank link symmetrically and, thus, splits the associated risk equally between them. This raises (lowers) the GCA Shapley value of an interbank borrower (lender) relative to the corresponding PA value.

Indeed, in *all* the stylised systems, an interbank borrower (lender) has a higher (lower) Shapley value under GCA than under PA (see the last two columns in Table 2). Importantly, this leads the alternative approaches to rank order differently individual banks' systemic importance. For example, if the centre bank only borrows from the periphery, PA Shapley values underplay this bank's role as a propagator of shocks and attribute lower systemic importance to it than to interbank lenders in the periphery (0.64% vs. 0.66%). By contrast, the centre bank has the highest GCA Shapley value in that system (0.9% vs. 0.56% for periphery lenders). This reflects the fact that, by being *the* only propagator of shocks, this bank is the main *contributor* to system-wide risk.

5.3 Comparing GCA and PA: Intermediaries

It is also of interest to compare the implications of GCA and PA for a centre bank that *intermediates* on the interbank market. Taken at face value, the results from the previous subsection seem to imply that GCA and PA should assign similar Shapley values to the intermediating bank, as its interbank lending exactly equals its interbank borrowing by construction (Table 1, right-hand panel). Such a conclusion ignores, however, that an intermediating centre bank creates *indirect* exposures between periphery lenders and borrowers. And by the fairness property in equation (9), part of the risk stemming from these exposures raises the GCA Shapley value of the centre bank. By contrast, PA attributes this risk entirely to periphery lenders. In the end, the PA Shapley value of the intermediating bank is lower than the GCA value (1.78% vs. 2.06%, bottom panel of Table 2).

The difference between PA and GCA becomes particularly stark when we assume that the centre bank acts as a central counterparty (CCP), thus *only intermediating* on the interbank market but neither lending to nor borrowing from non-banks. As explained in Section 3.1, the centre bank is then of zero size. Table 3 reports balance sheet information and risk measures for the system in which the centre bank acts as a CCP.

Table 3: Centre bank as a central counterparty

	Balance sheets[1]				Bank-specific risk measures[2] (system-wide ES = 4.85)			
							Shapley values	
	EQ	NBL	IBL	IBA	f. PD	c. PD	**GCA**	**PA**
Centre bank	3	0	32	32	0	0.19	**0.22**	**0**
PB lender	5	87	0	8	0.42	0.07	**0.58**	**0.70**
PB borrower	5	87	8	0	0.42	0	**0.58**	**0.52**

[1] PB: periphery bank; EQ: equity; NBL: non-bank liabilities (= size); IBL: interbank liabilities; IBA: interbank assets. To satisfy the balance sheet identity, we assume that the centre bank invests 3 units in a risk-free asset. [2] All values are in per cent. ES and Shapley values are expressed per unit of system size. ES pertains to the system as a whole. All other values pertain to a bank in the particular group. f.PD and c.PD are fundamental and contagion PDs, respectively.

Whereas the GCA Shapley values are in accordance with the intuition that the centre bank *contributes* to system-wide risk, the PA Shapley value of this bank is zero (Table 3, last two columns). The reason for the latter result is twofold. First, the centre bank does not lend to non-banks, therefore has a fundamental PD of zero. Second, since it does not borrow from non-banks, the centre bank cannot contribute *directly* to their credit losses and, thus, *never participates* in systemic events (in the sense introduced in Section 3.2.2). That said, this bank creates indirect links between lending and borrowing periphery banks, thereby *contributing* to system-wide risk. GCA captures this risk-transmission channel and assigns a Shapley value of 0.22% to the CCP.

While this is a stylised example, it highlights a key conceptual difference between the approach capturing *participation* in systemic events and that capturing *contribution* to system-wide risk. The example also underscores that there could be substantial quantitative differences between the measurements of the systemic importance of individual institutions under the two approaches.

6 Analysing the systemic importance of 20 large banks

In this section, we build on the insights of the stylized hypothetical networks in order to analyse the systemic importance of 20 large internationally active banks. Our balance sheet data for these 20 banks – provided by Bankscope for end-2009 – are summarised in Table 4. As reported by the first row in this table, the largest bank in our sample (bank C) is 3.5 times larger than the smallest (bank G). The table also shows the shares of each bank's interbank assets (*IBA* = loans and advances to banks) and interbank liabilities (*IBL* = deposits from banks) in the bank's total assets and liabilities, respectively. Both *IBA* and *IBL* average approximately 10% across banks.[11]

The lack of publicly available information on *bilateral* interbank exposures implies that we need to estimate the interbank network matrix. Most of the literature does so via a maximum entropy (ME) matrix of bilateral positions, which satisfies two conditions simultaneously. First, the sum of the entries in each row / column corresponding to a particular bank equals the aggregate level of this banks' liabilities / assets vis-à-vis the other banks in the system. Second, the interbank assets and liabilities of each bank are distributed as uniformly as possible across the other banks in the system. Unfortunately, the second assumption is clearly ad hoc, which suggests that focusing purely on the ME matrix could bias our results.

Table 4: Descriptive statistics of the 20 large internationally active banks, in per cent

	Bank																			
	A	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**	**K**	**L**	**M**	**N**	**O**	**P**	**Q**	**R**	**S**	**T**
Size	5.6	5.5	8.2	5	3.1	6.4	2.3	6.6	6.8	4.9	5.1	4.7	4.8	7.7	4.4	4.1	3.8	3.9	3.5	3.6
IBA	9.6	13.4	4.3	21	12.6	21.7	20.8	3.1	7.9	3.7	18.6	3.4	3.8	5.4	7.2	6.6	5.8	10.3	8.4	3.2
IBL	13	20.8	11.4	9.2	17.3	9	25.1	3.1	5.9	7.5	14.2	8.4	4	8.9	8.3	9.5	1.9	5	12.4	2.3

Note: Size: liabilities to non-banks divided by aggregate non-bank liabilities; IBA: a bank's interbank assets divided by its total assets; IBL: a bank's interbank liabilities divided by its total liabilities.

We, therefore, also randomly simulate other interbank matrices, all of which are consistent with the observed data. On the basis of 225 random perturbations around the ME matrix, we choose the one that differs the most (in terms of the 2-norm distance) from the ME matrix. As 75% of the off-diagonal entries of this matrix are zero – meaning that it concentrates all interbank positions in 25% of the possible links – we refer to it as a high-concentration (HC) matrix, Annex 2 contains

[11] When interpreting these shares, it is useful to keep in mind that (i) total interbank positions of any of the 20 banks in our sample include positions vis-à-vis banks that we abstract from; and (ii) part of the actual interbank links stem from off-balance sheet positions, which we cannot identify in our data. The former (latter) implies that we may overstate (understate) the importance of the network.

further detail on the construction of ME and HC matrices. In order to parallel the analysis of stylised networks, we also consider the zero matrix, which rules out interbank positions.

Next, we set bank-level PDs to the average value of each bank's monthly *one-year* expected default frequency (EDF), as estimated by Moody's KMV for 2006-2009. These PDs average 0.6% in the cross section, have a median of 0.25% and a standard deviation of 0.8%.

To calibrate the asset-return correlations across banks, we use the last three correlation matrices estimated by Moody's KMV: for 2006, 2007 and 2009. As explained in Annex 2, we condense these matrices to 20 bank-specific common-factor loadings, corresponding to ρ_i in equation (11). These loadings average *0.67*, implying an average correlation of exogenous shocks of roughly $0.67^2 = 0.45$.

Finally, we calibrate the distribution of the exogenous shocks in equation (11). We set the mean and variance of these shocks to be such that the frequencies of fundamental defaults plus the frequencies of contagion defaults, caused by the propagation of shocks through the interbank network *under the ME matrix*, match *exactly* the overall PDs we derive from the Moody's KMV data. In order to study how alternative network structures, as implied by the zero or the HC matrix, affect banks' PDs, we keep the distribution of exogenous shocks constant across matrices.

For each of the three interbank networks, Table 5 reports fundamental and contagion PDs. As we are not allowed to publish bank-specific PD estimates from Moody's KMV, we only report the rank of fundamental PDs (1 = highest and 20 = lowest). The values of these PDs remain the same across networks. By contrast, contagion PDs, for which we report actual values, do change with the structure of the network.

In the remainder of this section, we examine how system-wide risk and the systemic importance of each of the 20 banks depend on the interbank network. For the analysis of systemic importance, we first focus on results obtained under GCA and then compare these results to those obtained under PA. Of course, system-wide risk and systemic importance are also influenced by banks' size, PD and common factor loadings. We relegate the analysis of these drivers to Annex 3.

6.1 The impact of the network on system-wide risk

In this section, we examine how the measured level of system-wide risk changes when we replace the zero matrix of interbank exposures with either the ME or HC matrix.

Not surprisingly, interbank linkages raise system-wide ES, as they give rise to positive contagion PDs. In quantitative terms, the rise is substantial and roughly the same under the two network structures. Concretely, system-wide ES increases by roughly 40%, from 3.2% of the system size under the zero matrix to 4.4% under the ME matrix and to 4.6% under the HC matrix (Table 5).

The values of ES differ little between the ME and HC matrices, even though they distribute the aggregate interbank lending and borrowing of each bank differently across counterparties. This is because switching from the ME to the HC matrix sets two counteracting forces at work. To see this,

Table 5: System-wide risk and systemic importance in a system of 20 large banks
In per cent

		No interbank network ES = 3.2 median PD = 0.14			**ME matrix** ES = 4.4 median PD = 0.25			**HC matrix** ES = 4.6 median PD = 0.31		
			Shapley values			**Shapley values**			**Shapley values**	
Bank	**f.PD rank**	**c.PD**	**GCA**	**PA**	**c.PD**	**GCA**	**PA**	**c.PD**	**GCA**	**PA**
A	6	0	0.27	0.28	0.05	0.36	0.28	0.06	0.36	0.31
B	14	0	0.06	0.07	0.15	0.24	0.22	0.09	0.21	0.19
C	16	0	0.04	0.04	0.07	0.09	0.15	0.05	0.12	0.13
D	3	0	0.44	0.45	0.13	0.55	0.48	0.17	0.56	0.46
E	4	0	0.18	0.11	0.20	0.28	0.21	0.15	0.29	0.22
F	20	0	0.00	0.00	0.14	0.09	0.18	0.17	0.13	0.22
G	19	0	0.00	0.00	0.15	0.08	0.07	0.09	0.05	0.04
H	7	0	0.32	0.34	0.08	0.35	0.40	0.12	0.37	0.46
I	17	0	0.03	0.03	0.07	0.07	0.12	0.08	0.07	0.13
J	9	0	0.10	0.10	0.07	0.13	0.15	0.12	0.16	0.18
K	12	0	0.07	0.07	0.11	0.17	0.17	0.25	0.29	0.29
L	13	0	0.06	0.06	0.06	0.09	0.10	0.04	0.08	0.08
M	2	0	0.53	0.58	0.05	0.54	0.52	0.04	0.55	0.45
N	5	0	0.50	0.49	0.07	0.58	0.57	0.08	0.56	0.57
O	18	0	0.01	0.01	0.04	0.02	0.04	0.05	0.03	0.05
P	10	0	0.08	0.08	0.10	0.14	0.15	0.04	0.11	0.11
Q	1	0	0.38	0.37	0.07	0.38	0.31	0.04	0.36	0.32
R	11	0	0.04	0.03	0.14	0.09	0.13	0.29	0.13	0.21
S	15	0	0.03	0.02	0.06	0.06	0.07	0.04	0.06	0.05
T	8	0	0.10	0.07	0.02	0.09	0.07	0.02	0.10	0.08

Note: f.PD: fundamental PD; c.PD: contagion PD; GCA: generalised contribution approach; PA: participation approach; ME: maximum entropy; HC: high concentration. Shapley values are expressed per unit of system size.

recall that the ME matrix spreads interbank positions as widely as possible, whereas the HC matrix is constructed so that these positions are highly concentrated. Thus, if two banks are connected under both matrices, the shock caused by the default of one bank would tend to affect more strongly the other under the HC matrix, as the particular bilateral exposure would tend to be larger. On its own, this increases the probability of joint defaults under the HC matrix relative to that under the ME matrix, making the difference between the respective ESs positive. That said, each shock is propagated to fewer institutions under the HC than under the ME matrix. On its own, this leads to a negative difference between the respective ESs. In our particular case, the two effects roughly balance each other.

6.2 The impact of the network on systemic importance

As interbank linkages raise system-wide risk, they tend to elevate the systemic importance of each individual bank in the system. And the impact is strongest on banks that are most active in the interbank market. Consider banks *B*, *F*, *G* and *K*, for example, which feature the largest interbank assets and liabilities as a share in the respective balance sheets (see Table 4). These are the banks that experience the greatest rises in GCA Shapley values when interbank linkages are introduced (compare the left to the middle and right-hand panels of Table 5).

The two network structures HC and ME can lead to materially different levels of individual banks' systemic importance. This is despite the similarity of the corresponding levels of system-wide risk, reported in the previous subsection. For example, the GCA Shapley value of bank *K* is 70% higher when interbank markets are characterized by the HC matrix (0.29% of the system size) rather than the ME matrix (0.17%) (Table 5, middle and right-hand panels). This increase is largely driven by the substantial rise in the bank's contagion PD (from 0.11% to 0.25%), which in turn is caused by the greater concentration of its interbank exposures under the HC matrix. The picture is different for bank *G*, as the HC matrix concentrates the exposures of this bank to low-PD counterparties. As we keep the distribution of exogenous shocks fixed, this lowers the bank's PD and, ultimately, its GCA Shapley value relative to that under the ME matrix.

The impact of the network structure goes beyond contagion PDs. Bank *C*, for example, experiences a 33% *rise* in its GCA Shapley value (from 0.09% of the system size to 0.12%), even though its contagion PD *falls* from 0.07% to 0.05% when the interbank network changes from the ME to the HC matrix. Having the second largest level of interbank liabilities, this bank is a key propagator of shocks through the system. And the impact of shock propagation is greater, i.e. bank *C* increases the risk of its interbank creditors by more, when the interbank network is more concentrated. This is captured by GCA, thus underpinning a higher Shapley value for bank *C* under the HC matrix.

In order to study the impact of the network structure on systemic importance more formally, we construct two indicators. The first indicator gauges how the interbank network affects the extent to which a bank increases the risk of its *own* non-bank creditors. Given that the contagion PD reflects the impact of the network on the likelihood that a bank defaults and that the impact of a bank's default on its non-bank creditors is proportional to the bank's size, we construct the following lending indicator (*LI*) for a generic bank *i*:

$$LI_{nwk,i} = c.PD_{nwk,i} * S_i \tag{12}$$

where *nwk* indicates that the contagion PD, and thus the indicator, changes with the underlying network.

Our second indicator captures the notion that each interbank borrower is an *indirect* source of risk for the non-bank creditors of other banks. Paralleling *LI,* this indicator consists of two components. The first component reflects the impact that each bank *i* has on the risk of each other bank *j*. To measure this impact, we apply the Shapley value methodology to contagion PDs in order to allocate a portion of the contagion PD of bank *j* to bank i. We denote this component by $c.PD^{i}_{nwk,j}$. Note that this component captures the risk of direct contagion from bank *i* to bank *j,* as well as indirect channels of contagion from bank *i,* via a default of bank *k*, to bank *j*.

The second component of the indicator reflects the fact that the impact of bank i on the non-bank creditors of bank j would be higher if bank j itself borrowed heavily from non-banks, i.e. if bank j had a large size, S_j. Putting the two components together and summing across all the counterparties of bank i, leads to the borrowing indicator (BI_i):

$$BI_{nwk,i} = \sum_{j \neq i} c.PD^{i}_{nwk,j} * S_j \tag{13}$$

Given that a rise in either $LI_{nwk,i}$ or $BI_{nwk,i}$ signals a higher contribution to system-wide risk by bank i, we expect a positive relationship between GCA Shapley values and the values of either indicator, irrespective of the assumed interbank network. By extension, we also expect that the *difference* between the Shapley values under the ME network and those under the HC network is related positively to the two indicator differences, $LI_{ME,i} – LI_{HC,i}$ and $BI_{ME,i} – BI_{HC,i}$, respectively.

Table 6: Regression results

	GCA_{ME}-GCA_0	GCA_{HC}-GCA_0	GCA_{ME}-GCA_{HC}	PA_{HC}-GCA_{HC}
LI_{nwk} (1)	14.0***	10.4***		9.9***
BI_{nwk} (1)	5.2***	6.0***		-11.1***
LI_{ME} - LI_{HC}			9.5***	
BI_{ME} - BI_{HC}			9.0***	
R^2	0.77	0.77	0.89	0.82
# obs	20	20	20	20

Note: All regressions include a constant, which is sometimes significant but is not reported. (1) *nwk* is either ME for the regression of GCA_{ME}-GCA_0 or HC for the other two regressions.

Results from linear regressions confirm that LI_{nwk} and BI_{nwk} explain quite well the impact of the network structure on the systemic importance of individual banks (Table 6, first three columns).[12] Considering only HC or ME at a time, the corresponding indicators have positive and highly significant coefficients and lead to a very good fit of the regression (an R2 of 77%). Likewise, the differences $LI_{ME,i} – LI_{HC,i}$ and $BI_{ME,i} – BI_{HC,i}$ help explain the difference between ME and HC Shapley

[12] We consider linear regressions in order to study the explanatory power of the indicators under a simple specification. In general, the true relationship between an indicator and a measure of systemic importance would be non-linear. Tarashev et al (2010) derive this formally for the impact of bank size on systemic importance.

values. Both coefficients are with the expected positive sign, are highly significant and lead to an R^2 of 89%.

The findings of this and the previous subsections underscore the value of detailed information on interbank activity. Not only does the interbank network have a material impact on system-wide risk but different structures of this network – consistent with the same data on the aggregate interbank positions of each bank – can have very different implications for the systemic importance of individual institutions. It then follows that, in order to evaluate systemic importance, it is necessary to know more than just aggregate interbank positions. Data on *bilateral* positions would have a clear value added.

6.3 Comparing GCA and PA

GCA and PA can disagree substantially about the levels of systemic importance of individual banks (see Table 5). And even though the 20 banks we consider differ in a number of aspects – such as size, probability of default and exposure to exogenous common factors – a banks' role in the interbank network is the main driver of the difference between its GCA and PA Shapley values. This can be seen more formally by calculating the sum of absolute differences between GCA and PA Shapley values under the zero network matrix (left-hand panel of Table 5) and dividing it by the corresponding sum under the ME or the HC matrix (centre and right-hand panels). The resulting ratios equal 0.35 and 0.31, respectively, indicating that roughly two-thirds of the disagreement between GCA and PA is rooted in their different approaches to the risk associated with interbank links. This highlights that the choice of a particular approach to measuring systemic importance matters not only from a conceptual but also from an empirical point of view, and more so when interbank-market links are present.

In our analysis of hypothetical networks we have argued that different underlying versions of the fairness property result in PA Shapley values being larger than GCA ones for an interbank lender, and smaller for an interbank borrower. In this subsection, we show that this argument holds for the system of 20 real-world banks as well. For concreteness, we focus on results obtained under the HC matrix of interbank positions.

We start the comparison between PA and GCA Shapley values by zooming onto two banks. The first bank, *E*, has an average lending indicator (*LI*) in the cross-section, which indicates that the bank imposes an average level of risk on its direct non-bank creditors. However, the same bank has the third highest borrowing indicator (*BI*) and, thus, is a significant source of contagion in the interbank market, imposing significant risk on other banks. Since only GCA captures the latter characteristic of bank *E*, the GCA Shapley value of this bank is one-third higher than its PA Shapley value (0.29% vs. 0.22%). The relationship between GCA and PA Shapley values is reversed for the second bank, *R*, which has the third highest *LI* and the third lowest *BI*. Its high *LI* indicates that it is quite vulnerable to risk from the interbank market. This increases bank *R*'s participation in systemic events, boosting its PA Shapley value. GCA, however, attributes part of this risk to banks borrowing from bank *R*. The upshot is that PA attributes a 60% higher level of systemic importance to bank *R* than GCA: 0.21%, and 0.13% of the system size, respectively.

More generally, *BI* and *LI* exhibit statistically significant explanatory power for the difference between PA and GCA Shapley values (Table 6, right-most panel).[13] The positive sign of the *LI* coefficient indicates that, as anticipated, a PA Shapley value tends to be higher than the corresponding GCA Shapley value when the bank creates system-wide risk mainly by borrowing from non-banks in order to lend on the interbank market. Likewise, the negative coefficient of *BI* confirms that GCA tends to assign a higher Shapley value than PA to an interbank borrower.

7 Conclusion

In this paper we provide a framework for analysing the systemic importance of interconnected banks. We explore two approaches to measuring systemic importance, which allocate the same quantum of

[13] The results are equally pronounced under the ME matrix.

system-wide risk differently across banks. The reason is that the two approaches correspond to different concepts of systemic importance: (i) bank's participation in systemic events or (ii) banks' contributions to system-wide risk.

A key difference between the participation and the generalised contribution approach is the way in which they allocate the risk associated with an interbank transaction to different counterparties. The participation approach assigns this risk to the interbank lender because it is the direct source of risk to non-banks. In contrast, the generalised contribution approach splits the risk equally between the two counterparties. In this way, this approach captures the idea that the systemic importance of an interconnected bank depends not only on the risk it imposes directly on the real economy, but also on the risk it generates by both borrowing from and lending to other banks. Our empirical analysis shows that this is not only a theoretical consideration but that measurements of systemic importance for individual banks can differ materially across approaches, in particular when banks are interconnected.

Our findings highlight two policy messages. First, it is important that macro-prudential authorities possess data on bilateral interbank positions. In contrast to the aggregate data that are typically available at present, bilateral data accurately reveal the network structure, which is a key determinant of system-wide risk and the systemic importance of individual institutions.

Second, since alternative measures of systemic importance provide materially different results, macro-prudential authorities need to have a clear understanding which measure is designed to address the policy question at hand. If the goal is to design a scheme for insuring against losses in systemic events, then the participation approach provides the natural measure. Alternatively, if the authority's mandate is to ascertain the contribution of individual institutions to system-wide risk – which calls for analysing the propagation of shocks through the system – it is necessary to use the generalised contribution approach.

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Annex 1: Mechanical contribution approach

It is possible to design Shapley values that decompose system-wide ES differently from the participation approach (PA) and the generalised contribution approach (GCA), presented in Section 3.2. Gauthier et al (2010) do so by adopting what we refer to as the mechanical contribution approach (MCA). It is defined by the following characteristic function:

$$\vartheta^{MCA}\left(N^{sub}\right) \equiv E\left(\sum_{i \in N^{sub}} L_i^N \mid \tilde{e}\left(N^{sub}\right)\right). \tag{A1}$$

This characteristic function is somewhat of a cross between ϑ^{PA} and ϑ^{GCA}. First, like ϑ^{PA}, ϑ^{MCA} considers only the losses that bank *i* generates in the entire system, L_i^N. Second, like ϑ^{GCA}, ϑ^{MCA} allows the conditioning events, $\tilde{e}\left(N^{sub}\right)$, to change across subsystems. That said, these events are determined by the distribution of $\sum_{i \in N^{sub}} L_i^N$ and are thus different from the events $e\left(N^{sub}\right)$ incorporated in GCA, which are determined by the distribution of $\sum_{i \in N^{sub}} L_i^{N^{sub}}$.

These features of ϑ^{MCA} have two implications. First, MCA is computationally more efficient than GCA. This is because the former approach requires that the probability distribution of losses be calculated only once (for the overall system), whereas the latter requires the calculation of a loss distribution for each of the 2^n subsystems. Second, MCA does not capture fully the true contribution of a bank to system-wide risk. The reason is that, by keeping the risks associated with a particular bank fixed across subsystems, ϑ^{MCA} does not equate the removal of another bank from a (sub)system with the removal of the entire risk that this bank generates. And this creates a potential for material differences between the implications of MCA and those of GCA.

We use the hypothetical networks outlined in Section 5 to quantify the differences between MCA Shapley values and Shapley values under PA and GCA. The findings, reported in Table A1, reveal significant differences between the GCA and MCA Shapley values. Concretely, when the centre bank intermediates, the absolute differences between the MCA and GCA Shapley values equal on average one-third of GCA Shapley values. And the results are similar across all hypothetical systems.

That said, MCA Shapley values are much better aligned with PA Shapley values. In the light of equations (4), (6) and (A.1), this indicates that the assumption about stochastic losses (L_i^N vs. $L_i^{N^{sub}}$) has a larger impact on measured systemic importance than the assumption about conditioning events ($e(N)$ vs. $\tilde{e}(N^{sub})$).

Table A1: MCA results for the hypothetical banking systems

	No	CB borrows			CB lends			CB intermediates			CB as CCP		
	All	CB	PB lend	PB no	CB	PB borrow	PB no	CB	PB lend	PB borrow	CB	PB lend	PB borrow
GCA	0.45	0.9	0.56	0.45	1.06	0.57	0.45	2.06	0.71	0.71	0.22	0.58	0.58
PA	0.45	0.64	0.66	0.42	1.63	0.48	0.4	1.78	1	0.49	0	0.7	0.52
MCA	0.45	0.62	0.64	0.44	1.36	0.51	0.44	1.56	1.02	0.52	0	0.87	0.34

Note: All values are in per cent. CB = Centre bank; PB = periphery bank, which lends to or borrows from the centre bank or has no interbank links. "lend" and "borrow" indicate whether periphery banks are interbank lenders or borrowers. All systems are the same as those in Table 2.

Annex 2: Calibrating the system of 20 banks: some technical detail

This annex provides further information about the calibration of the system of 20 banks, which is introduced in Section 6. The annex has two parts. The first relates to the construction of interbank network matrices. The second part explains how we derive common-factor loadings on the basis of asset-return correlation matrices.

In the system of 20 banks, the sum of interbank assets is not equal to the sum of interbank liabilities because these banks have interbank exposures vis-à-vis banks outside our sample. In order to work with an internally consistent matrix of interbank positions, we create a "sink bank," which absorbs the excess amount of aggregate interbank assets or liabilities. We assume that this bank does not default and abstract from its potential losses on the interbank market. As a result, the sink bank does not create risk in the system.

In deriving the ME matrix and random perturbations around it, we proceed as follows. For the ME matrix, we use the RAS algorithm and, thus, start with the relative entropy matrix as a prior (see Upper (2011)). This matrix assumes that the exposure of bank *i* to bank *j* is equal to $a_i{*}l_j$ if $i{\neq}j$ and *0* for $i=j$, where a_i (l_j) are the normalised total interbank assets (liabilities). In turn, we generate random interbank matrices by treating each entry of the prior matrix as a uniform variable distributed between zero and twice its initial value. In addition, we restrict a random set of off-diagonal entries to be equal to zero. We then apply the RAS algorithm to this modified prior and only consider matrices for which the algorithm converges.

When working with the correlation matrix of banks' asset returns, we make two choices. First, we treat this matrix as equal to the correlation matrix of the returns on banks' non-bank assets. In principle, the correlation matrix should reflect the fact that the co-movement of banks' asset values would be caused by (i) exogenous common factors affecting assets vis-à-vis non-banks but also by (ii) interbank linkages. That said, since we model the latter linkages as driven exclusively by interbank credit exposures and banks' PDs are quite low, the impact of these linkages on asset-return correlations turns out to be negligible. Concretely, while asset return correlations estimated by Moody's KMV range between 0.30 and 0.60, interbank linkages increase correlations by roughly 0.01.

Second, we impose a single-common-factor structure on the correlation matrix, as done in Tarashev and Zhu (2008). This structure is not necessary for measuring system-wide risk and systemic importance but has the important expositional advantage of allowing us to describe the commonality of banks' exposures to non-banks with 20 bank-specific common-factor loadings. Reassuringly, the single-common-factor structure captures roughly 75% of the cross-sectional variability of pair-wise correlations.

Annex 3: Additional drivers of systemic importance

In the absence of an interbank network, the fundamental PD is the main driver of systemic importance in the system of 20 banks we consider. To illustrate this, we rank-order these banks according to the values of each of the three drivers – fundamental PD, size, and common factor loading – and according to their Shapley values (Table 6). The orderings of fundamental PDs and Shapley values are almost identical, having a rank correlation of 0.96. By contrast, the other two drivers provide virtually no information about systemic importance. Concretely, the rank correlation between GCA Shapley values, and size and common-factor loadings is 0.13 and -0.42, respectively.

Table A3: Shapley values and drivers of systemic importance: no network

	Bank																			
	A	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**	**K**	**L**	**M**	**N**	**O**	**P**	**Q**	**R**	**S**	**T**
GCA	6	11	14	3	7	19	20	5	17	9	12	15	1	2	18	10	4	13	16	8
f.PD	6	13	16	3	4	20	19	7	18	9	12	15	2	5	17	10	1	11	14	8
size	6	7	1	9	19	5	20	4	3	10	0	12	11	2	13	14	16	15	18	17
CF	15	4	3	17	13	1	6	9	7	10	11	8	16	5	14	2	20	19	12	18

Note: GCA Shapley values and drivers are ranked from 1 (= highest) to 20 (= lowest) within each category. GCA: generalised contribution approach; f. PD: fundamental PD; CF: loading on the common risk factor.