

*CoVaR**

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Abstract

We propose a measure for systemic risk: *CoVaR*, the Value at Risk (*VaR*) of the financial system conditional on institutions being under distress. We argue for regulatory requirements that are based on the difference between *CoVaR* and the financial system *VaR*, capturing an institution's (marginal) contribution to systemic risk. Countercyclical regulation should take into account institutions' characteristics such as leverage, maturity mismatch and size that predict systemic risk contributions. We also explore the extent to which market indicators such as credit default swap spreads and implied equity volatilities predict systemic risk contribution.

Keywords: Value at Risk, Systemic Risk, Adverse Feedback Loop, Endogenous Risk, Risk Spillovers, Financial Architecture

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1 Introduction

During times of financial crisis, losses tend to spread across financial institutions, threatening the financial system as a whole.¹ Measures of systemic risk that capture risk spillovers and tail risk correlations should become supervisory tools and form the basis of any macro-prudential regulation.

The most common measure of risk used by financial institutions—the Value at Risk (*VaR*)—focuses on the risk of an individual institution in isolation. The $q\%$ -*VaR* is the maximum dollar loss within the $(1 - q\%)$ -confidence interval; see, e.g., Jorion (2006). However, a single institution’s risk measure does not necessarily reflect systemic risk – the risk that the stability of the financial system as a whole is threatened. Following the classification in Brunnermeier, Crocket, Goodhart, Persaud, and Shin (2009), a systemic risk measure should identify the risk on the system by “individually systemic” institutions, which are so massively interconnected and large that they can cause negative risk spillover effects on others, but also by institutions which are “systemic as part of a herd.” A group of 100 institutions that act like identical clones can be as precarious/dangerous to the system as the large merged identity.

The objective of this paper is twofold: First, we propose a measure for systemic risk. Second, we outline a method that allows a countercyclical implementation of macro-prudential regulation by predicting future systemic risk using past variables like size, leverage, maturity mismatch etc. To emphasize systemic nature of our risk measure, we add to existing risk measures the prefix “*Co*”, which stands for *conditional*, *comovement*, *contagion*, or *contributing*. We primarily focus on *CoVaR*. For example,

¹Examples include the 1987 equity market crash which started by portfolio hedging of pension funds and led to substantial losses of investment banks; the 1998 crisis started with losses of hedge funds and spilled over to the trading floors of commercial and investment banks; and the 2007/08 crisis spread from SIVs to commercial banks and on to hedge funds and investment banks, see Brady (1988), Rubin, Greenspan, Levitt, and Born (1999), and Brunnermeier (2009).

institution i 's $CoVaR$ on the system is defined as the VaR of the whole financial sector conditional on this institution being in distress. The difference between the $CoVaR$ and the unconditional financial system VaR , $\Delta CoVaR$, captures the marginal contribution of a particular institution (in a non-causal sense) to the overall systemic risk and can be associated with this institution's externalities.

In practice, we argue for a change of the supervisory and regulatory framework that aims at internalizing externalities that an institution's risk taking imposes on the financial system rather than focusing bank's risk in isolation. More specifically, the degree an institution increases systemic risk as measured by $\Delta CoVaR$ should determine the macro-prudential regulation of that institution.

In contrast, current risk regulation focuses on the risk of an individual institution. This leads, in the aggregate, to excessive risk along the systemic risk factors. To see this more explicitly, consider two institutions, A and B , which report the same VaR , but for institution A the $\Delta CoVaR = 0$, while for institution B the $\Delta CoVaR$ is large (in absolute value). Based on their $VaRs$, both institutions appear to be equally risky. However, the high $\Delta CoVaR$ of institution B indicates that it contributes more to system risk. Since system risk might carry a higher risk premium, institution B might outshine institution A and competitive pressure will force institution A to follow suit. Imposing stricter regulatory requirements on institution B would break this herding tendency. One might argue that regulating institutions' VaR might be sufficient as long as each institution's $\Delta CoVaR$ goes hand in hand with its VaR . However, this is not the case, as (i) it is not desirable that institution A should increase its contribution to systemic risk by following a strategy similar to institution B and (ii) there is no one-to-one connection between an institution's $\Delta CoVaR$ (y-axis) and VaR (x-axis) as Figure 1 shows. Overall, Figure 1 questions the usefulness of current bank regulation,

such as Basel II, which relies primarily on VaR .

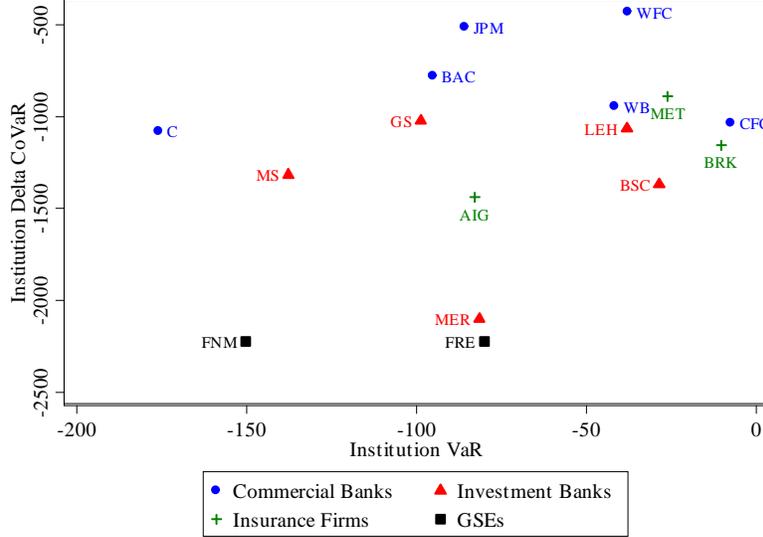


Figure 1: The scatter plot shows the weak link between institutions’ risk in isolation, measured by VaR^i (x-axis), and institutions’ contribution to system risk, measured by $\Delta CoVaR^i$ (y-axis). The VaR^i and $\Delta CoVaR^i$ are measured in 2007Q1 and are reported in Billions of dollars. The list with the names of the institutions corresponding to the tickers in this plot is given in Appendix B.

An additional advantage of our co-risk measure is that it is general enough to study the risk spillover effects across the whole financial network. For example, $\Delta CoVaR^{j|i}$ captures the increase in risk of individual institution j ’s when institution i falls in indistress. To the extent that it is causal, it captures the risk spillover effects that institution i causes on institution j . Of course, it can be that institution i ’s distress causes a large risk increase in institution j , while institution j causes almost no risk spillovers onto institution i . Similarly, there is no reason why $\Delta CoVaR^{j|i}$ should equal $\Delta CoVaR^{i|j}$. Figure 2 shows the directional effects for the top 5 investment banks.

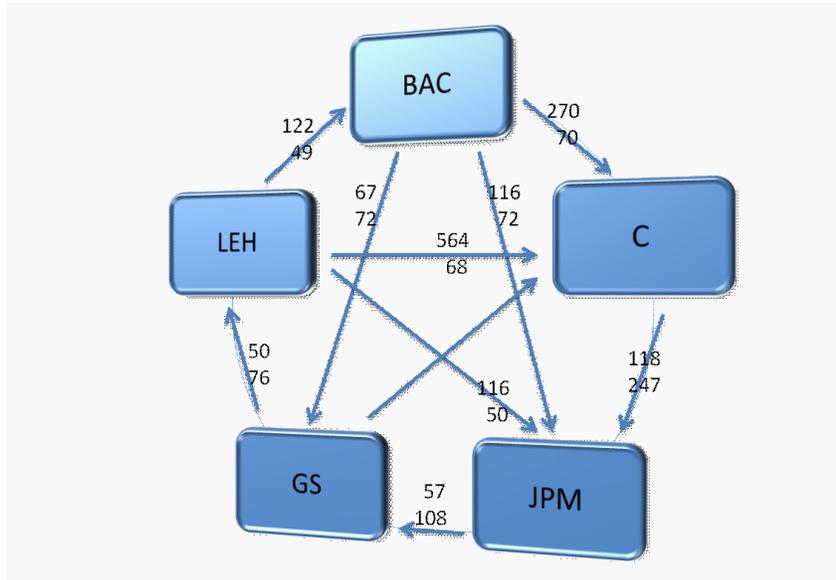


Figure 2: CoVaR network structure. The top number represents the CoVaR of the pointed institution conditional to the event that the institute at the origine of the arrow is in distress. The bottom number represents the CoVaR in the opposite direction.

So far, we deliberately have not specified how to estimate our *CoVaR* measure, since there are many possible ways. In this paper we primarily use quantile regressions which are appealing for their simplicity and efficient use of data. Since we want to capture all forms of risk, including not only the risk of adverse asset price movements, but—equally important—also funding liquidity risk, our estimates of $\Delta CoVaR$ in Figure 1 are based on (weekly) changes in the market valued assets of public financial institutions. Since the asset and liability composition of any particular financial institution may change rapidly over time (e.g., due to mergers, demergers, or ventures into new businesses), we estimate our risk measures over decile portfolios of financial intermediaries sorted based on leverage, maturity mismatch, size, and book-to-market. Ideally, one would like to base the risk measure on exact asset composition and funding structure, especially as both can change rapidly over time.

The second part of the paper addresses the problem that any (empirical) risk mea-

sure suffers from the fact that “tail observations” are—by definition—rare. After a string of good news, risk seems tamed, but, when a new tail event occurs, the estimated risk measure may sharply increase. This problem is most pronounced if the data samples are short. Hence, regulatory requirements that are based on estimated risk measures would be stringent during a crisis and lax during a boom. This introduces procyclicality – exactly the opposite of the goal of effective regulation. In order to derive a countercyclical risk measure, we derive the $\Delta CoVaR$ using the full length of available data. We first estimate it conditional on macro variables such as the slope of the yield curve, aggregate credit spread, and implied equity market volatility from VIX. Using panel regressions we then relate, in a predictive sense, these time-varying $\Delta CoVaR$ measures to each portfolio’s average maturity mismatch, leverage, book-to-market, and size. These predictive regressions allow the regulator to act in advance. The regression coefficients indicate how one should weigh the different funding liquidity measures in determining the capital charge or Pigouvian tax imposed on various financial institutions. Of course, any empirical analysis is limited and has to be complemented with “theorizing”, especially when the banking model changes.

Several authors have pointed out short-comings of the VaR and argued in favor of alternative risk measures. One of these measures is the expected short-fall (ES), which captures the expected loss conditional on being in the $q\%$ quantile. It is straightforward to extend our approach to other risk measures, e.g. the Co-Expected Shortfall ($Co-ES$). Just as ES is a sum of $VaRs$, $Co-ES$ is the sum of $CoVaRs$. The advantage of $Co-ES$ relative to $CoVaR$ is that it provides less incentive to load on to tail risk below the percentile that defines the VaR or $CoVaR$. In summary, the economic arguments of this paper are readily translatable to expected shortfall.

Related Literature. Our *co-risk measures* is related to theoretical research that points out externalities and liquidity spirals as well as to econometric work on contagion and spillover effects. A “fire-sale externality” gives rise to excessive risk taking and leverage. The externality arises since each individual trader takes potential fire-sale prices in the next period as given, while as a group the cause these low prices. In an incomplete market setting this precuniary externality leads to an outcome that is not even constrained Pareto efficient. This result was derived in banking context in Bhattacharya and Gale (1987), applied to international finance in Caballero and Krishnamurthy (2004) and most recently shown in Lorenzoni (2008). Stiglitz (1982) and Geanakoplos and Polemarchakis (1986) show it generically in a general equilibrium incomplete market setting. Runs on financial institutions are dynamic co-opetition games and lead to externalities and so does banks’ hoarding. While hoarding might be micro-prudent from a single bank’s perspective it need not be macro-prudent (fallacy of the commons). Network effects can also lead to externalities, as hiding one’s own contractual commitments increases the risk of one’s counterparties and the counterparties’ of one’s counterparties etc, Brunnermeier (2009). In Acharya (2009) banks do not fully take into account that they contribute to systematic risk.

Procyclicality occurs because risk measures and with them margin and haircut increase at time of crisis. The margin/haircut spiral outlined in Brunnermeier and Pedersen (2009) then forces financial institutions to delever at fire-sale prices. Adrian and Shin (2009) provide empirical evidence for the margin/haircut spiral for the investment banking sector.

Our work can also be related to the large literature in international finance that focuses on cross-country spillovers and contagion. Kyle and Xiong (2001) provide a model of contagion among financial institutions where the interaction of risk spillovers

and wealth effects leads to institutional contagion. Empirically, King and Wadhvani (1990) document an increase in correlation across stock markets during the 1987 crash, which, in itself, – as Forbes and Rigobon (2002) argue – is only evidence for interdependence, but not contagion, since estimates of correlation tend to go up when volatility is high. Claessens and Forbes (2001) and the articles therein provide an overview. In contrast to these papers, our analysis focuses on volatility spillovers. The most common method to test for volatility spillover is to estimate GARCH processes as, for example, Hamao, Masulis, and Ng (1990) do for international stock market returns. While GARCH processes allow for time-variation in conditional volatility, they assume that extreme returns follow the same return distribution as the rest of returns. Hartmann, Straetmans, and de Vries (2004) avoid this criticism by developing a contagion measure that focuses on extreme events. Building on extreme value theory, they estimate the expected number of market crashes given that at least one market crashes. However, extreme value theory works best for very low quantiles (see Danielsson and de Vries (2000)). This motivates Engle and Manganelli (2004) to develop *CAViaR* that – like our approach – makes use of quantile regressions as initially proposed by Koenker and Bassett (1978) and Bassett and Koenker (1978). While Engle and Manganelli’s *CAViaR* focuses on the evolution of quantiles over time, we study risk spillover effects across financial institutions as measured by our *CoVaR*. More recently, Rossi and Harvey (2007) estimate time-varying quantiles and expectiles using a state space signal extraction algorithm. The machinery developed by Engle and Manganelli (2004) and Rossi and Harvey (2007) could be used to study the time variation of *CoVaR*.

The remainder of the paper is organized in four sections. In Section 2, we outline the methodology. We define *CoVaR*, introduce time-variation and show how one could implement a countercyclical financial regulation. In Section 3, we relate *CoVaRs* to

macro risk factors. In Section 4, we show the degree to which *CoVaR* depends on institutional characteristics such as leverage, maturity mismatch, and size and show that these variables help to predict future *CoVaRs*. We conclude in Section 5.

2 *CoVaR* Methodology

In this section, we first introduce and define our systemic co-risk measure, *CoVaR*. Subsequently, we introduce time-varying *CoVaRs* by linking our *CoVaR* estimates to systemic risk factors, and we outline how countercyclical financial regulation can be achieved.

2.1 Definition of *CoVaR*

Recall that VaR_q^i is implicitly defined as the q quantile, i.e.

$$\Pr(X^i \leq VaR_q^i) = q,$$

where X^i is the variable of institution (or portfolio) i for which the VaR_q^i is defined. Note that VaR_q^i is typically a negative number. In practice, the sign is often switched, a sign convention we will not follow.

Definition 1 We denote by $CoVaR_q^{j|i}$ the VaR of institution j (or the financial system) conditional on $X^i = VaR_q^i$ of institution i . That is, $CoVaR_q^{j|i}$ is implicitly defined by the q -quantile of the conditional probability distribution:

$$\Pr(X^j \leq CoVaR_q^{j|i} | X^i = VaR_q^i) = q.$$

We denote institution i 's contribution to j by:

$$\Delta\text{CoVaR}_q^{j|i} = \text{CoVaR}_q^{j|i} - \text{VaR}_q^j.$$

Most part of the paper focuses on the case $j = \textit{system}$, i.e. when the portfolio of all financial institutions is at its VaR level. In this case, we drop the superscript j and hence, ΔCoVaR^i denotes the difference between the VaR of the financial system conditional on the distress of a particular financial institution i , CoVaR , and the unconditional VaR of the financial system, $\text{VaR}^{\textit{system}}$. It measures how much an institution adds to overall systemic risk. The measure captures externalities that arise because an institution is “too big to fail”, or “too interconnected to fail”, or takes on positions or relies on funding that can lead to crowded trades. Of course, ideally, one would like to have a co-risk measure that satisfies a set of axioms as e.g. the Shapley value does. Recall that the Shapley value measures the marginal contribution of a player to a grand coalition.

The more general definition of $\text{CoVaR}^{j|i}$ defined as the VaR of institution (portfolio) j conditional on that institution (or portfolio) i is at its VaR level allows us to study spillover effects across a whole financial network as illustrated in Figure 2. Moreover, we can also derive $\text{CoVaR}^{j|\textit{system}}$ which answers the questions which institutions are most at risk should a financial crisis occur. The corresponding $\Delta\text{CoVaR}^{j|\textit{system}}$ reports institution j 's increase in value-at-risk in the case of a financial crisis.

Properties of CoVaR Our CoVaR definition satisfies the desired property that after splitting one large “individually systemic” institution in, say n , identical clones, the CoVaR of the large institutions is exactly the sum of the CoVaRs of n identical clones. Put differently, conditioning on the distress on a large systemic institution is

the same as conditioning on one of the n identical clones and hence, the clone's *CoVaR* is simply the former merged entity's *CoVaR* divided by n .

Note that the $\Delta CoVaR$ measure does not distinguish whether the contribution is causal or simply driven by a common factor. We view this as a virtue rather than as a disadvantage. To see this, suppose a large number of small hedge funds hold similar positions and are funded in a similar way. That is, they are exposed to the same factors. Now, if only one of the small hedge funds falls into distress, this will not necessarily *cause* any systemic crisis. However, if this is due to a common factor, then all of the hedge funds, all of which are “systemic as part of a herd” will be in distress. Hence, each individual hedge fund's co-risk measure should capture this, even though there is no direct causal link, and the $\Delta CoVaR$ measure does so.

CoVaR focuses on the tail distribution and is more extreme than the unconditional *VaR* as *CoVaR* conditions on a “bad event”, a conditioning which typically shifts the mean downwards and increases the variance in an environment with heteroskedasticity. The *CoVaR*, unlike the covariance, reflects both shifts.

Note *CoVaR* conditions on the event that institution i is at its *VaR* level, which occurs with probability q . That is, the likelihood of the conditioning event is independent of the riskiness of i 's strategy. If we were to condition on an absolute return level of institution i , then more conservative institution could have a higher *CoVaR* since the conditioning event would be a more extreme event for less risky institutions.

In addition, *CoVaR* is directional. That is, the *CoVaR* of the system conditional on institution i does not equal the *CoVaR* of institution i conditional on the system.

Endogeneity of Systemic Risk Note that each institution's *CoVaR* is endogenous and depends on other institutions' risk taking. Hence, imposing a regulatory framework

that internalizes externalities alters the *CoVaR* measures. We view the fact that *CoVaR* is an equilibrium measure as a strength, since it adapts to changing environments and provides an incentive for each institution to reduce its exposure to certain risk factors if other institutions load excessively on it.

CoES Another attractive feature of *CoVaR* is that it can be easily adopted for other “corisk-measures”. One of them is the co-expected-shortfall, *Co-ES*. Expected shortfall has a number of advantages relative to *VaR* and can be calculated as a sum of *VaRs*. We denote the $CoES_q^i$, the *Expected Shortfall* of the financial system conditional on $X^i \leq VaR_q^i$ of institution i . That is, $CoES_q^i$ is defined by the expectation over the q -tail of the conditional probability distribution:

$$E [X^{system} | X^{system} \leq CoVaR_q^i]$$

Institutions i 's contribution to $CoES_q^i$ is simply denoted by:

$$\Delta CoES_q^i = E [X^{system} | X^{system} \leq CoVaR_q^i] - E [X^{system} | X^{system} \leq VaR_q^{system}].$$

Acharya, Pedersen, Philippon, and Richardson (2009) modify our approach by proposing the marginal expected shortfall as a measure of systemic risk.

2.2 Market value of total financial assets

Our analysis focuses on the VaR_q^i and $\Delta CoVaR_q^i$ of detrended changes in market valued total financial assets. We normalize changes in market value to take into account that the sizable total assets growth of the financial assets in the financial system over the last two decades. More formally, denote by ME_t^i the market value of an intermediary

i 's total equity, and by LEV_t^i the ratio of total assets to book equity. We define the normalized change in market value of total financial assets, X_t^i , by:

$$X_t^i = (ME_t^i \cdot LEV_t^i - ME_{t-1}^i \cdot LEV_{t-1}^i) \frac{\sum_i ME_{2006Q4}^i \cdot LEV_{2006Q4}^i}{\sum_i ME_{t-1}^i \cdot LEV_{t-1}^i} = (A_t^i - A_{t-1}^i) \frac{A_{2006Q4}^{system}}{A_{t-1}^{system}}, \quad (1)$$

where $A_t^i = ME_t^i \cdot LEV_t^i$. Note that the sum of the X_t^i across all institutions gives back the change of normalized market valued total assets for the financial system as a whole:

$$\sum_i X_t^i = (A_t^{system} - A_{t-1}^{system}) \frac{A_{2006Q4}^{system}}{A_{t-1}^{system}} = X_t^{system} \quad (2)$$

Our analysis is constrained by using publicly available data. In principal, a supervisor could compute the VaR_q^i and $\Delta CoVaR_q^i$ from a broader definition of total assets which includes off balance sheet items as well as derivative contracts. Such a more complete description of the assets and exposures of institutions would improve the measurement of systemic risk and systemic risk contribution. Conceptually, it is straightforward to extend the analysis to such a broader definition of total assets.

3 *CoVaR* Estimation

In this section, we outline one simple and efficient way to estimate *CoVaR* using quantile regressions, describe the data and then present our main empirical results. It should be noted that there is a large literature on the measurement on tail risk, and many alternative ways of estimating *CoVaR* are available.

3.1 Estimation Method: Quantile Regression

The *CoVaR* measure can be computed in various ways. Using quantile regressions is a particularly efficient way to estimate *CoVaR*, but by no means the only one. Alternatively, *CoVaR* can be computed from models with time varying second moments, from measures of extreme events, or by bootstrapping past returns.

To see the attractiveness of quantile regressions, consider the prediction of a quantile regression of the financial sector $\hat{X}^{system,i}$ on a particular portfolio i :

$$\hat{X}^{system,i} = \hat{\alpha}^i + \hat{\beta}^i X^i, \quad (3)$$

where $\hat{X}^{system,i}$ denotes the predicted value for a particular quantile conditional on institution i .² In principle, this regression could be extended to allow for nonlinearities by introducing higher order dependence of returns to style i as a function of returns to index j . From the definition of Value at Risk, it follows directly that:

$$VaR^{system}|X^i = \hat{X}^{system,i}. \quad (4)$$

That is, the predicted value from the quantile regression of the system on portfolio i gives the Value at Risk of the financial system conditional on i since the *VaR* given X^i is just the conditional quantile. Using a particular realization $X^i = VaR^i$ yields our *CoVaR* ^{i} measure. More formally, within the quantile regression framework our *CoVaR*

²Note that a median regression is the special case of a quantile regression where $q = 50\%$. We provide a short synopsis of quantile regressions in the context of linear factor models in the Appendix. Koenker (2005) provides a more detailed overview of many econometric issues.

While quantile regressions are regularly used in many applied fields of economics, their applications to financial economics are limited. Notable exceptions are econometric papers like Bassett and Chen (2001), Chernozhukov and Umantsev (2001), and Engle and Manganelli (2004) as well as the working papers by Barnes and Hughes (2002) and Ma and Pohlman (2005).

measure is simply given by:

$$CoVaR^i := VaR^{system} | VaR^i = \hat{\alpha}^i + \hat{\beta}^i VaR^i. \quad (5)$$

3.2 Financial institution asset data

We focus on publicly traded financial institutions, consisting primarily of commercial banks, investment banks and other security broker-dealers, and insurance companies. We start our sample in the beginning of 1986. We obtain the daily equity data from CRSP, and quarterly balance sheet data from COMPUSTAT. We also use the industry definitions for banking, security broker-dealers, insurance companies, and real estate corresponding to the four financial sector portfolios from the 49 industry portfolios by Kenneth French available at http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

Portfolio Sorts While we are interested in estimating the evolution of the risk measures VaR and $CoVaR$ for individual financial institutions, the nature of any particular institution might have changed drastically over the 1986-2008 sample period. In addition, many of the individual banks merged with other organizations, and some went out of business. One way to control for the changing nature of each individual institution is to form portfolios on particularly important balance sheet characteristics. In particular, we form the following sets of decile portfolios: maturity mismatch, leverage, book to market, and size. Maturity mismatch is measured as short-term debt - (cash + short-term investments), normalized by dividing by total assets. Leverage is the ratio of total assets to book equity. We form portfolios every quarter.

3.3 Time-variation associated with systematic risk factors

Applying our definitions directly, we can only estimate a single *CoVaR* for each institution that is constant over time. To overcome this limitation, we pursue two modifications. First, to reflect the fact that financial institutions' financing strategies change over time, we calculate the *CoVaRs* for portfolio sorts rather than for individual institutions. Second, to capture time variation that covaries with certain macro-variables and risk factors, we allow time-variation along these lines.

To allow for time-variation, we estimate the *CoVaR* and the *VaR* from systematic risk factors. We indicate time-varying $CoVaR_t$ and VaR_t with a subscript t . Taking time-variation into account leads to a panel data set of $CoVaR_t$ and VaR_t s and reduces the problem that tail correlation are overestimated when volatility is high (see e.g. Claessens and Forbes (2001)).

More specifically, we focus on the following systematic risk factors to estimate the variation of *VaRs* and *CoVaRs* across institutions and over time. The factors capture certain aspects of risks, and they are also liquid and easily tradable. We restrict ourselves to a small set of risk factors to avoid overfitting the data. Our factors are:

(i) *VIX* which captures the implied future volatility in the stock market. This implied volatility index is available on Chicago Board Options Exchange's website.

(ii) a short term "*liquidity spread*", defined as the difference between the 3-month repo rate and the 3-month bill rate measures the short-term counterparty liquidity risk. We use the 3-month general collateral repo rate that is available on Bloomberg, and obtain the 3-month Treasury rate from the Federal Reserve Bank of New York.

(iii) The level of the 3-month term Treasury bill rate.

In addition we consider the following two fixed-income factors that are known to be indicators in forecasting the business cycle and also predict excess stock returns:

(iv) The *slope of the yield curve*, measured by the yield-spread between the 10-year Treasury rate and the 3-month bill rate from the Federal Reserve Board's H.15 release.

(v) The *credit spread* between BAA rated bonds and the Treasury rate (with same maturity of 10 years) from the Federal Reserve Board's H.15 release.

We also control for the following equity market returns:

(vi) The weekly equity market return from CRSP.

(vii) The one year cumulative real estate sector return from Ken French's industry portfolio.

Let's denote the vector of risk factors by M_t . Then we run the following quantile regressions in the weekly data (where i is an individual institution or the whole system):

$$X_t^i = \alpha^i + \beta^i M_t + \varepsilon_t^i, \quad (6a)$$

$$X_t^i = \tilde{\alpha}^i + \tilde{\beta}^i M_t + \tilde{\gamma}^i X_t^{system} + \tilde{\varepsilon}_t^{i,system}, \quad (6b)$$

Then we generate the predicted values from the regressions (6a) to obtain:

$$VaR_t^i = \alpha^i + \beta^i M_t \quad (7a)$$

$$VaR_t^{system} = \alpha^{system} + \beta^{system} M_t \quad (7b)$$

$$CoVaR_t^i = VaR_t^{system} | [X^i = VaR_t^i] = \tilde{\alpha}^i + \tilde{\beta}^i VaR_t^i + \tilde{\gamma}^i M_t \quad (7c)$$

Finally, we compute $\Delta CoVaR_t^i$ for each institutions:

$$\Delta CoVaR_t^i = CoVaR_t^i - VaR_t^{system} \quad (8)$$

From these regressions, we obtain a series of weekly $\Delta CoVaR_t^i$ for each institution and

each portfolio. For the forecasting regressions, we generate a quarterly time series by taking averages within each quarter.

3.4 *CoVaR* summary statistics

Table 1 provides the estimates of our 1%-*CoVaR* measures that we obtain from using quantile regressions. Each of the summary statistics comprises 40 portfolios generated by forming deciles along four dimensions: leverage, maturity mismatch, book-to-market, and size. We also add four financial portfolios (commercial banks, security broker-dealers, insurance companies, and real estate). The first three lines give the summary statistics for the (normalized) market valued total asset changes, the next set of lines (6-9) give the summary statistics for the time-series / cross-section of VaR_t^i for each of the portfolios, and the last three lines give the summary statistics for the $\Delta CoVaR_t^i$. Recall that $\Delta CoVaR_t^i$ measures the marginal contribution of portfolio i to overall systemic risk and reflects the difference between two value at risks of the portfolio of the “financial universe”.

Estimates are based on a weekly frequency. We opt for a weekly horizon since we consider daily tail events as too short, while focusing on a monthly horizon would reduce the number of data points for our tail estimates. All portfolio data are weekly from 1986 - 2008. The summary statistics in Table 1 report overall results, as well as within and between statistics. All quantities are expressed in billions of dollars of total market valued financial assets as of 2006Q4.

TABLE 1: SUMMARY STATISTICS
WEEKLY, INDIVIDUAL INTERMEDIARIES

Variable		Mean	Std. Dev.	Obs
X^i	overall	0.20	10.18	N = 316697
	between		0.43	n = 430
	within		10.17	T-bar = 737
VaR^i	overall	-8.58	24.67	N = 316697
	between		19.69	n = 430
	within		12.38	T-bar = 737
$\Delta CoVaR^i$	overall	-578.41	572.54	N = 316697
	between		347.36	n = 430
	within		462.40	T-bar = 737

To capture time-variation of risk measures we relate them to macro factors as described in Section 2. More specifically, we quantile regress the weekly returns for each portfolio i on these macro variables.

TABLE 2: AVERAGE EXPOSURES TO RISK FACTORS
WEEKLY, INDIVIDUAL INTERMEDIARIES

COEFFICIENT	VaR^{system}	VaR^i	$CoVaR^i$
Repo spread (lag)	-1163***	-0.60	-877.94***
Credit spread (lag)	-107.75	-0.47	-226.75**
Term spread (lag)	128.71	0.64	18.80
VIX (lag)	-68.97***	-0.16***	-43.35***
3 Month Yield (lag)	118.73	0.42	15.95*
Market Return (lag)	242.74***	0.50***	196.00***
Housing (lag)	5.63	0.03	5.17

3.5 $CoVaR$ versus VaR

Figure 1 in the introduction shows that *across portfolios* there is only a very loose link between a portfolio's VaR^i and its contribution to systemic risk as measured by

$\Delta CoVaR^i$. Hence, imposing financial regulation that is solely based on the individual risk of an institution in isolation is not that useful. Figure 2 repeats the scatter plot of $\Delta CoVaR^i$ against VaR^i for the 44 portfolios.

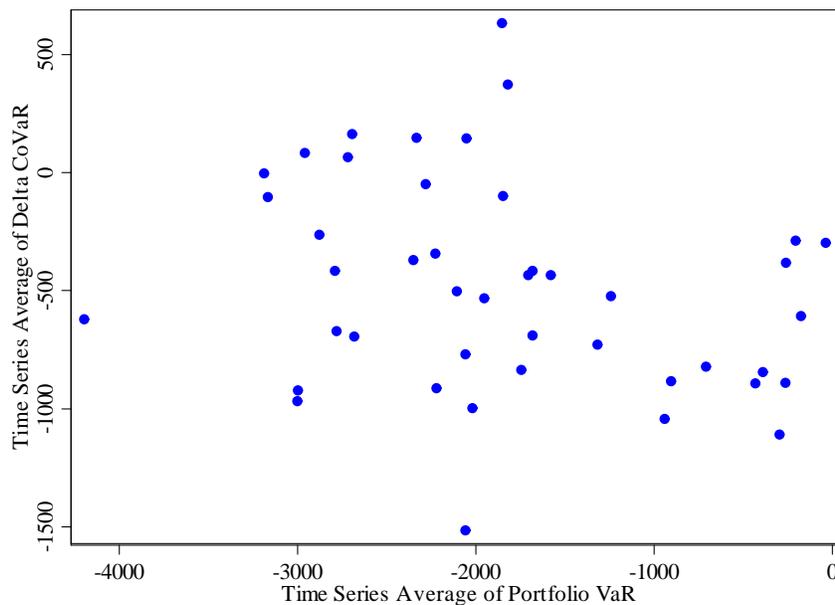


Figure 3: The scatter plot shows the weak link between a portfolio's risk in isolation, measured by VaR^i (x-axis), and the portfolio's contribution to system risk, measured by $\Delta CoVaR^i$ (y-axis). The VaR^i and $\Delta CoVaR^i$ are expressed in billions of dollars of total market valued financial assets as of 2006Q4.

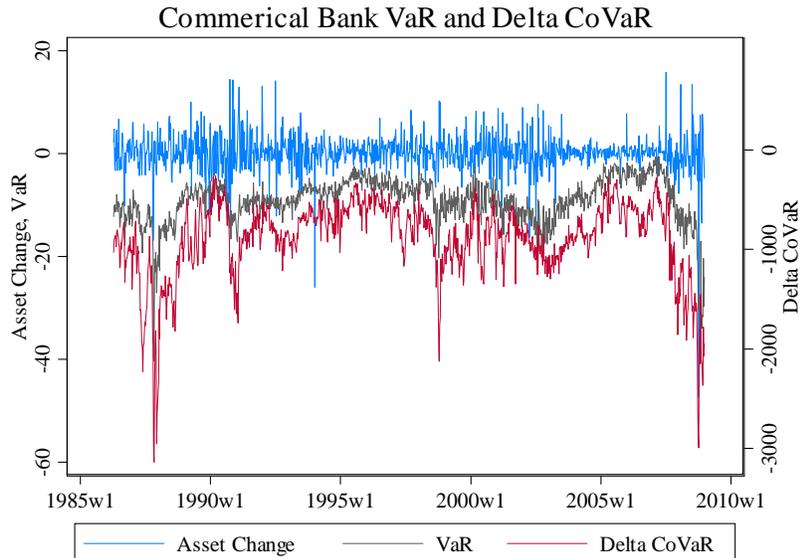


Figure 3 plots the evolution of the cross sectional average of $\Delta CoVaR^i$ and VaR^{system} over time. Figure 3 shows that the disconnect between VaR and $\Delta CoVaR$ in the cross-section is in contrast to a somewhat closer link in the *time series*. The figure shows that both the average contribution to systemic risk, and the overall financial sector risk increased during the financial crisis of 2008. However, in other instances, such as the LTCM crisis in 1998, the VaR increased substantially while average $\Delta CoVaR$ was not unusually negative.

4 *CoVaR* Forecasts

As explained in Section 2, (time-varying) tail risk measure estimates rely on relatively few observations. We therefore relate them to variables that are more readily observable. In the next subsection we do so by relating each portfolio's risk measure to the average maturity mismatch, leverage, book-to-market and relative size of its constituent institutions. We show that these variables help us to predict future tail

co-risk measures. Since most of these variables are only available at a quarterly basis, we aggregate weekly *CoVaR* and *VaR* measures to a quarterly frequency and run the forecasting regressions for both the cross section, and the time series.

4.1 Countercyclical regulation based on characteristics

Instead of relating financial regulation directly to our $\Delta CoVaR_t^i$ measure, we propose to link it to more frequently observed variables that predict the $\Delta CoVaR_t^i$ of a financial institution. This ensures that financial regulation is implemented in a pro-active and countercyclical way. Like any tail risk measure, *CoVaR*_{*t*} estimates rely on relatively few data points. Hence, adverse movements, especially following periods of stability, can lead to sizable increases in tail risk measures. Any regulation that naively relies on contemporaneous *VaR*_{*t*}^{*i*} and $\Delta CoVaR_t^i$ estimates would be unnecessarily tight after adverse events and unnecessarily loose in periods of stability. Capital regulation based on current risk measures thus amplify the adverse impacts after bad shocks, and amplify balance sheet expansions in good times (see Estrella (2004) and Gordy and Howells (2006)).

To overcome this procyclicality of capital regulation, we relate the *CoVaR* measures to characteristics of financial institutions. We focus, in particular, on institutions' maturity mismatch, leverage, book to market and relative size. Data limitations restrict our analysis, but supervisors can make use of a wider set of institution specific characteristics. We especially emphasize the predictive relationship between *CoVaR* and these characteristics since they allow supervisors to act before excessive financial sector leverage builds up. The coefficients for each of these characteristic variables determine how systemic risk capital charges should be imposed.

4.2 Forecasting *CoVaR* from cross-section of characteristics

Countercyclical regulation should tighten in booms, in advance of increases of risk. In Table 3, we ask whether systemic risk contributions can be forecasted, portfolio by portfolio, by the lagged characteristics at different time horizons.

Table 3 shows that portfolios with higher leverage, more maturity mismatch, larger size, and lower book-to-market tend to be associated with larger systemic risk contributions two years later. All coefficients are significant at the 1% level. While the *CoVaR* regressions have been run with the market valued asset changes normalized by total financial sector assets, we rescale the coefficients in Table 3 so as to correspond to \$ asset values as of 2006 Q4. For example, the coefficient of -5.42 for the leverage forecast at the two year horizon implies that an increase in leverage (say from 15 to 16) of one portfolio relative to other portfolios is associated with an increase in systemic risk of 5.42 Billion dollars (the $\Delta CoVaR^i$ becomes 5.42 billion more negative).

Table 3 can be understood as a "term structure" of systemic risk contribution by reading from right to left. The only variable that has a significant risk contribution at all frequencies is relative size: larger institutions tend to create more systemic risk. It should be noted that we include lagged variables of the $\Delta CoVaR^i$ and VaR^i in the regression so as to control for the persistence of systemic risk contribution.

TABLE 3: $\Delta CoVaR^i$ FORECASTS BY CHARACTERISTICS
 PANEL A: CROSS-SECTION, PORTFOLIOS, 1%

COEFFICIENT	2 Years	1 Year	1 Quarter
$\Delta CoVaR$ (lagged)	0.71***	0.80***	0.94***
VaR (lagged)	-1.99***	-2.27***	-0.47***
Leverage (lagged)	-9.43***	-10.73***	-2.53**
Maturity mismatch (lagged)	-0.89***	-0.30	-0.14
Relative size (lagged)	-170.84***	-161.99***	-38.58***
Book-to-market (lagged)	85.24***	87.65***	31.03**
Constant	-40.92**	-50.04***	-19.93*
Observations	3627	3805	3939
R-squared	0.62	0.69	0.89

*** p<0.01, ** p<0.05, * p<0.1

TABLE 3: $\Delta CoVaR^i$ FORECASTS BY CHARACTERISTICS
 PANEL B: CROSS-SECTION, PORTFOLIOS, 2 YEAR

COEFFICIENT	1%	5%	10%
$\Delta CoVaR$ (lagged)	0.71***	0.63***	0.70***
VaR (lagged)	-1.99***	-1.86***	-1.38***
Leverage (lagged)	-9.43***	-5.08***	-4.23***
Maturity mismatch (lagged)	-0.89***	-0.51***	0.10
Relative size (lagged)	-170.84***	-105.62***	-86.84***
Book-to-market (lagged)	85.24***	26.95***	-14.77**
Constant	-40.92**	14.70*	36.88***
Observations	3627	3627	3627
R-squared	0.62	0.62	0.70

*** p<0.01, ** p<0.05, * p<0.1

4.3 Forecasting $CoVaR$ from the time-series characteristics

Countercyclical regulation should tighten in booms, in advance of increases of risk. In Table 4, we ask whether systemic risk contributions can be forecasted, portfolio by portfolio, by the lagged characteristics at different time horizons. As in the cross sectional regressions of Table 3, we find in Table 4 that higher leverage and larger size significantly predict larger systemic risk contributions in the future. However, in the time series regressions of Table 4, maturity mismatch does not appear significant.

TABLE 4: $\Delta CoVaR^i$ FORECASTS BY CHARACTERISTICS
TIME SERIES / CROSS SECTION, PORTFOLIOS, 1%

COEFFICIENT	2 Years	1 Year	1 Quarter
$\Delta CoVaR$ (lagged)	0.41***	0.58***	0.86***
VaR (lagged)	-1.30***	-1.74***	0.06
Leverage (lagged)	0.92	-8.10***	-1.64
Maturity mismatch (lagged)	-0.31	0.53	0.33
Relative size (lagged)	-230***	-229***	-56***
Book to market (lagged)	29.25	42.69	29.36
Constant	-332.58***	-239.05***	-96.84***
Observations	3627	3805	3939
R-squared	0.69	0.73	0.89

*** p<0.01, ** p<0.05, * p<0.1

4.4 Forecasting VaR^{System} from average characteristics

For macroprudential purposes, a systemic risk regulator might want to judge the potential for systemic risk from average characteristics. In order to evaluate the degree to which it is possible to forecast overall systemic risk, we report regressions of the VaR^{system} on lagged average characteristics.

TABLE 5: VaR^{system} FORECASTS BY CHARACTERISTICS
TIME SERIES

COEFFICIENT	2 Years Ahead	1 Year Ahead	1 Quarter Ahead
VaR^{system} (lagged)	0.12	0.08	0.81***
Leverage (lagged)	-35.87	-312.10***	-40.12
Maturity mismatch (lagged)	-70.64**	-129.84***	-13.61
Book to market (lagged)	2,018.48***	1,693.89***	24.73
Constant	2,013.73	10,973.84***	1,123.30
Observations	83	87	90
R-squared	0.20	0.44	0.57

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

4.5 Forecasting $CoVaR$ from market variables

Options on the debt and equity of financial institutions provide information that allows an alternative way to assess the degree of systemic risk of financial institutions. In Table 6, we provide the results from a forecasting regression of the $\Delta CoVaR^i$ on lagged CDS spreads, CDS betas, equity implied volatilities, and equity implied volatility betas. The set of institutions used for this exercise consists of the five largest investment and commercial banks from Appendix B, for the 2004-2008 time horizon. We pull the CDS spreads and equity implied volatilities from Bloomberg. Betas are calculated by first extracting the principle component from CDS spread changes / implied volatility changes, within each quarter, from daily data, and then regressing each CDS spread change / implied volatility change on the first principal component. The regression is purely cross-sectional.

TABLE 6: $\Delta CoVaR^i$ FORECASTS BY MARKET VARIABLES
CROSS SECTION

COEFFICIENT	2 Years	1 Year	1 Quarter
$\Delta CoVaR$ (lagged)	0.60***	0.79***	0.94***
VaR (lagged)	-1.84	0.05	-0.08
CDS beta (lagged)	-1,727**	787.92	95.37
CDS (lagged change)	1,320	-2,211	-40.26
Implied Vol beta (lagged)	-8.30	-590.28**	-85.78
Implied Vol (lagged change)	-144.60	111.02	234.56***
Constant	-335.30	-147.72	-114.07*
Observations	114	154	184
R-squared	0.36	0.57	0.77

*** p<0.01, ** p<0.05, * p<0.1

5 Conclusion

During financial crises or periods of financial intermediary distress, tail events tend to spill over across financial institutions. Such risk spillovers are important to understand for supervisors of financial institutions.

The financial market crisis of 2007-2009 has underscored fundamental problems in the current regulatory set-up. When regulatory capital and margins are set relative to $VaRs$, forced unwinding of one institution tends to increase market volatility, thus making it more likely that other institutions are forced to unwind and delever as well. In equilibrium, such unwinding gives rise to a margin/haircut spiral triggering an adverse feedback loop. An economic theory of such amplification mechanisms are provided by Brunnermeier and Pedersen (2009) and ?. These “adverse feedback loops” were discussed by the Federal Open Market Committee in March 2008, and motivated

Federal Reserve Chairman Ben Bernanke to call for regulatory reform.³ Our *CoVaR* measure provides a potential remedy for the margin spiral, as the measure takes the risk spillovers which give rise to adverse feedback loops explicitly into account. We propose to require institutions to hold capital not only against their *VaR*, but also against their *CoVaR*. “Crowded trades” would also be penalized by capital requirements that rely on *CoVaR*.

For risk monitoring purposes, *CoVaR* is a parsimonious measure for the potential of systemic financial risk. Institutions that monitor systemic risk—for example, the Federal Reserve, other central banks around the world, the International Monetary Fund, and the Bank for International Settlement—have traditionally followed the evolution of *VaRs* of the financial sector. These institutions have also developed measures of systemic risk based on time varying second moments, estimates of exposures to different risk factors, and financial system tail risk measures. The advantage of using *CoVaR* is that it is tightly linked to *VaR*, the predominant risk measure. However, the logic of regulatory requirements based on *CoVaR* is straightforward to extend to alternative measures of risk, such as *CoES*, a measure of systemic expected shortfall.

³See <http://www.federalreserve.gov/monetarypolicy/fomcminutes20080318.htm>. and <http://www.federalreserve.gov/newsevents/speech/bernanke20080822a.htm>.

A Appendix: Quantile Regressions

This appendix is a short introduction to quantile regressions in the context of a linear factor model. Suppose that asset returns X_t have the following (linear) factor structure:

$$X_t = \gamma_0 + M_t\gamma_1 + (\gamma_2 + M_t\gamma_3)\varepsilon_t \quad (9)$$

where M_t is a vector of risk factors. The error term ε_t is assumed to be i.i.d. with zero mean and unit variance and is independent of M_t so that $E[\varepsilon_t|M_t] = 0$. Returns are generated by a process of the "location-scale" family, so that both the conditional expected return $E[X_t|M_t] = \gamma_0 + M_t\gamma_1$ and the conditional volatility $Vol_{t-1}[X_t|M_t] = (\gamma_2 + M_t\gamma_3)$ depend on the set of factors M_t . The coefficients γ_0 and γ_1 can be estimated consistently via OLS:⁴

$$\hat{\gamma}_0 = \alpha_{OLS} \quad (10)$$

$$\hat{\gamma}_1 = \beta_{OLS} \quad (11)$$

We denote the cumulative distribution function (cdf) of ε by $F_\varepsilon(\varepsilon)$, and the inverse cdf by $F_\varepsilon^{-1}(q)$ for percentile q . It follows immediately that the inverse cdf of R_t is:

$$\begin{aligned} F_{X_t}^{-1}(q|M_t) &= \gamma_0 + M_t\gamma_1 + (\gamma_2 + M_t\gamma_3)F_\varepsilon^{-1}(q) \\ &= \alpha(q) + M_t\beta(q) \end{aligned} \quad (12)$$

⁴The volatility coefficients γ_2 and γ_3 can be estimated using a stochastic volatility or GARCH model if distributional assumptions about ε are made, or via GMM. Below, we will describe how to estimate γ_2 and γ_3 using quantile regressions, which do not rely on a specific distribution function of ε .

where

$$\alpha(q) = \gamma_0 + \gamma_2 F_\varepsilon^{-1}(q) \quad (13)$$

$$\beta(q) = \gamma_1 + \gamma_3 F_\varepsilon^{-1}(q) \quad (14)$$

with quantiles $q \in (0, 1)$. We also call $F_{X_t}^{-1}(q|M_t)$ the conditional quantile function.

From the definition of VaR:

$$VaR_q|M_t = \inf_{VaR_q} \{ \Pr(X_t \leq VaR_q|M_t) \geq q \}, \quad (15)$$

it follows directly that:

$$VaR_q|M_t = F_{X_t}^{-1}(q|M_t). \quad (16)$$

The q -VaR in returns conditional on M_t coincides with conditional quantile function $F_{X_t}^{-1}(q|M_t)$. Typically, we are interested in values of q close to 0, or particularly $q = 1\%$.

We can estimate the quantile function via quantile regressions:

$$[\alpha_q, \beta_q] = \arg \min_{\alpha_q, \beta_q} \sum_t \begin{cases} q |X_t - \alpha_q - M_t \beta_q| & \text{if } (X_t - \alpha_q - M_t \beta_q) \geq 0 \\ (1 - q) |X_t - \alpha_q - M_t \beta_q| & \text{if } (X_t - \alpha_q - M_t \beta_q) < 0 \end{cases} \quad (17)$$

See Koenker and Bassett (1978), Koenker and Bassett (1978), and Chernozhukov and Umantsev (2001) for finite sample and asymptotic properties of quantile regressions.

B List of Financial Institutions

PANEL A: BANK HOLDING COMPANIES	PERMCO	TIC
BANK OF AMERICA CORP	3151	BAC
BANK OF NEW YORK MELLON CORP	20265	BIC
BANK ONE CORP	606	ONE
BANKERS TRUST CORP	20266	BT
CITIGROUP INC	20483	C
CONTINENTAL BANK CORP	20511	CBK
COUNTRYWIDE FINANCIAL CORP	796	CFC
FIRST CHICAGO CORP	20712	FNB
FIRST CHICAGO NBD CORP	3134	FCN
JPMORGAN CHASE & CO	20436	JPM

PANEL B: INVESTMENT BANKS	PERMCO	TIC
BEAR STEARNS COMPANIES INC	20282	BSC
SALOMON BROTHERS / CITIGROUP GLOBAL MARKETS	21556	CGM
GOLDMAN SACHS GROUP INC	35048	GS
LEHMAN BROTHERS HOLDINGS INC	21606	LEH
MERRILL LYNCH & CO INC	21190	MER
MORGAN STANLEY	21224	MS
PAINE WEBBER GROUP	21359	PWJ

PANEL C: GSEs	PERMCO	TIC
FANNIE MAE	20695	FNM
FREDDIE MAC	22096	FRE

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