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# Exchange Rate Dynamics Under Alternative Optimal Interest Rate Rules

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## Abstract

We explore the role of interest rate policy in the exchange rate determination process. Specifically, we derive exchange rate equations from interest rate rules that are theoretically optimal under a few alternative settings. The exchange rate equation depends on its underlying interest rule and its performance could vary across evaluation criteria and sample periods. The exchange rate equation implied by the interest rate rule that allows for interest rate and inflation inertia under commitment offers some encouraging results – exchange rate changes “calibrated” from the equation have a positive and significant correlation with actual data, and offer good direction of change prediction. Our exercise also demonstrates the role of the foreign exchange risk premium in determining exchange rates and the difficulty of explaining exchange rate variability using only policy based fundamentals.

**Keywords:** Taylor Rule, Exchange Rate Determination, Mean Squared Prediction Error, Direction of Change, Foreign Exchange Risk Premium

**JEL Classification:** F31, E52, C52

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## 1. Introduction

It is not an exaggeration to assert that modeling exchange rate behavior is notoriously difficult. After Meese and Rogoff (1983), there have been serious efforts devoted to constructing elaborate models on exchange rate behavior. For instance, by exploiting the notion of medium and long-run equilibrium conditions, the behavioral equilibrium exchange rate approach, and the fundamental equilibrium exchange rate approach introduce variables including productivity, government expenditure and net foreign assets into empirical exchange rate equations. These approaches appeared promising at the turn of the new millennium. One modelling strategy highlights the role of heterogeneity information among traders.<sup>1</sup> Under this framework, the exchange rate determination process depends on how traders perceive each other's information about future economic fundamentals.

The real triumph of Meese and Rogoff (1983) is that their result has largely withstood the challenge of sophisticated exchange rate models and elaborated econometric techniques developed after its publication. Even though the Meese and Rogoff result is typically framed in terms of forecasting performance, it signifies the inherent difficulty in modeling exchange rate behavior in general. The skeptic view on the performance of empirical exchange rate models is commonly found in the literature and we are still in search of an empirical model that could consistently and robustly describe exchange rate behavior of a wide spectrum of currencies.<sup>2</sup>

In a standard monetary model of exchange rate determination, the role of monetary policy is not explicitly incorporated. Engel and West (2005, 2006) explore the implications of monetary policy endogeneity for exchange rate determination. Specifically, they consider the exchange rate equation derived from the Taylor rule – an interest rate reaction function that is quite popular in the monetary policy literature and the uncovered interest parity. They showed that the real exchange rate generated from the Taylor rule based exchange rate equation has a quite high correlation (in the context of exchange rate economics) with the actual rate. By endogenizing monetary policy and explicitly introducing the interest rate rule, these authors advanced a new and promising approach to modeling exchange rate behaviors.

Their positive finding is affirmed by other studies pursuing a similar modelling strategy. For example, Mark (2009) studies a variant of the Taylor rule based exchange rate equation, and presents some encouraging results for the model.<sup>3</sup> Chinn (2008), Clarida and Waldman (2008), Molodtsova and Papell (2009), Molodtsova *et al.* (2008), and Wang and Wu (2008) also present favorable findings on the empirical performance of several variants of the Taylor rule based exchange rate specification.

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<sup>1</sup> See, for example, Bacchetta and van Wincoop (2006).

<sup>2</sup> See, for example, Bacchetta and van Wincoop (2006), Evans and Lyons (2002), and Frankel and Rose (1995). Cheung *et al.* (2005a), for instance, suggests that “no single model can forecast exchange rates for all currencies or at all times.”

<sup>3</sup> There are a few differences between the models examined in Mark (2009) and Engel and West (2006). For instance, the former study assumes the private sector has to learn about the Taylor rule over time and uses a Taylor rule that includes a lagged interest rate while the latter study involves no learning and has a real exchange rate variable in the Taylor rule specification.

Undeniably, these studies have narrowed the gap between exchange rate modelling and monetary policy practice, and demonstrated the potential of the policy rule based approach. One interesting observation is that these studies typically chose an empirical or operational Taylor rule from extant empirical monetary policy evaluation studies. While these rules are derived from empirical observations, they might not be the “optimal” interest rate rule in theory.

In the current exercise, we examine the exchange rate dynamics implied by a theoretically optimal interest rate policy rule. Compared with an empirical Taylor rule, an optimal interest rate rule derived from an economic model offers a normative policy perspective and is based on some explicit optimization behavior. To assess the role and usefulness of the policy based approach to modeling exchange rates, it is desirable to have a policy rule that has a firm theoretical foundation. It is believed that, in addition to intellectual curiosity, the assessment based on optimal interest rate policy rules should complement the extant studies based on empirical and operational rules.

A natural question to ask is: “What is the optimal interest rate policy rule?” Apparently, there is more than one optimal interest rate rule. The answer depends on a number of factors, including the structure of the economy under consideration, the central bank’s objective function, and the assumption about the adjustment mechanism. The form of an optimal policy rule changes as any one of these factors changes.

By comparing the performance of exchange rate behavior implied by different optimal interest rate rules, this study offers an alternative perspective on assessing the relevance of the policy rule approach to describe exchange rate behavior. Specifically, the performance could significantly depend on the functional form of optimal rule, which depends on the framework within which the rule is derived. The implication, albeit trivial, is that the relevance of the policy rule based approach depends on the (theoretical or empirical) rule selected for the analysis. In the current exercise, we consider a few canonical specifications to illustrate the point.

To put our exercise in perspective, we conduct our analysis within a standard and small scale new Keynesian macroeconomic framework that is commonly used in the monetary policy literature. We consider optimal interest rate rules obtained under a) alternative policy objectives including output and inflation stability, inflation targeting, and constant monetary growth, b) alternative adjustment mechanisms, and c) commitment and discretion assumptions.

We evaluate the empirical performance of exchange rate equations derived from different optimal interest rate rules by comparing the exchange rates implied by these equations with the actual exchange rates. The dollar exchange rate of the British pound is used to illustrate some issues that may be encountered in evaluating the performance of policy rule based exchange rate equations. Specifically, we examine the possible changes in policy behavior and in the relationship between exchange rates and its economic fundamentals when Britain adopted the inflation targeting policy in the early 1990s.

An issue of evaluating the empirical performance is related to the uncovered interest parity condition, which is a main component of the Taylor rule based approach. One of the stylized empirical facts in international finance is that the uncovered interest parity tends not to hold empirically. A foreign exchange risk premium term is usually introduced to bridge the difference between the expected change in the exchange rate and the interest differential. Indeed, it is known that the risk premium could be quite large and volatile. In our performance evaluation exercise, we attempt to shed some light on the role of risk premium.

To anticipate our results, we find that the policy rule based exchange rate equation could take a rather simple form or assume some complex dynamics depending on conditions under which the optimal interest rule is determined. As a consequence, the performance of these policy rule based exchange rate equations depends on their underlying optimal interest rate rules. In addition, the performance varies across evaluation criteria and sample periods. The inclusion of an estimated foreign exchange risk premium term substantially enhances the performance of all the policy rule based exchange rate equations.

The remainder of the paper is structured as follows. In Section 2, we introduce the canonical macroeconomic model, derive the optimal interest rate rules under alternative assumptions, and obtain the corresponding optimal policy rule implied exchange rate equations. Section 3 presents the performance analysis. Section 4 offers some concluding remarks.

## 2. Exchange Rates and Optimal Interest Rate Rules

In this section, we present the macroeconomic model, find the optimal interest rate policy rule under several scenarios, and derive the corresponding exchange rate equations. To facilitate presentation, we start with a simple macroeconomic system that is described in subsection 2.1. Then, a few variants of the basic structure and the corresponding policy rule based exchange rate equations are derived and discussed in sequence. The choice of a small-scale macroeconomic model is partly motivated by the observation that optimal policy rules for small models resemble simple rules that are commonly considered in monetary policy evaluation studies and these simple rules tend to have performance similar to optimal rules derived from large-scale models (Taylor and Williams, 2010).

Even though the new Keynesian model in the subsequent subsections is originally designed for a closed economy, it is relevant for our discussion. For instance, Clarida *et al.* (2001, 2002) show that the monetary policy design for a small open economy is the same for a closed economy. Further, Batini *et al.* (2003) and Leitimo and Söderström (2005) show that adding the exchange rate to the structure of economy and to the policy rule yields only a small welfare improvement.

A relatively *ad hoc* strategy is to add an exchange rate term to the interest rate rule – see subsection 2.2 below. Among extant studies, the exchange rate variable could take different forms – it could be the change in the nominal, real, effective, or real effective exchange rate, or the deviation from the

nominal, real, effective, or real effective equilibrium exchange rate. Taylor (2001), for example, expresses some skepticism on introducing an exchange rate variable, because the original rule already allows for exchange rate reaction via responding to both inflation and output variations. Theoretically, in the presence of perfect exchange rate pass-through, the central bank should target inflation rate and allow exchange rate to float. Engel (forthcoming) recently shows that, even for a policy target that includes currency misalignments, the interest rate instrument that responds to the CPI inflation rate could support the policy.

In view of these considerations, we adopt the new Keynesian model to illustrate the interactions between policy rules and exchange rate dynamics. Some discussions of the policy rule based exchange rate behavior are offered in the Subsection 2.8.

## 2.1 The Model

The subsequent discussions are based on the canonical new Keynesian model that includes an expectations-augmented Phillips curve and a forward-looking IS curve. The model could be written as (Clarida *et al.*, 1999):

$$\text{Phillips curve: } \pi_t = \beta E_t \pi_{t+1} + \kappa y_t + e_t, \quad (1)$$

$$\text{IS curve: } y_t = E_t y_{t+1} - \varphi [i_t - E_t \pi_{t+1}] + u_t, \quad (2)$$

where  $\pi_t$  is inflation rate,  $y_t$  is the output gap given by the deviation of the actual output from its natural level in logs,  $i_t$  is the nominal interest rate, and  $E_t$  is the expectations operator. The parameters  $\kappa$  and  $\varphi$  are assumed to be positive.  $\beta$  is the discount factor between zero and one. The cost push shock is given by  $e_t$  and  $u_t$  is a preference or demand shock.<sup>4</sup>

Despite its simplicity, the new Keynesian model is the work-horse and it occupies a core position of monetary policy analysis among academics and policymakers. Indeed, it has a micro foundation, and could be derived from household and firm optimization problems (Walsh, 2003; Woodford, 2003). It is forward looking and incorporates nominal rigidities that allow a monetary policy affects real economic activities in the short run.

We assume the central bank's loss function is given by

<sup>4</sup> If the aim is to find the solutions of  $\pi_t$  and  $y_t$  in term of shocks, then it is customary to assume the shocks follow an autoregressive process like  $e_t = \rho_e e_{t-1} + \varepsilon_{e,t}$  and  $u_t = \rho_u u_{t-1} + \varepsilon_{u,t}$ ; where  $0 \leq \rho_e, \rho_u < 1$ , and  $\varepsilon_{e,t}$  and  $\varepsilon_{u,t}$  are white noises.

$$L_t = \pi_t^2 + \lambda y_t^2, \quad (3)$$

where  $\lambda$  is the relative weight placed on stabilizing the output gap versus inflation. At times, equation (3) is known as the *period loss function*. Woodford (2001) argues that “both inflation and output-gap stabilization are sensible goals of monetary policy, as long as the ‘output gap’ is correctly understood.” In fact, in the context of the simple optimizing model underlying equations (1) and (2), a quadratic loss function could be motivated as a second order Taylor series approximation to the expected utility of the representative household. By varying the weight in the loss function, one could obtain either inflation targeting or output targeting, which are two other oft-discussed monetary policy targets.

The policy objective is to minimize the expected discounted sum of the losses given by

$$E_o \left[ \sum_{j=0}^{\infty} \beta^j L_t \right] \quad (4)$$

Equation (4) is often known as the *intertemporal loss function*. Given the policy objective, the question is: “What is policy instrument to be used?” In this exercise, the interest rate is the policy instrument. The optimal interest rate rule can be obtained by minimizing the objective function (4), subject to the two constraints represented by equations (1) and (2).

Should the optimal rule be derived under discretion or commitment? Under discretion, the central bank faces a single-period problem and set the optimal interest rate that does not depend on prior history (Walsh 2003; Woodford, 2003). Under commitment, the central bank has to choose a policy rule (and hence, the implied inflation and output gap paths) which minimizes (4) and entails trade-offs across all possible future scenarios.

Theoretically, the optimal rule under commitment should yield an equilibrium that is superior in terms of welfare. Also, with commitment policy, the central bank could enhance monetary policy credibility and affect the private sector’s inflation expectations (Clarida *et al.*, 1999; Walsh, 2003). On the other hand, some economists argue that a policy rule should not be followed mechanically, and that discretionary policy is relevant to the central bank decision making process. In making a decision, the central bank should take all the relevant information into account, rather than simply apply the reaction function for the entire time span. Once new information becomes available, the central bank will inevitably reformulate its policies and make new decisions (Taylor, 1993; Svensson, 1999).<sup>5</sup>

In the following exercise, we consider both types of optimal interest rate rules.

<sup>5</sup> In a speech delivered in March 2006, Fed Chairman Bernanke said: “Given this reality, policymakers are well advised to follow two principles familiar to navigators throughout the ages. First, determine your position frequently. Second, use as many guides or landmarks as available.” Also, there is an issue on whether to commit to the objective or the policy rule; see, for example, Svensson (1999).



The uncovered interest rate parity is a main component of the policy rule approach to model exchange rate behavior. It is used to connect the exchange rate to the economic variables that determine the interest rate setting behavior. The parity defines a relationship between the expected change in the nominal exchange rate and the home and foreign interest rate differential,

$$i_t - i_t^* = E_t \Delta s_{t+1} + \xi_t, \quad (5)$$

where  $s_t$  is the nominal exchange rate, in logs, expressed as units of domestic currency per one unit of foreign currency (i.e. an increase in  $s_t$  means a depreciation of the domestic currency),  $i_t^*$  is the foreign interest rate, and  $\xi_t$  is the foreign exchange risk premium term. The foreign exchange risk premium is included to account for the well-known weak empirical relationship between expected exchange rate changes and interest rate differentials.

## 2.2 The Taylor (1993) Rule

The Taylor rule comes in different forms and with alternative variable definitions in literature. The rule given in Taylor (1993) is a deterministic expression,

$$i_t = \pi_t + 0.5y_t + 0.5(\pi_t - 2) + 2 \quad (6)$$

where the policy instrument,  $i_t$ , is the federal fund rate. Note that the response coefficients take fixed values in the generic Taylor rule formulation. The target inflation rate is assumed to be 2 percent. According to the rule, when the real output is on target, that is  $y_t = 0$ , and the inflation is 2 percent then the federal fund rate should be 4 percent in nominal term, and 2 percent in real term. See Taylor (1993) for a detailed discussion of the rule and the related issues.

Subsequently, the rule has been modified in various ways. For instance, the numerical values in (6) are replaced by parameter estimates from regression analysis. A lagged interest rate is included in the right-hand-side to account for interest rate smoothing behavior. In some cases, an exchange rate variable is introduced in view of possible policy responses to exchange rate variability. For instance, in Engel and West (2006), the Taylor rules adopted by the home and foreign countries are given by

$$i_t = \gamma_q q_t + \gamma_\pi E_t \pi_{t+1} + \gamma_y y_t + z_t, \quad (6a)$$

and

$$i_t^* = \gamma_\pi E_t \pi_{t+1}^* + \gamma_y y_t^* + z_t^*, \quad (6b)$$

where “\*” denotes a foreign variable,  $q_t$  is the real exchange rate, and  $z_t$  and  $z_t^*$  are shocks to monetary policy rules.<sup>6,7</sup> For comparison purposes, we include the exchange rate equation implied by the policy rules (6a) and (6b) in the next section.

In the next five subsections, we derive alternative optimal interest rate rules and their implied exchange rate equations within the framework presented in subsection 2.1. Readers who are familiar with these rules could go directly to subsection 2.8, which offers some summary discussions on the derived exchange rate equations.

### 2.3 The Optimal Interest Rate Rule I – Learning

Under different monetary policy objectives and optimality conditions, one can have different optimal interest rate rules for the canonical new Keynesian model. In the current exercise, we consider the class of interest rate rules that imply equilibrium determinacy and lead to stability and learnability (Evans and Honkapohja, 2003). Determinacy rules out multiple stationary solutions including sunspot solutions.<sup>8</sup> Stability and learnability are important conditions regarding the identification and inflation determination with the Taylor rule in the new Keynesian framework – see the exchanges between McCallum (2008) and Cochrane (2007a, b). With these three conditions, we can derive the exchange rate behavior using the policy rule based approach in a system that has unique stationary rational expectations equilibrium.

The first optimal interest rate rule we consider is discussed in Evans and Honkapohja (2003). It allows for learning behavior and satisfies the determinacy and stability conditions. With the period loss function (3) and the infinite horizon loss function (4), the optimal interest rate rule under commitment is

$$i_t = \phi_\pi \widehat{E}_t \pi_{t+1} + \phi_y \widehat{E}_t y_{t+1} + \theta_y y_{t-1} + \varepsilon_t, \quad (7)$$

where the response coefficients are given by  $\phi_\pi = 1 + \kappa\beta / [\varphi(\lambda + \kappa^2)]$ ,  $\phi_y = 1/\varphi$ , and  $\theta_y = -\lambda / [\varphi(\lambda + \kappa^2)]$ . The innovation term,  $\varepsilon_t$ , is given by  $u_t / \varphi + \kappa e_t / \varphi(\lambda + \kappa^2)$  and  $\widehat{E}_t$  denotes the current expectations of the variable in the next period.<sup>9</sup> As expected, the response of the

<sup>6</sup> At the risk of repeating, see the discussion in the beginning of Section 2. The evidence on the empirical relevance of the exchange rate variable is mixed and could be country- and time-period-specific. See, for example, references cited in Taylor (2001), Clarida *et al.* (1998) and Mark (2009).

<sup>7</sup> Econometrically speaking, the I(1) and near-I(1) property of the real exchange rate documented in literature creates some technical issues in estimating the policy rule and the implied exchange rate equation. Taking first order differencing is a short-cut to remove the I(1) issue. Our experience, however, is that the change in real exchange rate variable is *typically* insignificant in our estimation.

<sup>8</sup> See, for example, Bernanke and Woodford (1997), Svensson and Woodford (2005), and Woodford (1999) for types of rules that lead to indeterminacy.

<sup>9</sup> Because the model allows for learning behavior, Evans and Honkapohja (2003) do not impose the rational expectation assumption in deriving the optimal rule.

policy rate depends on the underlying structural parameters ( $\beta$ ,  $\kappa$ ,  $\varphi$ ) of the economy and the trade-off parameter ( $\lambda$ ) between inflation and output stabilization.

The definitions of these response coefficients vary with changes in the model specification. In the following subsections, for notational simplicity, we will use similar notations for similar response coefficients, while being aware of the fact that the definitions of these coefficients could change across subsections.

The optimal interest rate rule (7) has two salient features of the original Taylor rule; namely a) the “Taylor principle” or the active Taylor rule,  $\phi_\pi > 1$ , and b)  $0 < \phi_y < 1$ . The parameter  $\theta_y < 0$  is in accordance with a mean reverting output gap variable – a large lagged output gap implies a small current output gap, and thus a small interest rate response. The derivation of equation (7) can be found in Evans and Honkapohja (2003). We included it in Appendix A for completeness.

The optimal interest rate rule under discretion is given by

$$i_t = \phi_\pi \widehat{E}_t \pi_{t+1} + \phi_y \widehat{E}_t y_{t+1} + \varepsilon_t. \quad (8)$$

It is noted that optimal rules under commitment are typically derived in some existing studies. The same could not be said for rules under discretion. Thus, we derived (8) and other discretion rules in the subsequent subsections and, for brevity, presented the derivation in Appendix A for completeness and easy references.

Under (8), the policy interest rate depends only on the expected values of inflation and the output gap in the next period. The result is consistent with the notion of discretion – the central bank sets the interest rate in response to the latest information to minimize the expected loss in the next period. The two response coefficients,  $\phi_\pi$  and  $\phi_y$ , are qualitatively comparable to the corresponding ones of the original Taylor rule. As expected, the optimal interest rate rule displays a less complex dynamic under discretion than under commitment.

The interest rate adjustment mechanism in both (7) and (8) is forward looking, while the interest rate in the generic Taylor rule (6) responds to contemporaneous inflation and the output gap. The difference is driven by the expectations mechanism included in the current model.

Assuming that both the home and foreign countries follow the same interest rate policy under commitment, the interest rate differential can be written as

$$\begin{aligned} i_t - i_t^* &= \phi_\pi \widehat{E}_t (\pi_{t+1} - \pi_{t+1}^*) + \phi_y \widehat{E}_t (y_{t+1} - y_{t+1}^*) + \theta_y (y_{t-1} - y_{t-1}^*) \\ &\quad + \varepsilon_t - \varepsilon_t^*, \end{aligned} \quad (9)$$

where “\*” indicates foreign variables.<sup>10</sup> Combining the uncovered interest parity (5) and (9), the exchange rate equation implied by the optimal interest rate rule under commitment is, thus, given by

$$E_t \Delta s_{t+1} = \phi_\pi \widehat{E}_t (\pi_{t+1} - \pi_{t+1}^*) + \phi_y \widehat{E}_t (y_{t+1} - y_{t+1}^*) + \theta_y (y_{t-1} - y_{t-1}^*) - \xi_t + \varepsilon_t - \varepsilon_t^*. \quad (10)$$

By construction, the role of fundamentals is dictated by the form of the interest rate rule. Note that  $\phi_\pi > 0$ ; that is, the exchange rate depreciates with an increase in expected inflation, *ceteris paribus*. The result is in contrast with the “bad news about inflation is good news for the exchange rate” noted in Clarida and Waldman (2008), Engel and West (2006) and Engel *et al.* (2008). The discrepancy is mainly due to differences in specifying the interest rate rules.<sup>11</sup> We return to the interaction between inflation and exchange rates below.

In the discretion case, the interest rate differential is given by

$$i_t - i_t^* = \phi_\pi \widehat{E}_t (\pi_{t+1} - \pi_{t+1}^*) + \phi_y \widehat{E}_t (y_{t+1} - y_{t+1}^*) + \varepsilon_t - \varepsilon_t^*, \quad (11)$$

and the corresponding exchange rate equation is given by

$$E_t \Delta s_{t+1} = \phi_\pi \widehat{E}_t (\pi_{t+1} - \pi_{t+1}^*) + \phi_y \widehat{E}_t (y_{t+1} - y_{t+1}^*) - \xi_t + \varepsilon_t - \varepsilon_t^*. \quad (12)$$

The main difference between equations (10) and (12) follows from that of the respective optimal interest rate rules. Specifically, because of commitment obligations, exchange rate changes respond to both the lead and the lag of output gap differentials in equation (10).

Henceforth, to simplify presentation, we will use the term “implied exchange rate equation” to denote the exchange rate equation derived from an optimal interest rate rule under commitment or under discretion.

<sup>10</sup> A less parsimonious model will be obtained if we assume the two countries follow different policy rules.

<sup>11</sup> To illustrate the point, we outlined the derivation of the policy rule based exchange rate equations from these papers in Appendix B.

## 2.4 The Optimal Interest Rate Rule II – Interest Rate Inertia

In the previous subsection, it is implicitly assumed that the interest rate could be adjusted at no or low costs. If interest rate volatility entails substantial economic costs, then the central bank would like to vary the interest rate gradually. One way to incorporate interest rate smoothing is to modify the central bank's loss function (3) to

$$L_t = \pi_t^2 + \lambda_y y_t^2 + \lambda_i (i_t - \tilde{i})^2, \quad (13)$$

where  $\lambda_y$  and  $\lambda_i$  are the weights on stabilizing the output gap and interest rate relative to stabilizing inflation. That is, the central bank penalizes interest rate volatility in addition to inflation and output variability. In doing so, we introduce an interest rate smoothing behavior and the interest rate will only partially adjust to deviations from  $\tilde{i}$ , the target interest rate.<sup>12</sup> For example, Woodford (1999) defines  $\tilde{i} = \log[(1 + \bar{r}^m) / (1 + \bar{r})]$ , where  $\bar{r}^m$  is the constant interest rate paid on the monetary base by the central bank and  $\bar{r}$  is the steady-state nominal interest rate.

Under the loss function (13), the optimal interest rate policy rule under commitment for this economy is given by

$$i_t = (1 - \rho_1)\tilde{i} + \rho_1 i_{t-1} + \rho_2 \Delta i_{t-1} + \phi_\pi \pi_t + \phi_y \Delta y_t, \quad (14)$$

where  $\rho_1 = 1 + \kappa\varphi / \beta$ ,  $\rho_2 = 1 / \beta$ ,  $\phi_\pi = \kappa\varphi / \lambda_i$ , and  $\phi_y = \varphi\lambda_y / \lambda_i$ . At the risk of repetition, we re-state again that, for notational simplicity, we use similar notations for similar response coefficients, while aware of the fact that definitions of these coefficients could change across model specifications. In general, the positivity of coefficients  $\phi_y$  and  $\phi_\pi$  is consistent with the generic Taylor rule (6). Additionally,  $\phi_\pi > 0$  and  $\rho_1 > 1$  satisfy the Taylor principle (Woodford, 2003).

One feature of equation (14) is that the rule is optimal regardless of the nature of shocks. However, the rule is backward looking because it depends on the histories of both the interest rate and the output gap. Giannoni and Woodford (2002) show that equilibrium determinacy is implied by a commitment to the rule (14). It is noted that, with interest rate smoothing, output gap variability could be sluggish because the interest rate responds to output gap changes rather than the output gap itself.

<sup>12</sup> Sack and Wieland (2000) review empirical studies and offer three main factors that account for interest rate smoothing; namely market participants' forward-looking behavior, measurement errors associated with key macroeconomic variables, and uncertainty regarding relevant structural parameters. Rudebusch (2002) does not find evidence from the yield curve in favor of the implication of interest rate smoothing for future interest rate predictability. Söderlind *et al.* (2005), on the other hand, argue that interest rate smoothing does not imply interest rate predictability, though it induces sluggish inflation and output gap movements.

Under discretion, the optimal interest rate rule is given by

$$i_t = \tilde{i} + \phi_\pi \pi_t + \phi_y y_t, \quad (15)$$

where the parameters,  $\phi_y$  and  $\phi_\pi$ , are equivalent to the ones in equation (14).

Compared with (14), the interest rate under the discretion assumption does not adjust to lagged interest rates, and reacts only to the current inflation and the output gap. Thus, the rule has a less complicated interest rate adjustment mechanism. Note that the rule bears some resemblance to the original Taylor rule (6).<sup>13</sup>

Again, if both the home and foreign countries follow the same interest rate policy, and the uncovered interest parity (5) is valid, the implied exchange rate equations under commitment and discretion are, respectively, given by

$$\begin{aligned} E_t \Delta s_{t+1} = & \rho_1 (i_{t-1} - i_{t-1}^*) + \rho_2 (\Delta i_{t-1} - \Delta i_{t-1}^*) \\ & + \phi_\pi (\pi_t - \pi_t^*) + \phi_y (\Delta y_t - \Delta y_t^*) - \xi_t, \end{aligned} \quad (16)$$

and

$$E_t \Delta s_{t+1} = \phi_\pi (\pi_t - \pi_t^*) + \phi_y (y_t - y_t^*) - \xi_t. \quad (17)$$

Comparing equations (16) and (17) with (10) and (12) in the previous subsection, we note that exchange rate changes could behave quite differently in the presence of interest rate smoothing. Specifically, in the commitment case, interest rate smoothing introduces complex interest rate effects and brings in the first difference of the output gap differential instead of the output gap differential itself into the exchange rate equation.

<sup>13</sup> The Giannoni and Woodford (2003) optimal interest rate rule does not include shock terms due to the way the model is setup.

## 2.5 The Optimal Interest Rate Rule III – Inflation Inertia

In the current subsection, we consider the possibility of partial inflation adjustment as in Giannoni and Woodford (2002) and Woodford (2003).<sup>14</sup> To accommodate this adjustment process, we follow these authors and modify the Phillips curve equation (1) to

$$\pi_t - \gamma\pi_{t-1} = \beta E_t(\pi_{t+1} - \gamma\pi_t) + \kappa y_t + e_t, \quad (18)$$

where  $\gamma \in (0,1)$  determines the inflation adjustment speed. In the presence of inflation inertia, the period loss-function is modified to

$$L_t = (\pi_t - \gamma\pi_{t-1})^2 + \lambda_y y_t^2 + \lambda_i (i_t - \tilde{i})^2. \quad (19)$$

The optimal instrument rule under commitment with inflation inertia is given by (Giannoni and Woodford, 2002; Woodford, 2003)

$$\begin{aligned} i_t = & (1 - \rho_1)\tilde{i} + (\phi_\pi - \theta_\pi)\bar{\pi} + \rho_1 i_{t-1} + \rho_2 \Delta i_{t-1} \\ & + \phi_\pi F_t(\pi) + \phi_y F_t(y) - \theta_\pi \pi_{t-1} - \theta_y y_{t-1}, \end{aligned} \quad (20)$$

where  $F_t(\cdot)$  denotes a linear combination of forecasts of the argument at various horizons,  $\tilde{i}$  and  $\bar{\pi}$  are the means of inflation and interest rates,  $\rho_1 = 1 + (\lambda_2 - 1)(1 - \lambda_1)$ ,  $\rho_2 = \lambda_1 \lambda_2$ ,  $\phi_\pi = \theta_\pi \{1 + (1 - \gamma)(1 - \beta\gamma) / [\gamma(1 - \lambda_3^{-1})]\}$ ,  $\theta_\pi = \kappa\varphi / \lambda_i \beta \lambda_3$ , and  $\phi_y = \theta_y = \lambda_y \varphi / \lambda_i \beta \gamma \lambda_3$ . See Appendix C for definition of the parameters  $\lambda_1$ ,  $\lambda_2$ , and  $\lambda_3$ , which are related to adjustments to inflation.

The optimal interest rate policy rule under discretion is

$$i_t = \tilde{i} + \phi_\pi \pi_t - \theta_\pi \pi_{t-1} + \phi_y y_t, \quad (21)$$

where  $\phi_\pi = \kappa\varphi / \lambda_i (1 + \beta\gamma)$ ,  $\theta_\pi = \kappa\varphi\gamma / \lambda_i (1 + \beta\gamma)$ , and  $\phi_y = \varphi\lambda_x / \lambda_i$ . It is evidenced that the inflation inertia assumption induces lagged inflation to the interest rate rule under both commitment and discretion.

In this case, the implied exchange rate equations under commitment and discretion are given,

<sup>14</sup> For example, Woodford (2003) provides a model with partial inflation inertia under the presumption that “.... prices are not held constant between the dates at which they are re-optimized, but instead are automatically adjusted on the basis of the most recent quarter’s increase in the aggregate price index, by a percentage that is a fraction  $\gamma$  of the percentage increase in the index.” (pp. 1432-1433).

respectively, by

$$\begin{aligned}
E_t \Delta s_{t+1} = & (1 - \rho_1)(\tilde{i} - \tilde{i}^*) + (\phi_\pi - \theta_\pi)(\bar{\pi} - \bar{\pi}^*) + \rho_1(i_{t-1} - i_{t-1}^*) + \rho_2(\Delta i_{t-1} - \Delta i_{t-1}^*) \\
& + \phi_\pi [F_t(\pi) - F_t^*(\pi)] + \phi_y [F_t(y) - F_t^*(y)] \\
& - \theta_\pi(\pi_{t-1} - \pi_{t-1}^*) - \theta_y(y_{t-1} - y_{t-1}^*) - \xi_t,
\end{aligned} \tag{22}$$

and

$$E_t \Delta s_{t+1} = \phi_\pi(\pi_t - \pi_t^*) - \theta_\pi(\pi_{t-1} - \pi_{t-1}^*) + \phi_y(y_t - y_t^*) - \xi_t. \tag{23}$$

These two equations allow us to explore the roles of forward, current, and lagged inflation.

## 2.6 The Optimal Interest Rate Rule IV – Inflation Targeting

Since it has been adopted by New Zealand in 1990, inflation targeting is gaining acceptance by both developed and developing countries.<sup>15</sup> To illustrate the strict inflation targeting case, we modify the loss function (13) by setting the weight on output gap stabilization to zero. In doing so, we implicitly assume the inflation target is zero and retain the role of interest rate stabilization. The resulting loss function is

$$L_t = \pi_t^2 + \lambda_i(i_t - \tilde{i})^2, \tag{24}$$

where  $\lambda_i > 0$ .<sup>16</sup> Thus, with the loss function (24), the optimal interest rate policy rules under commitment and discretion requirements are

$$i_t = (1 - \rho_1)\tilde{i} + \rho_1 i_{t-1} + \rho_2 \Delta i_{t-1} + \phi_\pi \pi_t, \tag{25}$$

and

$$i_t = \tilde{i} + \phi_\pi \pi_t, \tag{26}$$

<sup>15</sup> See Rose (2007) for a list of inflation targeters. Mishkin (2004) and Svensson (1999), for example, list the characteristics of an inflation targeting regime. Bernanke and Woodford (2005) discuss various aspects of the inflation targeting. Mishkin and Schmidt-Hebbel (2007) document the benefits of inflation targeting, and Walsh (2009) surveys the effects of inflation targeting on macroeconomic performance and assesses their implications for the design of monetary policy.

<sup>16</sup> Svensson (1999), for example, refers flexible inflation targeting to the case of a non-zero  $\lambda$  in (3) or a non-zero  $\lambda_y$  in (13).



respectively, where  $\rho_1 = 1 + \kappa\varphi / \beta$ ,  $\rho_2 = 1 / \beta$ , and  $\phi_\pi = \kappa\varphi / \lambda_i$ . Expectedly, the policy rule under strict inflation targeting does not respond to the output gap.

The implied exchange rate equations under commitment and discretion rules are

$$E_t \Delta s_{t+1} = \rho_1(i_{t-1} - i_{t-1}^*) + \rho_2(\Delta i_{t-1} - \Delta i_{t-1}^*) + \phi_\pi(\pi_t - \pi_t^*) - \xi_t, \quad (27)$$

and

$$E_t \Delta s_{t+1} = \phi_\pi(\pi_t - \pi_t^*) - \xi_t. \quad (28)$$

Thus, with the modified loss function under strict inflation targeting, we removed the output gap variable from the exchange rate determination process.

## 2.7 The Optimal Interest Rate Rule V – Constant Money Growth

One competitor to inflation targeting is money growth targeting. Specifically, the money growth rate is used as a target through which some intermediate targets, such as price stability can be achieved. The money growth rule can be discussed in the context of the Friedman's money supply rule in which the money supply is increased by a certain percentage at each period to facilitate economic growth and maintain price stability.<sup>17</sup>

Under a constant money growth target, the period loss function is modified to (Svensson, 1999):

$$L_t = (k_t - \tilde{k})^2, \quad (29)$$

where  $k_t$  is the growth rate of money supply, and  $\tilde{k}$  is the target growth rate. Assume the money process follows the law of motion,

$$m_t = m_{t-1} + k_t + \omega_t, \quad (30)$$

where  $m_t$  is the log of money supply and  $\omega_t$  is the innovation term. The demand for real balances is a function of the output and nominal interest rate, and the money market equilibrium is given by

$$m_t - p_t = \nu \tilde{y}_t - i_t / \eta + \varsigma_t, \quad (31)$$

<sup>17</sup> See, for example, Nelson (2008) for a comparison of the Friedman and Taylor policy rules. Taylor (1999) notes the link between money-growth targeting and the interest rate function.

where  $p_t$  is the log price level and  $\tilde{y}_t$  is the log nominal output. The interest rate rule which incorporates the money growth rate is given by

$$\dot{i}_t = \dot{i}_{t-1} + \eta(\pi_t - k_t) + \nu\eta g_{y,t} + \chi_t, \quad (32)$$

where  $\pi_t = p_t - p_{t-1}$ ,  $g_{y,t} = \tilde{y}_t - \tilde{y}_{t-1}$ , and  $\chi_t = \eta(\varsigma_t - \varsigma_{t-1} - \omega_t)$ .

Two remarks are in order: First, under the current setting, both the commitment and discretion assumptions give the same interest rate rule. Second, strictly speaking, equation (32) is the optimal rule if  $k_t$  is replaced by  $\tilde{k}$ , the (optimal) target money growth rate. Since we do not know the optimal rate, we keep  $k_t$  in equation (32) and call it the interest rate rule under money growth consideration. In this case the implied exchange rate equation is given by

$$\begin{aligned} E_t \Delta s_{t+1} &= (\dot{i}_{t-1} - \dot{i}_{t-1}^*) + \eta[(\pi_t - k_t) - (\pi_t^* - k_t^*)] \\ &\quad + \nu\eta(g_{y,t} - g_{y,t}^*) - \xi_t + \chi_t - \chi_t^*. \end{aligned} \quad (33)$$

The presence of money factors is an obvious difference between (33) and the other implied exchange rate equations listed in the previous subsection. Again, when we modify the loss or objective functions, the implied exchange rate equation changes accordingly. In this case, the shift to money growth brings money back into the exchange rate determination process.

## 2.8 Discussion

In the previous subsections, we considered interest rate rules and the corresponding implied exchange rate equations under five different scenarios, and obtained some markedly different interest rate rules and a diverse group of policy rule based exchange rate equations. The optimal interest rate rule changes with the period loss function, which reflects assumptions about the model structure and policy preferences. Even with the same period loss function, the functional form of the policy rule depends upon the assumption about time consistency; that is, whether it is a commitment or discretion equilibrium. Usually, a rule is optimal only within the framework in which it is derived. For instance, the optimal rule in Subsection 2.3 may not be robustly-optimal for the model considered in Subsection 2.7.

It is known that the optimal interest rate rule could be different from the rules estimated from empirical data, or used in empirical studies. The Taylor rules (6), (6a) and (6b), for example, are quite different from the optimal rules reported in the last few subsections. The exchange rate equation implied by equations (6a) and (6b) is

$$E_t \Delta s_{t+1} = \gamma_q q_t + \gamma_\pi E_t (\pi_{t+1} - \pi_{t+1}^*) + \gamma_y (y_t - y_t^*) - \xi_t + z_t - z_t^*, \quad (34)$$

which differs from the exchange rate equations listed in the previous subsections.<sup>18</sup> Table 1 presents the implied exchange rate equations discussed so far.

With the exception of the money growth case, the money variable does not directly enter into these implied exchange rate equations. That is, money plays a subtle role at best in determining exchange rates.<sup>19</sup> This result represents a significant departure from the monetary approach to exchange rate determination. The output gap effect is mostly positive in these exchange rate equations and, thus, is comparable to the output effect under the monetary approach.

In our implied exchange rate equations, inflation in general has a positive effect on exchange rate and the negative inflation effect is mainly associated with the lagged inflation rate (see (22) and (23)). The result is different from the observation of “bad news about inflation is good news for the exchange rate”; that is, a high (unexpected) inflation leads to a strong currency, *ceteris paribus*, noted in some studies.<sup>20</sup> The discrepancy possibly reflects the difference between implications from theory and properties of empirical data.

As an aside, we note that a negative inflation effect could be built into a monetary model. While the interest rate differential has a positive effect on exchange rates in the flexible price monetary model, it has a negative effect in the well-known overshooting model. Gourinchas and Tornell (2004) and Bacchetta and van Wincoop (2010) offer alternative models to illustrate that an increase in interest rate could lead to exchange rate appreciation. By appealing to the positive association between interest rate and inflation differentials, inflation could have a negative effect in these models.

In sum, a policy rule based exchange rate equation depends on how the policy rule is defined. The disparate exchange rate behaviors implied by these different interest rate rules can be a reminder of the complexity surrounding exchange rate modeling. Of course, these optimal interest rate rules are derived under specific assumptions about the structure of economy and data dynamics, and there is no guarantee that these optimal rules are relevant for real world policy issues. Thus, the performance of a policy rule based exchange rate equation depends on, among other things, the relevance of the underlying policy rule.

In the next section, we assess the performance of exchange rate equations based on these optimal interest rate rules.

<sup>18</sup> Chinn (2008), Molodtsova and Papell (2009), Molodtsova *et al.* (2008), Wang and Wu (2008), and Engel *et al.* (2008) used variants of equation (34) in their studies of Taylor rule based exchange rate models.

<sup>19</sup> McCallum (1994), for example, illustrates that the long-run exchange rate behavior can be independent of money supply policy.

<sup>20</sup> See, for example, Clarida and Waldman (2008), Engel and West (2006) and Engel *et al.* (2008). Molodtsova and Papell (2009) and Chinn (2008), on the other hand, imposed the negative inflation effect on their policy rule based exchange rate equations.

### 3. A Performance Analysis

The illustrious Meese and Rogoff result raises a fundamental issue in exchange rate modeling – do we want to build an exchange rate model to explain in-sample data behavior or to generate superior forecasts?

A related question is: “Does a model that gives a good in-sample description of exchange rate behavior also generate good exchange rate forecasts?” Cheung *et al.* (2005b), for instance, show that there is little correspondence between how well a model conforms to theoretical priors and its predictive power. Clements and Hendry (2001), on the other hand, show that an incorrect but simple model may outperform a correct model in forecasting. In other words, in-sample and out-of-sample analyses could offer information on different aspects of a model’s performance.

Between these two choices, our inclination is to gauge a model’s in-sample performance and leave forecast evaluation as a future exercise.<sup>21</sup>

#### 3.1 Procedures and Data

We follow the spirit of Engel and West (2006) to assess the performance of the implied exchange rate equations. The focus is on exchange rate changes because the exchange rate itself is typically a non-stationary  $I(1)$  process – a property that could lead to complex empirical issues that are unrelated to the theme of the current study. Specifically, we generate exchange rate changes from individual implied exchange rate equations derived in Section 2 and summarized in Table 1. The implied exchange rate changes are then compared with the actual changes. To generate these implied rates, we have to determine the individual components of these implied exchange rate equations.

For each equation, we impose coefficient values that are taken from extant studies on optimal interest rate rules using a similar new Keynesian framework. The parameter values are presented in Table 2, and briefly discussed in Appendix D. Engel and West (2006), for example, also adopted the calibration approach. Using *a priori* parameter values, the calibration approach avoids the well-known difficulty of estimating an exchange rate equation.<sup>22</sup>

Some implied exchange rate equations involve the one-period-ahead forecasts of inflation and output gap variables. We generate these forecasts from the trivariate vector autoregressive model,

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<sup>21</sup> One motivation for using out-of-sample performance is to minimize the possibility of data mining. Nonetheless, out-of-sample analyses are subject to similar criticisms. Without any connotations, we note that researchers are free to explore different ways to generate forecasts in various sample periods and for different currencies. To be sure, it is not easy to replicate the reported good forecasting performance using different forecasting horizons, different forecasting periods, and different currencies. See, for example, Inoue and Kilian (2004) for additional discussions on in-sample and out-of-sample comparisons.

<sup>22</sup> Chinn and Meese (1995) and Mark (1995), for example, also imposed values regarding the long run relationship between exchange rates and their fundamentals in evaluating exchange rate models.

$$x_t = c + A_1 x_{t-1} + \dots + A_p x_{t-p} + \varrho_t, \quad (35)$$

where  $x_t = (\pi_t, y_t, i_t)'$ ,  $c = (c_\pi, c_y, c_i)'$  is a vector of intercepts, and  $\varrho_t$  is the innovation vector with a nonsingular covariance matrix,  $\Sigma_\varrho$ .<sup>23</sup> A 10-year constant rolling window is used to obtain the required forecast estimates. See Appendix E for a more detailed discussion.

It is noted that the simple parity,  $i_t - i_t^* = E_t \Delta s_{t+1}$ , without  $\xi_t$  is typically not supported by data; see Engel (1996) for a survey of earlier studies. The existing empirical findings suggest that  $\xi_t$  is highly variable. Thus, the risk premium term,  $\xi_t$ , included in the uncovered interest parity condition (5) could have substantial implications for modeling exchange rate behavior. The risk premium term indeed appears in all the implied exchange rate equations.

To shed some light on the role of the risk premium, we construct estimates of  $\xi_t$  using the Kalman filter setup adopted by Wolff (1987) and Cheung (1993). The Kalman filter extracts risk premium estimates by exploiting an *identity* between the forward and spot rates, the risk premium, and the unanticipated exchange rate change. Specifically, the risk premium is given by the difference between the error of using the forward rate to forecast the spot rate and the unexpected change in the spot rate. The risk premium is, thus, theoretically not related to unexpected shocks to exchange rates. The procedure is discussed in Appendix F.

In the following subsections, we consider the dollar-pound exchange rates, and take the United Kingdom (UK) as the home country and the United States (US) as the foreign country. The choice of British pound is motivated by the fact that UK is one of the developed countries that pursues an interest rate rule type monetary policy and is considered in existing studies on Taylor rule exchange rate models (Clarida *et al.*, 1998; Nelson, 2001; Bank of England, 1999). Further, UK is among the first group of countries adopting inflation targeting (in 1992). Thus, it offers a good case study on the effects of policy change on exchange rate behavior.

The data ranges from January 1974 to December 2008. Since a 10-year rolling window is used to generate inflation and output gap forecasts, the sample for generated exchange rate changes is from January 1984 to December 2008. Because of the inflation targeting adoption in 1992, we also consider the two subsample periods: January 1984 to December 1991 and January 1992 to December 2008.

Monthly data on the spot exchange rate, the one-month forward exchange rates, the consumer price index, the industrial production index, the US federal fund rate, and the UK money market rate were retrieved from the International Financial Statistics, Congressional Budget Office, the Bank of England,

<sup>23</sup> This empirical strategy is commonly used in the literature even though the generated expectations variables may not represent the model-consistent expectations.

and DataStream. See Appendix H for data description.

Both consumer price and industrial production indexes are seasonally adjusted. Inflation is given by the month-over-month growth rate of the consumer price index. The proxy for the output gap is given by the difference between the industrial production index (in logs) and the index's trend, which is obtained using the Hodrick-Prescott filter with lambda sets to 14400. The interest rate series are scaled by 1/12 to get the monthly rates equivalence.

### 3.2 In-Sample Correlation

Table 3 presents the bivariate sample correlation coefficients between exchange rate changes, interest rate differentials, inflation differentials, and output gap differentials. Four sample periods are considered – a) the entire sample period; January 1974 to December 2008, b) the period in which we have the constructed implied exchange rate changes (recall that a 10-years rolling window is used to generate inflation and output gap forecasts); January 1984 to December 2008, c) the pre-inflation-targeting period; January 1984 to December 1991, and d) the inflation targeting period; January 1992 to December 2008.

The correlation between exchange rate changes and macroeconomic variables displays a considerable degree of variation across these sample periods. For instance, exchange rate changes were positively associated with interest rate differentials only in the subsample starting 1992, the year UK officially adopted inflation targeting. It does not give strong support for the *ex post* uncovered interest rate parity. The positive association between exchange rate changes and inflation differentials in the entire sample is in accordance with most of implied exchange rate equations in Table 1, while a negative relationship in the three subsample periods lends support to the “bad news about inflation is good news for the exchange rate” view. Nonetheless, these correlation estimates are quite small. A puzzling observation is that exchange rate changes have a negative correlation with output gap differentials for most of the sample periods while the implied exchange rate equations usually suggest a positive correlation.

Relatively speaking, the 1984 to 1991 period gives the strongest bivariate correlation between the three differential variables while the 1992 to 2008 sample gives the weakest correlation coefficients. An obvious caveat is that these sample coefficient estimates do not capture the dynamic interactions and, thus, have to be interpreted with caution. Nonetheless, they are suggestive of some non-stable relationships between exchange rates and their fundamentals.

The bivariate correlation coefficients between the actual exchange rate change series and individual implied exchange rate change series and their robust standard errors are reported in Table 4. The results pertaining to implied exchange rate changes without the risk premium term are given in

columns (2) to (4), and those with the risk premium term are in the last three columns.<sup>24</sup>

In the absence of the risk premium term, the performance of these implied exchange rate equations is not impressive. For the 1984 to 2008 sample, eight of the ten generated series have a negative, albeit insignificant, correlation with the actual data series. An encouraging result is that the only statistically significant correlation coefficient is positive, and has a magnitude of 0.212. The significant result is given by the implied exchange rate equation based the interest rate rule that allows for inflation inertia and with commitment (Model III-inflation inertia, commitment). In view of the difficulty of explaining exchange rates, a correlation coefficient over 20 percent is quite high for monthly data.

The performance of these exchange rate equations can be quite different across the two subsample periods 1984-1991 and 1992-2008. In general, these implied exchange rate equations perform better in the later sample period than the earlier one – with the exception of Model III with commitment. In both sample periods, the implied exchange rate equation derived from Model III with commitment gives a significant and positive correlation coefficient. On the other hand, the implied exchange rate equations from Model II and Model IV with commitment yield a significantly negative correlation coefficient in the 1984 to 1991 period.

It is worth noting that an implied exchange rate equation derived from a commitment interest rate rule does not necessarily perform better than one derived from a discretion rule in terms of its association with the actual exchange rate change. Also, Model V, the model based on the constant money growth assumption, yields implied exchange rate changes that have a large positive, though insignificant, correlation coefficient in the 1992-2008 sample period. On the other hand, the correlation coefficients are quite small for the implied exchange rate equation derived from Model VI, though it is reported to work quite well with the real Deutschemark-US dollar exchange rate data (Engel and West, 2006).

These results, obtained without the risk premium term, suggest that the performance of these implied exchange rate equations depends on the specification of the underlying optimal interest rate rule and also on the sample period. The model that incorporates inflation inertia and with a commitment rule yields exchange rate changes that are significantly and positively correlated with the actual exchange rate changes.

The inclusion of risk premium has a discernable positive impact on all the correlation coefficients between the implied and the actual exchange rate change. All the correlation coefficient estimates reported in the last three columns are quite significant and large – they range from the low of 0.641 (Model IV, 1984-1991) to the high of 0.942 (Model V, 1984-2008 and 1984-1991). These results assert the role of risk premiums and indirectly affirm the difficulty of explaining exchange rate variability using only information on fundamentals.

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<sup>24</sup> The use of the non-parametric Pearson statistics instead of heteroscedasticity-and-autocorrelation consistent statistics gives qualitatively similar inferences.

### 3.3 In-Sample Prediction

Table 5 presents the results of assessing the in-sample prediction performance. Specifically, we compare the in-sample squared prediction error of a given implied exchange rate equation relative to that of a driftless random walk model, which is a commonly used benchmark. The null hypothesis of the two models have the same prediction power is evaluated using the Diebold and Mariano (1995) test statistic, which tests if the implied exchange rate equation and the random walk model yield the same in-sample squared prediction error. The test statistic is described in Appendix G.<sup>25</sup> A significant and negative statistic means that the implied exchange rate equation performs better than the random walk model.

In the absence of a risk premium term, only six out of 30 statistics in Table 5 are negative. These negative statistics are, however, statistically insignificant – indicating that the corresponding implied exchange rate equations do not yield an in-sample mean squared prediction error that is *significantly* smaller than a random walk model. Three of these negative statistics are from the implied exchange rate equation under Model 3 with a commitment rule, which performs quite well in Table 4. On the other hand, the test indicates that the implied exchange rate equations under Model II, Model IV, Model V, and Model VI can yield an in-sample mean squared prediction error that is significantly larger than that of a random walk specification.

Similar to the results in Table 4, the inclusion of risk premiums substantially enhances the performance of these implied exchange rate equations. All the test statistics reported in the last three columns in Table 5 are negative and highly significant. Thus, compared with a random walk specification, an implied exchange rate equation that incorporates risk premiums has a significantly smaller in-sample mean squared prediction error. Again, the results attest to the difficulty of modeling exchange rates using only economic fundamentals.

In addition to in-sample squared prediction errors, we assess the ability of these implied exchange rate equations to predict the direction of change. Table 6 reports the proportions of in-sample predictions that have the same sign as that of the corresponding exchange rate changes, the test statistics for the hypothesis that the reported proportion is significantly different from  $\frac{1}{2}$ , and the associated (asymptotic) p-values. We consider the cases in which the proportion of “correct” predictions that is larger than  $\frac{1}{2}$  contain usual useful information about exchange rate movements.

Again, consider the case without the risk premium. The implied exchange rate equations under Model I, Model III, and Model IV have a more than 50% chance of predicting the direction of exchange rate movement. In total, there are 13 out of 30 cases in which the proportion of correct predictions is larger than  $\frac{1}{2}$ . Nonetheless, only two prediction statistics from Model III with commitment and one statistic from Model I with commitment are statistically significant. Note that Model III with commitment also gives a strong performance in Tables 4 and 5.

<sup>25</sup> The choice of the Diebold and Mariano procedure is motivated by the work of Rogoff and Stavrageva (2008).



Even though the direction of change test offers evidence some what more favorable to the implied exchange rate equation than the mean squared prediction error, the results do not suggest these equations in general have a superior in-sample direction of change prediction ability. In fact, Model II and Model IV with a commitment rule have significantly negative proportion statistics; that is, these models tend to yield in-sample predictions of exchange rate changes with the wrong sign. These two specifications also give mean squared prediction errors that are larger than those of a random walk model in Table 5.

The results in the last three columns of Table 6 indicate that the inclusion of the risk premium term substantially improves the ability of these implied exchange rate equations to predict in-sample direction of change. All the implied exchange rate equations have the ability to predict the direction of change in a statistically significant manner; indeed, the proportion of correct predictions ranges from 71 to 93 percent.

### 3.4 Discussion

There are a few observations on these empirical findings. For the results that do not incorporate a risk premium term, the empirical performance of an implied exchange rate equation depends on its underlying optimal interest rate rule, the evaluation criterion, and the sample period. Among those considered in our exercise, the implied exchange rate equation derived from Model III with a commitment rule offers the “best” performance.<sup>26</sup> It gives the highest (positive and significant) correlation with the actual exchange rate changes, and has the ability to predict the direction of change. It also gives a mean squared prediction error that is less than that of a random walk model, though the improvement is not statistically significant.

Incidentally, this implied exchange rate equation has the most complex dynamics, and includes linear combinations of forecasts of variables (Table 1 and Appendix C). Its success in the current exercise may well reflect the role of expectations in determining exchange rate behavior discussed in the asset pricing approach (Engel and West, 2006). Nonetheless, its performance is not consistent across different sample periods.

In general, the performance measures of these exchange rate equations could differ quite substantially across the subsample periods 1984-1991 and 1992-2008; the difference could either be the sign or the magnitude. The differences in performance across the two subsamples could result from the change in the UK monetary policy. Admittedly, our empirical setting is not a rigorous design to investigate effects of policy changes. The circumstantial evidence reported in the previous subsections, nonetheless, are indicative of the possibility that the policy change to inflation targeting could have implications for the observed correlations between the exchange rate and its fundamentals, and the performance of policy rule based exchange rate equations. Incidentally, the entire period of 1984-2008 covers the arrival of great moderation and the related change in the US monetary policy.

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<sup>26</sup> Apparently, the implied exchange rate equations from Model II give the worse performance.

Thus, further analyses of the policy change effects on exchange rate behavior are warranted.

The inclusion of the estimated risk premium term could substantially improve the performance of these implied exchange rate equations. One possible interpretation is that a large proportion of exchange rate changes is not explained by the fundamental variables included in these policy rules, but by other factors that we “conveniently” called risk premium. The interpretation is in line with the weak explanatory power of fundamentals reported in the literature. Our risk premium estimates were generated using the data-driven Kalman filter, which does not cast light on their underlying economic forces. It will be of interest to consider alternative measures of the unobservable risk premium and the underlying economic factors.

## 4. Concluding Remarks

We study exchange rate determination through the prism of interest rate policy. Instead of selecting a specific interest rate rule, we consider exchange rate equations that are derived from a few interest rate rules, which are (theoretically) optimal under specific conditions. Our exercise illustrates the role of defining/selecting an interest rate rule in establishing the link between monetary policy and exchange rate dynamics. While exchange rate equations implied by alternative interest rate rules offer differing abilities to describe exchange rate variability, it is encouraging to observe that, at least, one implied exchange rate equation shows a strong correlation with the actual data and has a predictive power.

At the same time, our exercise re-affirms the difficulty of explaining exchange rates. The performance of a policy rule based exchange rate equation depends on, in addition to its underlying interest rate rule, the evaluation criterion, and the evaluation period. The finding echoes the view that it is hard to find an exchange model that performs well across different evaluation criteria and in different sample periods. In the current content, the pertinent question is “(W)hich is the interest rate rule that should be used to model exchange rate behavior?”

The results pertaining to the inclusion of risk premium estimates highlight the limitation of using only fundamentals to explain exchange rates. Indeed, our proxy for the unobservable risk premium substantially enhances the performance of implied exchange rate equations.

To shed some insight on the role of the risk premium term, we compared the results from estimating the two specifications  $\Delta s_{t+1} = \alpha(i_t - i_t^*) + \varepsilon_t$  and  $\Delta s_{t+1} = \alpha(i_t - i_t^*) + \beta \hat{\xi}_t + \varepsilon_t$ . In regressing  $\Delta s_{t+1}$  on only  $(i_t - i_t^*)$ , the  $\alpha$ -estimate is either insignificant or significantly negative; a result that is consistent with those in the literature. In the presence of the estimated risk premium,  $\hat{\xi}_t$ , the  $\alpha$ -estimates are all positively significant around one. The  $\beta$ -estimates are always significantly negative but are less than minus one. Thus, the estimated interest differential effect is more aligned with the theoretical prediction in the presence of risk premium.

While these results are quite rudimentary, they suggest that further analyses on the role of risk premium (estimated via the Kalman filter or other methods) and its underlying economic factors are warranted.

Overall, our results show that the role of monetary policy in describing exchange rate behavior depends on the choice of policy rule. While the exchange rate equation based on the optimal interest rate allowing for interest rate and inflation inertia offers the best in-sample performance, we note that its performance could vary across evaluation methods and sample periods. Further, there is still a large portion of exchange rate variability left unexplained by the policy rule fundamentals.

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**Table 1. Interest Rate Rule Based Exchange Rate Models****Model 1: Learning**

$$\text{Commitment} \quad E_t \Delta s_{t+1} = \phi_\pi \widehat{E}_t(\pi_{t+1} - \pi_{t+1}^*) + \phi_y \widehat{E}_t(y_{t+1} - y_{t+1}^*) + \theta_y (y_{t-1} - y_{t-1}^*) - \xi_t + \varepsilon_t - \varepsilon_t^* \quad (10)$$

$$\text{Discretion} \quad E_t \Delta s_{t+1} = \phi_\pi \widehat{E}_t(\pi_{t+1} - \pi_{t+1}^*) + \phi_y \widehat{E}_t(y_{t+1} - y_{t+1}^*) - \xi_t + \varepsilon_t - \varepsilon_t^* \quad (12)$$

**Model 2: Interest Rate Inertia**

$$\text{Commitment} \quad E_t \Delta s_{t+1} = \rho_1(i_{t-1} - i_{t-1}^*) + \rho_2(\Delta i_{t-1} - \Delta i_{t-1}^*) + \phi_\pi(\pi_t - \pi_t^*) + \phi_y(\Delta y_t - \Delta y_t^*) - \xi_t \quad (16)$$

$$\text{Discretion} \quad E_t \Delta s_{t+1} = \phi_\pi(\pi_t - \pi_t^*) + \phi_y(y_t - y_t^*) - \xi_t \quad (17)$$

**Model 3: Inflation Inertia**

$$\begin{aligned} \text{Commitment} \quad E_t \Delta s_{t+1} = & (1 - \rho_1)(\bar{i} - \bar{i}^*) + (\phi_\pi - \theta_\pi)(\bar{\pi} - \bar{\pi}^*) + \rho_1(i_{t-1} - i_{t-1}^*) + \rho_2(\Delta i_{t-1} - \Delta i_{t-1}^*) \\ & + \phi_\pi[F_t(\pi) - F_t^*(\pi)] + \phi_y[F_t(y) - F_t^*(y)] - \theta_\pi(\pi_{t-1} - \pi_{t-1}^*) - \theta_y(y_{t-1} - y_{t-1}^*) - \xi_t \end{aligned} \quad (22)$$

$$\text{Discretion} \quad E_t \Delta s_{t+1} = \phi_\pi(\pi_t - \pi_t^*) - \theta_\pi(\pi_{t-1} - \pi_{t-1}^*) + \phi_y(y_t - y_t^*) - \xi_t \quad (23)$$

**Model 4: Inflation Targeting**

$$\text{Commitment} \quad E_t \Delta s_{t+1} = \rho_1(i_{t-1} - i_{t-1}^*) + \rho_2(\Delta i_{t-1} - \Delta i_{t-1}^*) + \phi_\pi(\pi_t - \pi_t^*) - \xi_t \quad (27)$$

$$\text{Discretion} \quad E_t \Delta s_{t+1} = \phi_\pi(\pi_t - \pi_t^*) - \xi_t \quad (28)$$

**Model 5: Money Growth**

$$E_t \Delta s_{t+1} = (i_{t-1} - i_{t-1}^*) + \eta[(\pi_t - k_t) - (\pi_t^* - k_t^*)] + \nu\eta(g_{y,t} - g_{y,t}^*) - \xi_t + \chi_t - \chi_t^* \quad (33)$$

**Model 6: Taylor Rule**

$$E_t \Delta s_{t+1} = \gamma_q q_t + \gamma_\pi E_t(\pi_{t+1} - \pi_{t+1}^*) + \gamma_y (y_t - y_t^*) - \xi_t + z_t - z_t^* \quad (34)$$

Note: The table collects the policy rule implied exchange rate equations discussed in the text. See the text for variable definitions. The equation numbers are those used in the text.

Table 2. Interest Rate Rule Parameters

	$\phi_\pi$	$\phi_y$	$\rho_1$	$\rho_2$	$\theta_\pi$	$\theta_y$	$\eta$	$\nu$	$\gamma_q$	$\gamma_\pi$	$\gamma_y$
Model 1: Learning											
Commitment	2.08	0.16				0.15					
Discretion	2.08	0.16									
Model 2: Interest Rate Inertia											
Commitment	0.42	0.05	1.10	1.03							
Discretion	0.42	0.05									
Model 3: Inflation Inertia											
Commitment	1.00	0.60	0.91	0.47	0.36	0.60					
Discretion	0.26	0.05		0.16							
Model 4: Inflation Targeting											
Commitment	0.42		1.10	1.03							
Discretion	0.42										
Model 5: Money Growth											
							0.09	0.93			
Model 6: Taylor Rule											
									0.10	1.50	0.50

Note: The table lists the parameter values of the interest rate rules used to generate the corresponding implied exchange rate changes considered in the text and Table 1. A discussion on the parameter values is given in the Appendix.

**Table 3. Bivariate Sample Correlation Coefficients**

	$\Delta S_{t+1}$	Interest Rate Diff.	Inflation Diff.	Output Gap Diff.
<i>A. 1974-2008</i>				
$\Delta S_{t+1}$	1.000			
Interest Rate Diff.	-0.062	1.000		
Inflation Diff.	0.068	-0.061	1.000	
Output Gap Diff.	-0.063	0.245	0.175	1.000
<i>B. 1984-2008</i>				
$\Delta S_{t+1}$	1.000			
Interest Rate Diff.	-0.078	1.000		
Inflation Diff.	-0.045	0.194	1.000	
Output Gap Diff.	-0.097	0.259	0.042	1.000
<i>C. 1984-1991</i>				
$\Delta S_{t+1}$	1.000			
Interest Rate Diff.	-0.257	1.000		
Inflation Diff.	-0.037	0.243	1.000	
Output Gap Diff.	-0.180	0.597	0.071	1.000
<i>D. 1992-2008</i>				
$\Delta S_{t+1}$	1.000			
Interest Rate Diff.	0.140	1.000		
Inflation Diff.	-0.032	0.005	1.000	
Output Gap Diff.	-0.002	0.004	0.061	1.000

Note: The table presents the bivariate correlation coefficients between exchange rate changes and the fundamentals included in interest rate policy rules.

**Table 4. Correlation between Model Based and Actual Exchange Rate Changes**

	Without Risk Premium			With Risk Premium		
	1984-2008	1984-1991	1992-2008	1984-2008	1984-1991	1992-2008
<b>Model 1: Learning</b>						
Commitment	0.025 (0.068)	0.011 (0.100)	0.079 (0.072)	0.862*** (0.120)	0.853*** (0.150)	0.872*** (0.190)
Discretion	-0.037 (0.073)	-0.096 (0.110)	0.058 (0.076)	0.851*** (0.130)	0.838*** (0.160)	0.862*** (0.190)
<b>Model 2: Interest Rate Inertia</b>						
Commitment	-0.104 (0.077)	-0.208* (0.110)	0.038 (0.110)	0.926*** (0.140)	0.930*** (0.150)	0.920*** (0.220)
Discretion	-0.0894 (0.071)	-0.142 (0.098)	-0.0282 (0.100)	0.940*** (0.140)	0.940*** (0.150)	0.933*** (0.210)
<b>Model 3: Inflation Inertia</b>						
Commitment	0.212*** (0.067)	0.257*** (0.095)	0.171* (0.093)	0.790*** (0.110)	0.771*** (0.120)	0.807*** (0.190)
Discretion	-0.0240 (0.067)	0.0486 (0.110)	-0.0653 (0.079)	0.938*** (0.130)	0.930*** (0.140)	0.938*** (0.210)
<b>Model 4: Inflation Targeting</b>						
Commitment	-0.119 (0.076)	-0.233** (0.110)	0.026 (0.110)	0.928*** (0.140)	0.934*** (0.150)	0.921*** (0.220)
Discretion	-0.044 (0.064)	-0.037 (0.086)	-0.031 (0.091)	0.941*** (0.130)	0.938*** (0.150)	0.937*** (0.210)
<b>Model 5: Money Growth</b>						
	-0.0210 (0.079)	-0.144 (0.100)	0.145 (0.120)	0.942*** (0.140)	0.942*** (0.150)	0.937*** (0.220)
<b>Model 6: Taylor Rule</b>						
	-0.093 (0.078)	-0.173 (0.120)	0.015 (0.092)	0.704*** (0.130)	0.641*** (0.170)	0.757*** (0.180)

Note: Bivariate correlation coefficients are reported. Robust standard errors are given in parentheses. "\*\*\*\*", "\*\*\*" and "\*\*" indicate significance at the 1%, 5% and 10% level.

Table 5. In-Sample Mean Squared Prediction Error

	Without Risk Premium			With Risk Premium		
	1984-2008	1984-1991	1992-2008	1984-2008	1984-1991	1992-2008
<b>Model 1: Learning</b>						
Commitment	0.167 (0.220)	0.561 (0.610)	-0.019 (0.150)	-4.490*** (0.680)	-6.819*** (1.270)	-3.389*** (0.780)
Discretion	0.370 (0.240)	1.086 (0.660)	0.0314 (0.170)	-4.355*** (0.680)	-6.488*** (1.310)	-3.346*** (0.770)
<b>Model 2: Interest Rate Inertia</b>						
Commitment	0.314 (0.190)	0.987** (0.480)	-0.005 (0.160)	-4.460*** (0.680)	-6.678*** (1.090)	-3.411*** (0.830)
Discretion	0.0981 (0.063)	0.235* (0.140)	0.0333 (0.067)	-4.581*** (0.660)	-7.107*** (1.190)	-3.387*** (0.760)
<b>Model 3: Inflation Inertia</b>						
Commitment	-0.308 (0.330)	-0.614 (0.760)	-0.164 (0.330)	-4.655*** (0.770)	-7.267*** (1.290)	-3.419*** (0.930)
Discretion	0.0325 (0.049)	0.0117 (0.130)	0.0424 (0.041)	-4.619*** (0.650)	-7.227*** (1.150)	-3.386*** (0.750)
<b>Model 4: Inflation Targeting</b>						
Commitment	0.329* (0.190)	1.011** (0.460)	0.007 (0.150)	-4.450*** (0.670)	-6.664*** (1.090)	-3.403*** (0.830)
Discretion	0.0465 (0.047)	0.0367 (0.084)	0.0511 (0.057)	-4.602*** (0.650)	-7.206*** (1.190)	-3.371*** (0.750)
<b>Model 5: Money Growth</b>						
	0.097 (0.140)	0.533* (0.320)	-0.109 (0.130)	-4.616*** (0.680)	-6.976*** (1.140)	-3.500*** (0.830)
<b>Model 6: Taylor Rule</b>						
	1.184** (0.460)	2.974** (1.250)	0.338 (0.290)	-3.686*** (0.750)	-5.016*** (1.580)	-3.058*** (0.810)

Note: The table reports the Diebold-Mariano test statistics. Robust standard errors are given in parentheses. "\*\*\*\*", "\*\*\*" and "\*\*" indicate significance at the 1%, 5% and 10% level.

Table 6. In-Sample Direction of Change Prediction

		Without Risk Premium			With Risk Premium		
		1984-2008	1984-1991	1992-2008	1984-2008	1984-1991	1992-2008
<b>Model 1: Learning</b>							
Commitment	(a)	0.512	0.531	0.502	0.826	0.865	0.808
	(b)	0.405	0.612	0.070	11.277	7.144	8.773
	(c)	(0.345)	(0.270)	(0.472)	(0.000)	(0.000)	(0.000)
Discretion	(a)	0.508	0.479	0.522	0.826	0.875	0.803
	(b)	0.289	-0.408	0.632	11.277	7.348	8.633
	(c)	(0.389)	(0.340)	(0.264)	(0.000)	(0.000)	(0.000)
<b>Model 2: Interest Rate Inertia</b>							
Commitment	(a)	0.431	0.469	0.414	0.856	0.906	0.833
	(b)	-2.371	-0.612	-2.457	12.318	7.961	9.475
	(c)	(0.009)	(0.270)	(0.007)	(0.000)	(0.000)	(0.000)
Discretion	(a)	0.445	0.427	0.453	0.866	0.906	0.847
	(b)	-1.908	-1.429	-1.334	12.665	7.961	9.896
	(c)	(0.281)	(0.076)	(0.092)	(0.000)	(0.000)	(0.000)
<b>Model 3: Inflation Inertia</b>							
Commitment	(a)	0.569	0.625	0.542	0.769	0.813	0.749
	(b)	2.371	2.449	1.193	9.311	6.124	7.089
	(c)	(0.009)	(0.007)	(0.117)	(0.000)	(0.000)	(0.000)
Discretion	(a)	0.468	0.510	0.448	0.880	0.906	0.867
	(b)	-1.099	0.204	-1.474	13.128	7.961	10.458
	(c)	(0.138)	(0.420)	(0.071)	(0.000)	(0.000)	(0.000)
<b>Model 4: Inflation Targeting</b>							
Commitment	(a)	0.421	0.458	0.404	0.860	0.906	0.837
	(b)	-2.718	-0.816	-2.737	12.434	7.961	9.616
	(c)	(0.003)	(0.206)	(0.003)	(0.000)	(0.000)	(0.000)
Discretion	(a)	0.515	0.521	0.512	0.863	0.896	0.847
	(b)	0.520	0.408	0.351	12.549	7.757	9.896
	(c)	(0.302)	(0.341)	(0.363)	(0.000)	(0.000)	(0.000)
<b>Model 5: Money Growth</b>							
	(a)	0.468	0.427	0.488	0.903	0.938	0.887
	(b)	-1.099	-1.429	-0.351	13.937	8.573	11.019
	(c)	(0.138)	(0.764)	(0.363)	(0.000)	(0.000)	(0.000)
<b>Model 6: Taylor Rule</b>							
	(a)	0.468	0.469	0.468	0.722	0.740	0.714
	(b)	-1.099	-0.612	-0.912	7.692	4.695	6.106
	(c)	(0.138)	(0.271)	(0.181)	(0.000)	(0.000)	(0.000)

Note: Row (a) in the table presents the proportion of correct sign predictions. Row (b) presents the Diebold-Mariano direction of change (DOC) statistics, and Row (c) presents p-values of the DOC statistics.

## Appendix A. Derivation of Optimal Interest Rules

As noted in the text, the derivation of optimal interest rate rules under commitment is usually presented in some existing studies. However, the derivation of the rules under discretion is not that commonly discussed. For completeness and easy references, the Appendix outlines the derivation of the optimal rule under commitment for Model I and the optimal rules under discretion for all the models considered in the text. We also discuss the optimal rule under commitment for Model IV as its derivation is not available in the existing studies.

### A1. The Optimal Interest Rate Rule I - Learning

By substituting the Phillips curve and the IS curve into the *intertemporal loss function*, the objective function of the optimizing exercise can be written as

$$B_t = \sum_{j=0}^{\infty} \beta^j \left\{ \frac{1}{2} (\pi_{t+j})^2 + \lambda y_{t+j}^2 \right\} + \theta_{t+j} [y_{t+j} - y_{t+j+1} + \varphi(i_{t+j} - \pi_{t+j+1}) - u_{t+j}] \\ + \psi_{t+i} [\pi_{t+j} - \beta \pi_{t+j+1} - \kappa y_{t+j} - e_{t+j}].$$

Under commitment, the first order conditions are

$$\begin{aligned} \varphi E_t(\theta_{t+j}) &= 0 \quad \text{if } j \geq 0, \\ \pi_t + \psi_t &= 0 \quad \text{if } j = 0, \\ \beta^j E_t(\pi_{t+j} + \psi_{t+j} - \psi_{t+j-1}) - \varphi \beta^{j-1} \theta_{t+j-1} &= 0 \quad \text{if } j \geq 1, \\ \lambda y_t + \theta_t - \kappa \psi_t &= 0 \quad \text{if } j = 0, \\ \beta^j E_t(\lambda y_{t+j} + \theta_{t+j} - \kappa \psi_{t+j}) + \beta^{j-1} \theta_{t+j-1} &= 0 \quad \text{if } j \geq 1. \end{aligned}$$

From the first order condition, the IS curve does not impose any constraint since  $\theta_t = 0$ . In this case, we have a compact form given by

$$\begin{aligned} \pi_t + \psi_t &= 0 \quad j = 0, \\ E_t(\pi_{t+j} + \psi_{t+j} - \psi_{t+j-1}) &= 0 \quad j \geq 1, \\ E_t(\lambda y_{t+j} - \kappa \psi_{t+j}) &= 0 \quad j \geq 0. \end{aligned}$$



Suppose the expectations/decision pertaining to the last equation was formed in distant past, which Woodford (1999) calls the “timeless-perspective,” we have  $\psi_t = \frac{\lambda}{\kappa} y_t$ . Therefore,  $\pi_t = -\frac{\lambda}{\kappa}(y_t - y_{t-1})$  gives the first order optimal condition. Using the inflation equation, the solution for the output gap (not imposing rational expectations) is

$$y_t = -\frac{\kappa\beta}{\lambda+\kappa^2} \widehat{E}_t \pi_{t+1} + \frac{\lambda}{\lambda+\kappa^2} y_{t-1} - \frac{\kappa}{\lambda+\kappa^2} e.$$

Thus, the rule under commitment can be derived by substituting this equation into the IS curve and obtained

$$i_t = \left(1 + \frac{\kappa}{\varphi(\lambda+\kappa^2)}\right) \beta \widehat{E}_t \pi_{t+1} - \frac{\lambda}{\varphi(\lambda+\kappa^2)} y_{t-1} + \frac{1}{\varphi} \widehat{E}_t y_{t+1} + \frac{1}{\varphi} u_t + \frac{\kappa}{\varphi(\lambda+\kappa^2)} e_t.$$

The first order optimality condition under discretion is of the form  $\pi_t = -\frac{\lambda}{\kappa} y_t$ . In this case, the inflation equation can be used to find the solution for the output gap (not imposing the rational expectations) that is given by  $y_t = -\frac{\kappa\beta}{\lambda+\kappa^2} \widehat{E}_t \pi_{t+1} - \frac{\kappa}{\lambda+\kappa^2} e$ .

Using this equation and the IS equation, the rule under discretion is given by

$$i_t = \left(1 + \frac{\kappa}{\varphi(\lambda+\kappa^2)}\right) \beta \widehat{E}_t \pi_{t+1} + \frac{1}{\varphi} \widehat{E}_t y_{t+1} + \frac{1}{\varphi} u_t + \frac{\kappa}{\varphi(\lambda+\kappa^2)} e_t.$$

## A2. The Optimal Interest Rate Rule II - Interest Rate Inertia

The solution for the optimal interest rate rule under commitment can be found in Giannoni and Woodford (2002, 2003) and Woodford (2003). Here we derive the solution for the optimal interest rate rule under discretion. Given the *intertemporal loss function* for the model with interest rate inertia, the Phillips curve, and the IS curve, the objective function of the optimizing problem can be written as

$$\begin{aligned} \mathcal{L}_t = & \frac{1}{2} \{(\pi_t)^2 + \lambda_y y_t^2 + \lambda_i (i_t - \tilde{i})^2\} + \theta_t [y_t - E_t y_{t+1} + \varphi [i_t - E_t \pi_{t+1}] - u_t] \\ & + \psi_t [\pi_t - \beta E_t \pi_{t+1} - \kappa y_t - e_t]. \end{aligned}$$

The first order conditions are

$$\begin{aligned}\pi_t + \psi_t &= 0, \\ \lambda_y y_t + \theta_t - \kappa \psi_t &= 0, \\ \lambda_i (i_t - \tilde{i}) + \theta_t \varphi &= 0.\end{aligned}$$

These three equations could then be used to solve for  $\theta_t$ ,  $\psi_t$ , and  $i_t$ . Thus, the optimal interest rate rule under discretion is

$$i_t = \tilde{i} + \frac{\kappa \varphi}{\lambda_i} \pi_t + \frac{\varphi \lambda_y}{\lambda_i} y_t.$$

### A3. The Optimal Interest Rate Rule III - Inflation Inertia

As for the optimal interest rate rule with interest rate inertia, the solution for the optimal rule with inflation inertia under commitment can be found in Giannoni and Woodford (2002, 2003) and Woodford (2003). Given the *intertemporal loss function* for the model with inflation inertia, the Phillips curve, and the IS curve, the objective function of the optimizing problem under discretion can be written as

$$\begin{aligned}\mathcal{L}_t = & \frac{1}{2} \{ (\pi_t - \gamma \pi_{t-1})^2 + \lambda_y y_t^2 + \lambda_i (i_t - \tilde{i})^2 \} + \theta_t [y_t - E_t y_{t+1} + \varphi [i_t - E_t \pi_{t+1}] - u_t] \\ & \psi_t [(\pi_t - \gamma \pi_{t-1}) - \beta E_t (\pi_{t+1} - \gamma \pi_t) - \kappa y_t - e_t].\end{aligned}$$

The first order conditions are

$$\begin{aligned}\pi_t - \gamma \pi_{t-1} + \psi_t (1 + \beta \gamma) &= 0, \\ \lambda_y y_t + \theta_t - \kappa \psi_t &= 0, \\ \lambda_i (i_t - \tilde{i}) + \theta_t \varphi &= 0.\end{aligned}$$

Following the same solution procedure for the case of interest rate inertia, the optimal interest rate rule for the model with inflation inertia can be shown to follow

$$i_t = \tilde{i} + \frac{\kappa \varphi}{\lambda_i (1 + \beta \gamma)} \pi_t - \frac{\kappa \varphi \gamma}{\lambda_i (1 + \beta \gamma)} \pi_{t-1} + \frac{\varphi \lambda_y}{\lambda_i} y_t.$$

#### A4. The Optimal Interest Rate Rule IV - Inflation Targeting

As mentioned in the text, we illustrate the strict inflation targeting case by modifying the intertemporal loss function under interest rate inertia such that the weight on output gap stabilization is zero. Incorporating the Phillips curve and the IS curve into the *intertemporal loss function* gives the objective function

$$\begin{aligned}\mathcal{L}_t = & E_t \sum_{j=0}^{\infty} \beta^j \left\{ \frac{1}{2} (\pi_{t+j})^2 + \lambda_i (i_{t+j} - \tilde{i})^2 \right. \\ & + \theta_{t+j} [y_{t+j} - y_{t+j+1} + \varphi (i_{t+j} - \pi_{t+j+1}) - u_{t+j}] \\ & \left. + \psi_{t+i} [\pi_{t+j} - \beta \pi_{t+j+1} - \kappa y_{t+j} - e_{t+j}] \right\}.\end{aligned}$$

From a "timeless perspective" (and for  $j \geq 1$ ), the first order conditions are

$$\begin{aligned}\pi_t + \psi_t - \psi_{t-1} - \varphi \beta^{-1} \theta_{t-1} &= 0, \\ \theta_t - \beta^{-1} \theta_{t-1} - \kappa \psi_t &= 0, \\ \lambda_i (i_t - \tilde{i}) + \theta_t \varphi &= 0.\end{aligned}$$

The optimal interest rate rule can be obtained by solving for the two Lagrange multipliers,  $\theta_t$  and  $\psi_t$ . Under commitment, thus, the optimal interest rate rule can be expressed as

$$i_t = (1 - \rho_1) \tilde{i} + \rho_1 i_{t-1} + \rho_2 \Delta i_{t-1} + \phi_\pi \pi_t,$$

where  $\rho_1 = 1 + \kappa \varphi / \beta$ ,  $\rho_2 = 1 / \beta$ , and  $\phi_\pi = \kappa \varphi / \lambda_i$ .

Under discretion, on the other hand, the objective function of the optimization problem can be written as

$$\begin{aligned}\mathcal{L}_t = & \frac{1}{2} \{ (\pi_t)^2 + \lambda_i (i_t - \tilde{i})^2 \} + \theta_t [y_t - E_t y_{t+1} + \varphi [i_{t+j} - E_t \pi_{t+1}] - u_t] \\ & + \psi_t [\pi_t - \beta E_t \pi_{t+1} - \kappa y_t - e_t].\end{aligned}$$

The first order conditions are

$$\begin{aligned}\pi_t + \psi_t &= 0, \\ \theta_t - \kappa \psi_t &= 0,\end{aligned}$$

$$\lambda_i(i_t - \tilde{i}) + \theta_t \varphi = 0.$$

The optimal interest rate rule, after eliminating the two Lagrange multipliers,  $\theta_t$  and  $\psi_t$ , is given by

$$i_t = \tilde{i} + \frac{\kappa \varphi}{\lambda_i} \pi_t.$$

## Appendix B. “Bad News about Inflation Is Good News for the Exchange Rate”

This appendix illustrates the negative relationship between the real exchange rate and inflation.

Engel and West (2006), for example, consider the case in which the home country reacts to the real exchange rate while the foreign country does not. The asymmetric Taylor rules adopted by the home and foreign countries, repeated here for convenience, are given by

$$i_t = \gamma_q q_t + \gamma_\pi E_t \pi_{t+1} + \gamma_y y_t + z_t,$$

and

$$i_t^* = \gamma_\pi E_t \pi_{t+1}^* + \gamma_y y_t^* + z_t^*.$$

The interest rate differential from the two rules takes the form

$$i_t - i_t^* = \gamma_q q_t + \gamma_\pi E_t (\pi_{t+1} - \pi_{t+1}^*) + \gamma_y (y_t - y_t^*) + z_t - z_t^*.$$

Using the uncovered interest parity condition in real terms, the expected real exchange rate change is given by

$$E_t \Delta q_{t+1} = (i_t - E_t \pi_{t+1}) - (i_t^* - E_t \pi_{t+1}^*),$$

where the risk premium term is set to zero. Thus, the real exchange rate could be written as

$$q_t = [E_t q_{t+1} + (1 - \gamma_\pi) E_t (\pi_{t+1} - \pi_{t+1}^*) - \gamma_y (y_t - y_t^*) + z_t - z_t^*] / (1 + \gamma_q),$$

where  $(1 - \gamma_\pi) < 0$  since they assume that  $\gamma_\pi > 1$ . Thus, a higher home inflation relative to the foreign implies an appreciation of the real exchange rate. The underlying assumption that derives such a conclusion is that the home central bank follows an active Taylor rule, where  $\gamma_\pi > 1$ .

Clarida and Waldman (2008) introduce shocks to the model so that

$$y_t = -(i_t - E_t \pi_{t+1}) + q_t,$$

and

$$\pi_t = \pi_{t-1} + y_t + \varepsilon_t,$$

where  $\varepsilon_t$  is a white noise shock to the supply curve, and the other variables are defined as in the text. The home country is assumed to be a small open economy and takes the world interest rate and world inflation as given. The Taylor rule is defined to be

$$i_t = E_t \pi_{t+1} + b(\pi_t - \tilde{\pi}) + ay_t.$$

Assuming the world interest rate and world inflation are zero, the uncovered interest parity condition in real term can be written as

$$q_t = E_t q_{t+1} - (i_t - E_t \pi_{t+1}) = -E_t \sum_{k=0}^{\infty} (i_{t+k} - E_t \pi_{t+1+k}).$$

Suppose the *ex-ante* real interest rate,  $r_t = i_t - E_t \pi_{t+1}$ , follows a zero mean AR(1) process

$E_t r_{t+j} = d^j r_t$ , where  $0 < d < 1$ . Then  $q_t$  and  $y_t$  could be written as

$$q_t = -(i_t - E_t \pi_{t+1}) / (1 - d),$$

and

$$y_t = (2 - d)q_t.$$

With the Taylor rule, we have

$$q_t = -b(\pi_t - \tilde{\pi}) / [a(2 - d) + (1 - d)],$$

and the real exchange rate and inflation are negatively correlated.

## Appendix C. Parameter Values for the Model III under Commitment

For convenience, we reproduce the loss function and the Phillips curve for the model under inflation inertia here:

$$\pi_t - \gamma\pi_{t-1} = \beta E_t(\pi_{t+1} - \gamma\pi_t) + \kappa y_t + e_t,$$

and

$$L_t = (\pi_t - \gamma\pi_{t-1})^2 + \lambda_y y_t^2 + \lambda_i (i_t - \tilde{i})^2.$$

Under commitment, the first order conditions are

$$(\pi_t - \gamma\pi_{t-1}) - \beta\gamma E_t(\pi_{t+1} - \gamma\pi_t) - \varphi\beta^{-1}\theta_{t-1} - \beta\gamma E_t\psi_{t+1} + (1 + \beta\gamma)\psi_t - \psi_{t-1} = 0,$$

$$\lambda_y y_t + \theta_t - \beta^{-1}\theta_{t-1} - \kappa\psi_t = 0,$$

$$\lambda_i (i_t - \tilde{i}) + \theta_t \varphi = 0.$$

Using the last two equations to eliminate the Lagrange multipliers, Woodford (2003) writes down the Euler equation

$$E_t \left[ A(L)(i_t - \tilde{i}) \right] = -f_t,$$

where

$$A(L) = \beta\gamma - (1 + \gamma + \beta\gamma)L + (1 + \gamma + \beta^{-1}(1 + \kappa\sigma))L^2 - \beta^{-1}L^3,$$

and

$$f_t \equiv \kappa\varphi[(\pi_t - \gamma\pi_{t-1} + \lambda_y \Delta y_t / \kappa) - \beta\gamma E_t(\pi_{t+1} - \gamma\pi_t + \lambda_y \Delta y_{t+1} / \kappa)] / \lambda_i.$$

To solve for the roots of the polynomial  $A(L)$ , we need an estimate for the indexation parameter  $\gamma$ . We obtained a  $\gamma$  estimate of 0.6, which is an approximate value for both the US and UK inflation rates over the entire period of interest in our study by fitting an AR(1) to the inflation data. Given the other parameters specified in the text and the appendix, we are able to find the roots for the  $A(L)$  polynomial. Specifically, the polynomial is found to be  $A(L) = 0.5808 - 2.1808L + 2.6511L^2 - 1.0331L^3$

with the roots  $\lambda_1 = 0.643$ ,  $\lambda_2 = 0.737$ , and  $\lambda_3 = 1.186$ . Given three real roots, Woodford (2003) define the interest rate rule to be

$$i_t = (1 - \rho_1)\bar{i} + (\phi_\pi - \theta_\pi)\bar{\pi} + \rho_1 i_{t-1} + \rho_2 \Delta i_{t-1} + \phi_\pi F_t(\pi) + \phi_y F_t(y) - \theta_\pi \pi_{t-1} - \theta_y y_{t-1},$$

where  $\rho_1 = 1 + (\lambda_2 - 1)(1 - \lambda_1)$ ,  $\rho_2 = \lambda_1 \lambda_2$ , and the rest of parameters are defined in the text. As mentioned before,  $F_t(\pi)$  and  $F_t(y)$  denote a linear combination of inflation and the output gap forecasts, respectively, at various horizons with normalized weights. In our study, we set the maximum forecast horizon for inflation and the output gap to twelve months. The upper bound of forecast the horizon is chosen according to the conventional inflation targeting horizon for most central banks. Additionally, the forecast horizon choice is also consistent with Svensson's (1997) argument that the optimal inflation targeting forecast horizon should coincide with the lag in the monetary policy transmission mechanism. The normalized weights for the first to the twelfth period forecasts are, respectively, 0.154, 0.141, 0.128, 0.115, 0.103, 0.09, 0.077, 0.064, 0.051, 0.038, 0.026, and 0.013 – these weights are declining with an increase in the forecast horizon and reflect the priori believe that distant forecasts carry a smaller weight.



## Appendix D. Calibration Values

The parameters used for the interest rate rules and, thus, for generating the predicted exchange rate changes are mainly from the calibrated parameters suggested by Giannoni and Woodford (2002, 2003) and McCallum and Nelson (1999). The parameter values are  $\beta = 0.968$ ,  $\varphi^{-1} = 0.16$ ,  $\nu = 0.93$ , and  $\eta = 0.09$ . Since we use monthly data, some of the parameters are adjusted to their corresponding monthly values.

The price stickiness parameter assumes the value given by Nakamura and Steinsson (2008). These authors find that an uncensored median duration of 8-11 months for the U.S. prices. Additionally, Hall *et al.* (2000) find that the U.K. firms, on average, reviewed prices monthly but changed them only twice during 1994. We take the average duration of 9.5 months, which is larger than Hall *et al.*'s finding. However, given the recent evidence on inflation stickiness in a more stable macroeconomic environment in the U.K. after the mid-90s, we expect a duration of prices longer than the one reported in Hall *et al.* (2000).

Following Walsh (2003) and Woodford (2003), we construct  $\kappa$  using  $\kappa = \frac{(1-\omega)(1-\omega\beta)}{\omega} \varpi$ , where  $\omega$  is the fraction of firms that do not adjust their prices, and  $\varpi$  is a measure of the strategic complementarities, which is assumed to be equal to one. Hence, given the fraction of non-adjuster firms, the expected time between price adjustments is  $1 / (1 - \omega)$ . Therefore, the duration of prices being 9.5 implies  $\kappa = 0.0157$ .

The values of the parameters  $\lambda_y$  and  $\lambda_i$ , which give the relative weights the central bank places on output and interest rate stabilization are chosen according to Giannoni and Woodford (2003). Specifically, we set  $\lambda_i = 0.236$  and  $\lambda_y = 0.002$ . The weight on the output gap stabilization is lower than those in Giannoni and Woodford (2003) because they scaled the parameters to be consistent with the annualized growth rate of prices.

For the parameters in the empirical Taylor rule given in (6a) and (6b), we set the inflation and the output gap coefficients to  $\gamma_\pi = 1.5$  and  $\gamma_y = 0.5$ . The real exchange rate coefficient is  $\gamma_q = 0.1$ , which is comparable to the estimate 0.09 reported for the Bank of England's reaction function (Clarida *et al.*, 1998).

## Appendix E. Inflation and Output Gap Forecasts

The forecast values of the relevant variables in the exchange rate equations are estimated with a  $p$ -th order vector autoregression ( $VAR(p)$ ) model using 10-year constant rolling windows. Since forecasting is the objective, we select  $p$ , the order of the  $VAR$  model, using the final prediction error criterion, which optimizes the forecast precision (Lütkepohl, 2005). Thus, the estimate of  $p$ ,  $\hat{p}$ , could vary across subsamples in the rolling regression. For each country, we fit a  $VAR(p)$  to the trivariate system that comprises inflation ( $\pi_t$ ), the output gap ( $y_t$ ), and the interest rate ( $i_t$ ) and generate the required forecasts.

## Appendix F. Estimating Risk Premium

Following Wolff (1987) and Cheung (1993), we construct the risk premium term using the Kalman filter setup. The state space model used to estimate the risk premium is

$$\Xi_t = \xi_t + v_{t+1},$$

$$\xi_t = \phi \xi_{t-1} + a_t,$$

where  $\Xi_t \equiv f_t - s_{t+1}$ ,  $\xi_t \equiv f_t - E_t s_{t+1}$  and  $v_{t+1} \equiv E_t s_{t+1} - s_{t+1}$ .  $f_t$  is the one month forward exchange rate (in log),  $s_t$  is the spot exchange rate (in log), and  $\Xi_t$  is error of using  $f_t$  to predict  $s_{t+1}$ . The variable of interest,  $\xi_t$ , is the unobservable risk premium, and  $v_t$  is the unexpected change in the spot rate. As in the standard state space models, we assume that  $a_t \sim iidN(0, \sigma_a^2)$ ,  $v_t \sim iidN(0, \sigma_v^2)$ , and the covariance between  $a_t$  and  $v_t$  is zero,  $\sigma_{va} = 0$ . The risk premium is then estimated by maximizing the likelihood function constructed from the Kalman filter algorithm. See Wolff (1987) and Cheung (1993) for a more detailed discussion.

## Appendix G. Evaluating In-Sample Prediction Accuracy

The Diebold-Mariano statistics (Diebold and Mariano, 1995) are used to evaluate the in-sample prediction performance of implied exchange rate changes obtained from alternative optimal interest rate rules relative to that of the driftless random walk. Given the actual exchange rate change  $a_t$  and the implied exchange rate change  $b_t$ , the loss function  $L$  for the mean square error is defined as

$$L(b_t) = (b_t - a_t)^2.$$

Testing whether the performance of the implied exchange rate equation is different from a driftless random walk  $c_t$  is equivalent to testing whether the population mean of the loss differential series  $d_t$  is zero. The loss differential is defined as

$$d_t = L(b_t) - L(c_t).$$

Under the assumptions of covariance stationarity and short-memory for  $d_t$ , the large-sample statistic for the null of equal forecast performance is distributed as a standard normal, and can be expressed as

$$\bar{d} \left\{ 2\pi \sum_{\tau=-(T-1)}^{(T-1)} l(\tau / S(T)) \sum_{t=|\tau|+1}^T (d_t - \bar{d})(d_{t-|\tau|} - \bar{d}) \right\}^{-1/2},$$

where  $l(\tau / S(T))$  is the lag window,  $S(T)$  is the truncation lag, and  $T$  is the number of observations. Different lag-window specifications can be applied, such as the Barlett or the quadratic spectral kernels, in combination with a data-dependent lag-selection procedure (Andrews, 1991).

For the direction of change statistic, the loss differential series is defined as follows:  $d_t$  takes a value of one if the implied exchange rate series correctly predict the direction of change, otherwise it will take a value of zero. Hence, a value of  $\bar{d}$  significantly larger than 0.5 indicates that the implied exchange rate equation has the ability to predict the direction of change; on the other hand, if the statistic is significantly less than 0.5, the implied rate tends to give the wrong direction of change. In large samples, the studentized version of the test statistic,

$$(\bar{d} - 0.5) / \sqrt{0.25 / T},$$

is distributed as a standard normal.

## Appendix H. Data Description

### U.S. Data:

Federal Fund Rate: Weighted average rate at which banks borrow funds through New York brokers. Monthly rate is the average of rates of all calendar days. Source: International Financial Statistics-IFS (line 60b)

Industrial Production: Seasonally adjusted monthly series. Source: IFS (line 66c)

Inflation Rate: First difference of consumer price index (in log), seasonally adjusted. Source: Congressional Budget Office

Money Growth Rate: Monthly growth rates of seasonally adjusted M2 Money Stock. Source: Board of Governors of the Federal Reserve System

Output Growth Rate: The first difference of industrial production (in log).

### U.K. Data:

Money Market Rate: The interbank offer rate for overnight deposits. Source: IFS (line 60b)

Industrial Production: Seasonally adjusted monthly series. Source: IFS (line 66c)

Inflation Rate: First difference of consumer price index (in log). Source: IFS (line 64c). X-12-ARIMA is used for seasonal adjustment.

Money Growth Rate: Monthly growth rate of seasonally adjusted M4 money stock (monetary financial institutions' sterling M4 liabilities to private sector). Source: The Bank of England

Output Growth Rate: The first difference of industrial production (in log).

### Exchange Rate Data:

Nominal Exchange Rate (NER): U.S. Dollars per National Currency (i.e. how many pound sterling a dollar can buy), end of period. Source: IFS (line *ae*).

Forward Rates: U.S. Dollar to U.K. pound one-month forward exchange rate. Source: DataStream (Code=BBGBP1F).