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Monetary Policy Transmission in China: A DSGE Model with Parallel Shadow Banking and Interest Rate Control

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Abstract

The paper sheds light on the interplay between monetary policy, the commercial banking sector and the shadow banking sector in mainland China by means of a nonlinear stochastic general equilibrium (DSGE) model with occasionally binding constraints. In particular, we analyze the impacts of interest rate liberalization on monetary policy transmission as well as the dynamics of the parallel shadow banking sector. Comparison of various interest rate liberalization scenarios reveals that monetary policy results in increased feed-through to the lending and investment under complete liberalization. Furthermore, tighter regulation of interest rates in the commercial banking sector in China leads to an increase in loans provided by the shadow banking sector.

Keywords: DSGE Model, Monetary Policy, Financial Market Reform, Shadow Banking, China

JEL Classifications: E32, E42, E52, E58

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1. Introduction

The Chinese financial system has undergone gradual refoms since the mid 1990s. In the wake of the Asian crisis of 1997, the Chinese authorities recognized that structural reforms and better regulation were necessary to tackle the growing systemic risks of the Chinese financial system. At that time more than 20% of loans were nonperforming, which implied potential losses in excess of banks' net assets. The banking cleanup lasted more than a decade and achieved considerable success. The bad debts have been replaced a decade later by highly profitable and well-capitalized banks. A concomitant effect, however, has been a policy of allowing large-scale interest rate distortions. This policy prevented banks from collapsing. But China´s policy of financial repression, whose main feature is a regulated interest rate system, forces households to endure artificially low interest rates on bank deposits. Another direct consequence of the tight interest rate regulation is that access to bank loans tends to be limited and uneven across borrowers. This has led to the emergence of a shadow banking system as an important channel for alternative funding. The superficial reason is that Chinese banks are not extending enough credit to small- and medium-sized enterprises (SMEs), but are focusing instead on lending to established large Chinese firms.

This distorted interest rate structure has discouraged marginal investment and is a significant obstacle to sustaining China´s rapid economic growth. The global recession of 2008 - 2009 aggravated these difficulties in China´s financial system, as the government´s huge stimulus package in response to the crisis emphasized bank loans rather than direct government spending which entails a sizable risk of non-performing loans, and impaired bank balance sheets in the future. In addition, new financing channels outside the well-regulated banking system have subsequently developed and expanded further aggravating the risk management challenges for monetary policy and regulators. Thus, the Chinese financial system again stands at a cross-roads and requires a new round of reforms to address the challenges that have accumulated over the past several years.[1]

Against this background, our paper addresses the Chinese shadow bank issue and contributes to the literature on modelling parallel shadow banks and interest rate control in micro-founded dynamic stochastic general equilibrium (DSGE) frameworks. Few theoretical analyses exist to guide policymakers in this way. This paper is most closely related and complementary to three recent papers modelling a shadow banking sector, but differs in several respects. Verona et al. (2013) consider a financial accelerator DSGE model for the US economy with investment banks investing in less risky projects while formal retail banks provide funding to riskier firms. They are mainly concerned with the adverse effects of shadow banking for boom-bust events caused by a level of interest rates that is too low for too long. Meeks et al. (2014) are concerned about financial instability due to commercial banks unloading risky loans to off-balance sheet shadow banks via securization. Mazelis

[1] In line with this, the third plenum of the Chinese Communist Party in November 2013 has called for equal competition where firms must freely make resource allocation decisions considering market-based input prices. However, the Chinese State Council said the shifts would be carried out in an "orderly way" - usually a buzzword for moving slowly. Thus the Chinese authorities will most likely employ a piecemeal approach where those tools for which the impacts are well known are frequently used while others will be put on hold.

(2014) has investigated the impact of monetary policy shocks on aggregate loan supply in a DSGE framework with commercial banks and shadow banks. In contrast to Meeks et al. (2014), Mazelis (2014) does not assume that shadow banks are funded by the commercial banking sector; instead, shadow banks have to acquire deposits from the markets in order to function as intermediaries. The funding market is modelled via search and matching by shadow banks for available deposits of households. Our paper differs from the existing papers in a number of ways. None of the above papers focuses on the multifaceted interactions between nonstandard monetary policy, the traditional banking sector and the shadow banking sector in China. In our DSGE framework, in contrast, we analyze monetary policy transmission with parallel shadow banking and different degrees of interest rate control. This means, as a corollary, that we also investigate the impacts of financial liberalization and regulatory change in China on shadow banking.

The remainder of the paper is organized as follows. Section 2 provides a brief overview of shadow banking activities in China: the products, the range of participants, and the reasons behind their rapid increase. We devote section 3 to the careful construction of a tractable DSGE model with a parallel shadow banking sector. Section 4 presents the model calibration. Section 5 presents impulse response functions and model simulations and analyze the main channels at work. Finally, Section 6 concludes. Omitted modelling and calibration details are provided in three appendices. To economize on space, the complete set of equilibrium conditions is available in an Online Appendix on our website.

2. Shadow Banks and Financial Repression in China

What is shadow banking? The definition of shadow banking is itself shadowy. According to the Financial Stability Board (FSB), shadow banking is "credit intermediation involving entities and activities outside the regulated banking system". In other words, off balance sheet shadow banking moves financial intermediation (fully or partially) outside of regular banking and thus circumvents safeguards such as capital requirements, loan-loss provisions, loan-to-deposit ratios, and well-established supervision and regulation. The FSB also suggests a narrow definition of shadow banking as a "subset of non-bank credit intermediation where there are (i) developments that increase systemic risk (in particular maturity/liquidity transformation, imperfect credit risk transfer, and/or leverage), and/or (ii) indications of regulatory arbitrage that is undermining the benefits of financial regulation."[2]

The definition and the development of shadow banking are country-specific. In China, shadow banking activity emerged in the wake of a "dual-track" reform strategy in interest rate liberalization. As a background information, note that interest rates are heavily regulated in China. In 2004, the central bank removed lower bound restrictions on deposit rates and upper bound restrictions on lending rates, but maintained upper bound restrictions on deposit rates and lower bound restrictions on lending

[2] See FSB (2013) and Li (2013) for an overview of definitions used in the literature. FSB (2014) monitors financial stability risks using end-2013 data. The definition implies that shadow banking entities do not include equity-based funds and venture capital companies, which do not make use of credit instruments in the financing process.

rates.

The PBoC has gradually eased interest rate controls in recent years. On the deposit rate side, it introduced as a maximum a 10% premium above benchmark deposit rates in June 2012 and raised it further to 20% in November 2014. Despite this liberalization, the deposit rate ceiling still appears to be binding, as deposit rates remain clustered at their upper bound. On the lending rate side, People's Bank of China (PBoC) raised the maximum discount from the benchmark lending rate from 10% to 20% in June 2012, then to 30% in July 2012, and it finally removed lending rate control in July 2013. At end-2013, 24% of bank loans offered were at discounts from the benchmark lending rates and 63% were at premium. Table 1 indicates that in practice there is no longer a strict enforcement mechanism.

The PBoC also controls bank credit through its administrative window guidance policy on commercial bank lending. This quantity-based non-price instrument is an important tool in the conduct of monetary policy and can be understood as gentle coersion through formal statements or private discussions. Under this policy, the PBoC persuades banks to lend according to the guideline. The guidance typically covers the level of loan growth and sectors to which bank lending should be directed. Such window guidance has been important in driving bank loan growth in recent years, which was 32% in 2009 and 20% in 2010 in support of the large stimulus package, and continued to decelerate after 2011 amid the central bank's efforts to normalize monetary policy (loan growth was 13.6% in 2014). Furthermore, since 2012 the bank regulator has restricted bank lending to local government financing platforms and the real estate sector, and has encouraged bank lending to SMEs and to rural sectors.

China´s shadow banking initially emerged to support interest rate liberalization, a "dual-track" reform strategy which aims to develop market-based deposit and lending rates outside the banking system. For instance, wealth management products (WMP) are a result of the search-for-yield effect and the endeavour to bypass regulation on maximum deposit rates. WMPs are typically short term (usually less than 6 months) and marketed as high-yield alternatives to bank deposits. Separately, trust loans are alternatives to bank loans, in which a trust company invests client funds according to a pre-specified objective, purpose, amount, maturity, and interest rates (which is not subject to interest rate control).

The other, and perhaps more important, reason for the rapid growth in China's shadow banking is regulatory arbitrage. This is a major reason for the rapid growth of shadow banking in China since 2012, when the Chinese authorities started to counter inflation after the large-scale stimulus program in response to the global financial crisis 2008-2010. Furthermore, PBoC raised the bank reserve requirement ratios 12 times in 2010 and 2011 to a record high of 21.5 percent for large institutions in June 2011. In response, WMP and trusts driven by investors' quest for higher yields formed a parallel lending channel to support those borrowers with limited access to bank loans. In a typical shadow banking credit chain, a trust company received funds via WMPs and then lent to these borrowers (Figure 2). Because WMPs are banks' off-balance sheet items and can offer attractive yields to individual investors, while trust companies do not face interest rate restrictions, loan quotas, or loan-to-deposit ratio requirements, and are subject to lighter regulation, these parallel lending channels have grown rapidly and support economic growth. In a nutshell, the growth of the Chinese shadow banking system results in distortions in the formal financial system as well as in elements of the monetary and regulatory policy framework.[3]

Figure 3 shows the rapid growth in China's shadow banking activity. Between 2010 and 2014Q1, WMP increased from RMB 2.8 trillion to RMB 12.2 trillion, trust asset under management increased from RMB 3.04 trillion to RMB 11.7 trillion, and asset management products (AMPs) by security firms increased from RMB 200 billion to RMB 6.08 trillion. In 2014Q1, the amount of WMP was equivalent to 12.2% of bank deposits and trust assets under management (AUM), and the total of AMP was equivalent to 23.8% of bank loans. The yields on WMP were about 200bp higher than on 1-year benchmark deposit rates, and trust yields were also higher than average bank lending rate (the cost to trust borrowers is usually 200-300bp higher than trust yields, the latter of which do not take into account the fees for financial intermediaries).

3. Modelling China’s Financial System with Parallel Shadow Banking

In order to approach the problem of quantifying effects of policy changes, a structural model is needed (in the absence of a natural experiment at hand). Against this background, the contribution of this paper is to shed light on the interplay between the commercial banking sector and the shadow banking sector in mainland China by means of a conceptual DSGE framework, identifying separate factors which help to explain the dynamics of the parallel shadow banking sector.[4] The modelling setup also facilitates a discussion of the monetary policy implications of the parallel shadow banking sector. There is a number of methodological considerations that arise in developing such a DSGE framework. The literature has not yet presented an all-encompassing DSGE model appropriate for modelling China’s shadow banking sector, but several elements have been developed, and we naturally build on them. The papers by Chen et al. (2012) and Funke and Paetz (2012) develop a nonlinear DSGE model that captures China’s nonstandard monetary policy toolkit. In this paper we augment that framework with a shadow banking sector, along the lines of Verona et al. (2013). The latter is a simplified version of the financial accelerator model proposed by Christiano et al. (2010). We deliberately adopted a modelling approach that considers only a simplified version of the interbank market. This choice has the virtue of keeping the model simple without changing the nature of our modelling results. In simple terms, our goal is to capture, for China, the interface between

3 The WMP vehicles enhance the tradability of credit portfolios, thereby allowing shadow banks to free up resources by selling loans. This in itself can give shadow banks greater scope for lending. See Altunbas et al. (2009).

4 Our model does not attempt to capture the full complexity of the Chinese economy. For simplicity, we disregard fiscal policy and the economy is closed. By focusing on the essential monetary transmission channel, the dimensionality of the DSGE model can be greatly reduced.

qualitative and quantitative monetary policy versus shadow banking.[5] A diagrammatic drawing of the main elements of the modelling framework is given in Figure 4.

The modelling setup assumes homogeneity in the household sector and heterogeneity among banks and firms. Next we sketch the representation of the banking and firm sectors. The model is populated by two types of banks: a commercial bank and a shadow bank. China's shadow banking activities typically involve direct lending to firms with unmet demand for loans. The so-called "trusts" pool money from investors promising a state-contingent return. There are two types of firms in the economy: perceived low-risk large private and state-owned firms (SOEs) and perceived high-risk SMEs. It is widely taken as a fact that the Chinese government implicitly guarantees much, if not all, of SOEs' debt. Accordingly, the large and state-owned firms have access to cheap funding from the commercial banking sector. In contrast, the SMEs fraught with risk find it difficult to borrow from the formal banking sector. In addition, high-risk firms are not able to self-finance their capital purchases and households cannot lend to SMEs directly. All this confers that the SMEs interact with the shadow banking sector. Shadow banking finance typically carries a higher interest rate than commercial bank finance.[6] In what follows, superscripts *se* and *re* stand for large private and state-owned firms and small and medium-sized firms, respectively. Finally, a non-standard Chinese monetary policy rule completes the model. Next we describe the ingredients of the modelling environment in more detail.

3.1 Commercial Banks and Low-Risk Firms

To fix the modelling ideas and notation, we start with low-risk firms. Low-risk firms' role in this model is to purchase physical capital from capital producers and provide it to intermediate-good firms. The timing of this process goes as follows. At the beginning of period *t* the low-risk entrepreneur provides capital services to the intermediate-good firms. Capital services are related to the stock of physical capital as $K_t^{se} = u_t^{se}\bar{K}_t^{se}$ where u_t^{se} stands for capital utilization. The latter faces an increasing and convex cost function of the form

$$a(u_t^{se}) = \frac{\bar{r}^{k,se}}{\sigma_a^{se}}[\exp(\sigma_a^{se}(u_t^{se}-1))-1] \quad (1)$$

where $\bar{r}^{k,se}$ is the steady state value of the rental rate of physical capital provided by the risky entrepreneur and σ_a^{se} gives the curvature of the cost function.[7]

5 It is worth bearing in mind that we do not attempt to model the process of financial innovation and deregulation which lies behind the rapid expansion of the Chinese shadow banking sector. Instead, we focus upon the policy issues of nonstandard monetary policy tools, shadow banking activities and further interest rate liberalization in China.

6 The higher shadow banking lending rates are consistent with the Berlin and Mester (1999) model considering the contracting relationship between a firm and a bank. The core feature of the model is the setting of lending rates subject to the liability structure of the bank.

7 Bars over a variable without a time index generally denote its steady-state or long-run value. It is worth mentioning that an equivalent way of modelling the costs associated with a higher utilization rate has been suggested by Gertler and

Then, at the end of period *t*, the entrepreneur sells the undepreciated capital to capital producers at price $Q_{\widetilde{K},t}$ and pays interest on the loan provided by the commercial bank. The profit function of the low-risk firm is given by the expression

$$\Pi_t^{se} = \left[u_t^{se} r_t^{k,se} - a(u_t^{se})\right] P_t \widetilde{K}_t^{se} + (1-\delta) Q_{\widetilde{K},t} \widetilde{K}_t^{se} - Q_{\widetilde{K},t} \widetilde{K}_{t+1}^{se} - r_t^l (Q_{\widetilde{K},t-1} \widetilde{K}_t^{se} - N_t^{se}) \qquad (2)$$

where $r_t^{k,se}$ is the rental price of physical capital at time *t*, P_t is the price of the final good and δ is the rate of depreciation. The last term in the profit function of the low-risk firm denotes the interest rate payable on the loan amount borrowed from the commercial bank where credit value is given by

$$L_t^{se} = Q_{\widetilde{K},t-1} \widetilde{K}_t^{se} - N_t^{se} \qquad (3)$$

The above equation emphasizes that the firm finances the acquisition of capital by means of both equity and debt. That is, the present model follows the standard assumption in the literature that the firm is not able to fully finance its projects by simply using their net worth.

In period *t* the firm faces both a static and a dynamic optimization problem, that is, it determines the utilization rate u_t^{se} and the demand for physical capital $\widetilde{K}_{t+1}^{se}$ to be used in period *t*+1. The first-order conditions give rise to the following relationships:

$$r_t^{k,se} = a'(u_t^{se}) \qquad (4)$$

$$Q_{\widetilde{K},t} = \beta E_t\left\{[u_{t+1}^{se} r_{t+1}^{k,se} - a(u_{t+1}^{se})] P_{t+1} + (1-\delta) Q_{\widetilde{K},t+1} - r_{t+1}^l Q_{\widetilde{K},t}\right\} \qquad (5)$$

Equation (4) represents the rental rate a low-risk firm would charge an intermediate-good producer. It says that the firm would choose such a rate such that the marginal gain (profit) is equal to the marginal cost of renting out capital, that is, the extra utilization cost. Equation (5) is the capital Euler equation of the low-risk firm and shows that the opportunity cost of renting out capital (the price of capital today) must equal the discounted marginal benefit tomorrow. The latter is given by the nominal value of the return on capital in period *t* +1 net of depreciation and interest payments.

In line with DSGE models containing a banking sector, the current paper assumes that firms cannot accumulate enough net worth so that in the future they are able to finance their projects solely by means of their own equity. Hence, in each period a certain percentage of the firms exit the economy with probability $1-\gamma^{se}$. The leaving firms transfer their equity back to households since the latter are the owners of all firms and banks in the economy. Therefore the amount transferred back to

Karadi (2011) and Iacoviello (2014), among many others. They assume that the depreciation rate is an increasing function of capital utilization.

households is $(1-\gamma^{se})V_t^{se}$ where the last term is the low-risk firm´s equity in period *t* and is given by:

$$V_t^{se} = \{[u_t^{se} r_t^{k,se} - a(u_t^{se})]P_t + (1-\delta)Q_{\widetilde{K},t}\}\widetilde{K}_t^{se} - (1+r_t^l)(Q_{\widetilde{K},t-1}\widetilde{K}_t^{se} - N_t^{se}) \tag{6}$$

In order to keep the number of firms constant, it is assumed that in each period a new firm is born with probability $1-\gamma^{se}$. Hence, the total net wealth of the low-risk firm is equal to the remaining equity plus the initial transfer from the households and evolves according to

$$N_{t+1}^{se} = \gamma^{se} V_t^{se} + W^{e,se} \tag{7}$$

The commercial banks in the present setup are assumed to have some market power in setting interest rates. Furthermore, banks' decision-making process is linked to the cost-minimization problem of the firm. That is, at the end of period *t* the low-risk firm minimizes the total repayment due:

$$\min_{\{L_{t+1}^{se}(j)\}} \int_0^1 [1 + r_{t+1}^l(j)] L_{t+1}^{se}(j) dj \tag{8}$$

subject to the Dixit-Stiglitz agregator

$$L_{t+1}^{se} = \left\{ \int_0^1 [L_{t+1}^{se}(j)]^{\frac{\varepsilon_{t+1}^{l,op}-1}{\varepsilon_{t+1}^{l,op}}} dj \right\}^{\frac{\varepsilon_{t+1}^{l,op}}{\varepsilon_{t+1}^{l,op}-1}} \tag{9}$$

where $r_{t+1}^l(j)$ is the lending rate charged by bank *j* and $\varepsilon_{t+1}^{l,op}$ is the time-varying interest elasticity of demand for loans. The latter essentially determines the mark-up banks charge over the deposit rate due to monopolistic competition. The solution is characterized by the following condition which is standard in the DSGE literature:

$$L_{t+1}^{se}(j) = \left\{ \frac{1 + r_{t+1}^l(j)}{1 + r_{t+1}^l} \right\}^{-\varepsilon_{t+1}^{l,op}} L_{t+1}^{se} \tag{10}$$

where r_{t+1}^l is the average lending rate and is given by

$$1 + r_{t+1}^l = \left\{ \int_0^1 [1 + r_{t+1}^l(j)]^{1-\varepsilon_{t+1}^{l,op}} dj \right\}^{\frac{1}{1-\varepsilon_{t+1}^{l,op}}} \tag{11}$$

After determining the low-risk firm's demand for loans from bank *j* as a function of the total loan demand in the commercial banking sector, it is important to shed light on the bank´s profit maximization problem. The reason for embracing this framework is that it allows banks to have a mark-up on the lending rate and a mark-down on the deposit rate. Differently put, commercial banks enjoy some market power in setting interest rates. The banking sector is composed of a continuum of financial intermediaries where $j \in [0,1]$. Furthermore, each bank consists of two main branches: a wholesale and a retail branch where the latter is composed of both a deposit retail and a lending retail branch. The deposit retail branch is responsible for pooling deposits from households promising a certain (risk-free) return. Then it provides these deposits to the wholesale branch in return for an interest income. Afterwards, the wholesale branch generates new deposits in accord with the money multiplier, amounting to $\frac{D_t}{v}$, and provides all these newly generated assets to the lending retail branch. Finally, the latter uses the newly generated deposits to provide loans to the low-risk firms. The wholesale branch is assumed to be in a situation of perfect competition and as a result it takes interest rates as given. In contrast, the retail banking sector operates in a monopolistically competitive environment and thus the two retail branches have some market power when setting lending and respectively deposit rates. As will become clear later on, this framework is very convenient for incorporating all particularities of the commercial banking sector in China such as monopolistic competition, interest rate caps and floors and last but not least loan quotas.[8]

Next the text describes the link between the wholesale and retail branches of the commercial bank and all maximization problems. First the maximization problem of the deposit branch is presented where at the end of time *t* it sets the deposit rate in order to maximize its profits for the following period subject to household's demand for deposits:

$$(1 + R_{t+1}^d)D_{t+1}(j) - [1 + r_{t+1}^d(j)]D_{t+1}(j) \tag{12}$$

subject to

$$D_{t+1}(j) = \left(\frac{1 + r_{t+1}^d(j)}{1 + r_{t+1}^d}\right)^{\varepsilon^d} D_{t+1} \tag{13}$$

The solution to this maximization problem, after imposing symmetric equilibrium, leads to the first-order condition:

$$1 + r_{t+1}^d = \frac{\varepsilon_d}{\varepsilon_d - 1}(1 + R_{t+1}^d) \tag{14}$$

where $\frac{\varepsilon_d}{\varepsilon_d - 1}$ is the mark-down on the deposit rate set by the retail bank. Likewise, the retail loan branch of bank *j* faces a similar maximization problem:

8 These features draw on elements of the DSGE models in Gerali et al. (2010), Chen et al. (2012) and Funke and Paetz (2012) .

$$[1+r_{t+1}^{l}(j)]L^{se}{}_{t+1}(j)-[1+R_{t+1}^{l}]L^{se}{}_{t+1}(j)-\frac{\kappa^{l}}{2}\left(\frac{r_{t+1}^{l,cb}-r_{t+1}^{l}(j)}{r_{t+1}^{l}}\right)^{2}r_{t+1}^{l}L_{t+1}^{se} \tag{15}$$

subject to

$$L^{se}{}_{t+1}(j)=\left(\frac{1+r_{t+1}^{l}(j)}{1+r_{t+1}^{l}}\right)^{-\varepsilon_{t+1}^{l,op}}L_{t+1}^{se} \tag{16}$$

As can be seen from equation (15), the maximization problem of the loan branch differs slightly from that of the deposit branch. That is, we assume that the lending facility of the bank incurs costs for negative deviations of lending rate from benchmark one set by the PBoC. In order to circumvent the problem of penalizing the bank for deviations from the benchmark lending rate, we set $\kappa^{l}>0$ only in scenarios where the lending rate falls and stays below the steady state for some time. In all other cases we assume $\kappa^{l}=0$. The reason why we use two different methods for modelling deposit and lending rate controls relates to the nature of regulation. In particular, the PBoC sets only a cap on deposit rates, and commercial banks are not allowed to offer return on deposits higher than this rate while the lending rate floor is not strictly binding.

The section on the model's properties provides a detailed discussion of the lending rate distribution in China. Again, taking the first-order condition with respect to $r_{t+1}^{l}(j)$ and imposing symmetric equilibrium yields

$$1+r_{t+1}^{l}=\frac{1}{\varepsilon_{t+1}^{l,op}-1+\kappa^{l}}[\varepsilon_{t+1}^{l,op}(1+R_{t+1}^{l})+\kappa^{l}(1+r_{t+1}^{l,cb})] \tag{17}$$

In the extreme case where $\kappa^{l}=0$ we obtain the standard expression saying that the marginal gain is equal to the marginal cost times the mark-up (time-varying in this case, due to optimism):

$$1+r_{t+1}^{l}=\frac{\varepsilon_{t+1}^{l,op}}{\varepsilon_{t+1}^{l,op}-1}(1+R_{t+1}^{l}) \tag{18}$$

Last but not least, we employ the framework developed by Chen et al. (2012) and Funke and Paetz (2012) for modelling the wholesale branch of the commercial banking sector. The advantage of their banking model is the ability of the bank to create money so as to incorporate the money multiplier which is usually ignored in the DSGE literature. Accordingly, the wholesale branch in the present model takes in deposits from the retail deposit branch, creates new deposits, given the required reserve ratio ν, and provides those new deposits to the retail loan branch, taking all prices as given. Hence, the balance sheet of the wholesale branch in time *t* is given by

$$IB_t + \frac{D_t}{\nu} = L_t^{se} + D_t \tag{19}$$

where IB_t denotes the bank's position in the interbank market. Here it should be noted that in equilibrium the net supply of interbank loans is zero. Furthermore, we assume that the money creation process entails a quadratic cost function given by

$$C_t = \frac{1}{2\bar{Y}}\left\{c_d\left[\left(\frac{D_t}{\nu}\right)^2 - \left(\frac{\bar{D}}{\nu}\right)^2\right] + c_l[(L_t^{se})^2 - (\bar{L}^{se})^2]\right\} \tag{20}$$

As already mentioned, the wholesale branch also incurs quadratic costs for deviating from the benchmark loan target L_t^{cb} specified by the central bank and given by $\frac{\kappa^w}{2}(L_t^{se} - L_t^{cb})^2$. Having all the above-mentioned factors in mind, we are now able to formulate the maximization problem of the wholesale branch:

$$\begin{aligned}\max_{\{L_t^{se},D_t\}} E_0 \sum_{t=0}^{\infty} \beta_{wb}^t \{ & (1+R_t^l)L_t^{se} - L_{t+1}^{se} + (1+R_t^R)D_t - D_{t+1} \\ & - \left(\frac{(1+R_t^d)D_t}{\nu} - \frac{D_{t+1}}{\nu}\right) - [(1+R_t^{IB})IB_t - IB_{t+1}] - \frac{\kappa^w}{2}(L_t^{se} - L_t^{cb})^2 - C_t\}\end{aligned} \tag{21}$$

Substituting the period budget constraint in equation (21) yields the following periodic profit maximization problem.

$$F_t^{wb} = (R_t^l - R_t^{IB})L_t^{se} + [(R_t^R - R_t^{IB}) + \frac{1}{\nu}(R_t^{IB} - R_t^D)]D_t - \frac{\kappa^w}{2}(L_t^{se} - L_t^{cb})^2 - C_t \tag{22}$$

The optimal amount of deposits and loans is given by the first-order conditions and illustrates the fact that the marginal benefit from each asset is equal to the opportunity cost of holding it:

$$R_t^D = R_t^{IB} + \nu(R_t^R - R_t^{IB}) - \frac{c_d}{\bar{Y}}\frac{D_t}{\nu} \tag{23}$$

$$R_t^L = R_t^{IB} + \kappa^w(L_t^{se} - L_t^{cb}) + \left(\frac{c_l}{\bar{Y}}\right)L_t^{se} \tag{24}$$

Equation (23) reveals that the opportunity cost of holding deposits is equal to the interbank rate adjusted for the return on required reserves and the management cost entailed by the production of new deposits. Similarly, Equation (24) illustrates that the opportunity cost for loans is given by the interbank rate, the deviation from the window guidance loan quota, and the management cost. Finally, closing the model entails a rule for the interbank rate. In line with Gerali et al. (2010), we assume that the wholesale branch of the commercial bank has unlimited access to the lending facility of the central bank. As a result, arbitrage ensures that $R_t^{IB} = R_t$.

3.2 Shadow Banking and High-Risk Small and Medium-Sized Firms

This section introduces our approach to modelling shadow banking in the Chinese economy. The framework for modelling shadow banks' behaviour in the present setup follows Bernanke et al. (1999) and takes into account the Fisher-deflation effect emphasized by Christensen and Dib (2008) and Iacoviello (2005), among many others. That is, we assume the frictions arise on the firm level rather than on the side of the shadow bank. Nonetheless, bank´s net worth plays an important role in the current framework as the accumulation of profits (losses) is possible due to biased expectations regarding high-risk firm's productivity level. This way of introducing bank equity in a DSGE model with perfectly competitive financial sector has been proposed by Zhang (2010). The next paragraph sheds light on the interaction between a perceived high potential high-risk firm and the shadow banking sector.

High-risk firms, like their low-risk peers, own a share of the economy´s physical stock of capital. They purchase it from capital producers and provide it to intermediate-good firms for the production of intermediate goods. Moreover, since high-risk firms are not able to self-finance their capital purchases and do not have access to the commercial banking sector, they seek financing from the economy's shadow banks. In the model, the high-risk firms are fraught with risk because their own capital is subject to idiosyncratic random productivity shocks in period *t*+1 equal to ω_{t+1}. The latter is a random variable assumed to be log-normally distributed:

$$\log(\omega) \sim N\left(-\frac{1}{2}{\sigma_s}^2, {\sigma_s}^2\right) \tag{25}$$

Equation (25) implies $\mathbb{E}(\omega) = 1$. At the end of period *t* the high-risk firm decides on the loan amount needed to purchase new capital, which is equal to the difference between the expenditure on physical capital and the firm´s own net worth:

$$L_{t+1}^{re} = \mathrm{Q}_{\widetilde{K},t} \widetilde{K}_{t+1}^{re} - N_{t+1}^{re} \tag{26}$$

At the end of period *t* the shadow bank offers a debt contract to the high-risk firm which specifies the lending rate R^{sb} and the loan value L^{re}.[9] In period *t*+1, the firm sees whether the idiosyncratic shock to its capital stock is below or above a threshold level $\bar{\omega}_{t+1}$, defined by

9 R^{sb} is the only interest variable in the model and is to be interpreted as an overall return, i.e. $R^{sb} = 1 + r^{sb}$.

$$\bar{\omega}_{t+1}(1+R^{k,re}_{t+1})Q_{\tilde{K},t}\tilde{K}^{re}_{t+1} = R^{sb}_{t+1}L^{re}_{t+1} \tag{27}$$

If $\omega_{t+1} > \bar{\omega}_{t+1}$ the firm remains solvent and pays the lender the principal as well as the interest due on the loan, $R^{sb}_{t+1}L^{re}_{t+1}$. Accordingly the borrower is able to keep the value of the remaining capital stock, given by $(\omega_{t+1} - \bar{\omega}_{t+1})(1+R^{k,re}_{t+1})Q_{\tilde{K},t}\tilde{K}^{re}_{t+1}$. On the other hand, if $\omega_{t+1} < \bar{\omega}_{t+1}$, the firm declares bankruptcy and so receives nothing. Furthermore, the insolvent firm undergoes monitoring by the bank, which appropriates what is left of the capital stock after the occurrence of the shock. Hence, the shadow bank's revenue in the case of default of the firm is $(1-\mu)(1+R^{k,re}_{t+1})\omega_{t+1}Q_{\tilde{K},t}\tilde{K}^{re}_{t+1}$. Unlike commercial financial intermediaries, shadow banks operate in perfectly competitive environment. The advantage of this assumption is that it renders the model simple enough and yet does not leave out any of the important implications of the current paper. Then the zero-profit condition of the bank is given by

$$[1-F_t(\bar{\omega}_{t+1})]R^{sb}_{t+1}L^{re}_{t+1} + (1-\mu)\int_0^{\bar{\omega}_{t+1}} \omega dF(\omega)(1+R^{k,re}_{t+1})Q_{\tilde{K},t}\tilde{K}^{re}_{t+1} = (1+r^E_{t+1})L^{re}_{t+1} \tag{28}$$

where $F(\omega)$ is the cumulative distribution function of ω. Unlike Bernanke et al. (1999), the opportunity cost of lending for the financial intermediary is not the risk-free rate. Rather, shadow banks pay interest to their shareholders equal to r^E_{t+1}. The latter will be higher than the risk-free rate so long as the probability of default in the shadow banking sector is positive. This framework is also employed by Zhang (2010) and Suh (2012). The risky rate is equal to $r^E_{t+1} = \frac{(1+r^d_{t+1})}{(1-\phi_{t+1})} - 1$. Using the fact that $G_t(\bar{\omega}_{t+1}) = \int_0^{\bar{\omega}_{t+1}} \omega dF(\omega)$ and $\Gamma_t(\bar{\omega}_{t+1}) = \bar{\omega}_{t+1}[1-F_t(\bar{\omega}_{t+1})] + G_t(\bar{\omega}_{t+1})$, the shadow bank´s maximization problem is given by

$$\max_{\{k^{re}_{t+1},\bar{\omega}_{t+1}\}} \mathbb{E}_t\left\{[1-\Gamma_t(\bar{\omega}_{t+1})]\frac{1+R^{k,re}_{t+1}}{1+r^E_{t+1}}k^{re}_{t+1}\right\} \tag{29}$$

subject to

$$[\Gamma_t(\bar{\omega}_{t+1}) - \mu G_t(\bar{\omega}_{t+1})]\frac{1+R^{k,re}_{t+1}}{1+r^E_{t+1}}k^{re}_{t+1} = k^{re}_{t+1} - 1 \tag{30}$$

where $\Gamma_t(\bar{\omega}_{t+1})$ is the share of entrepreneurial profits received by the bank and $\mu G_t(\bar{\omega}_{t+1})$ is the monitoring cost the bank expects to incur. Hence, $1-\Gamma_t(\bar{\omega}_{t+1})$ is the share of the profits received by the entrepreneur. Finally, $k^{re}_{t+1} = \frac{Q_{\tilde{K},t}\tilde{K}^{re}_{t+1}}{N^{re}_{t+1}}$ stands for the leverage ratio of the risky firm. As in Bernanke et al. (1999) the solution to (30) leads to the expression for the financial accelerator, given by

$$\frac{\mathbb{E}_t(1+R_{t+1}^{k,re})}{1+r_{t+1}^{E}} = \Psi\left(\frac{Q_{\widetilde{K},t}\widetilde{K}_{t+1}^{re}}{N_{t+1}^{re}}\right) \tag{31}$$

This result implies that the external finance premium (LHS term) is positively correlated with the entrepreneur's leverage ratio. That is, the lower the leverage ratio, the lower the probability of default of the firm and hence the lower the bank's lending rate. In period *t* the high-risk firm´s equity is a predetermined variable in the model and is dependent on the settling of the debt contract in period *t*-1. It is given by

$$V_t^{re} = \left(1+R_{t+1}^{k,re}\right)Q_{\widetilde{K},t-1}\widetilde{K}_t^{re} - \left[1+r_t^E+\frac{\mu\int_0^{\bar{\omega}_t}\omega dF_{t-1}(\omega)\left(1+R_t^{k,re}\right)Q_{\widetilde{K},t-1}\widetilde{K}_t^{re}}{Q_{\widetilde{K},t-1}\widetilde{K}_t^{re}-N_t^{re}}\right]\left(Q_{\widetilde{K},t-1}\widetilde{K}_t^{re}-N_t^{re}\right) \tag{32}$$

where the first term gives the gains from selling undepreciated capital to capital producers and the term in square brackets represents the gross return firms must pay to the shadow bank for period *t*-1 loans. Like the low-risk firm, it is assumed that the high-risk firm exits the economy with probability $1-\gamma^{re}$. In this case, the entrepreneur rebates its equity to the household. Hence, the transfer amounts to $(1-\gamma^{re})V_t^{re}$. Moreover, to keep the population constant, a high-risk firm is born with probability $1-\gamma^{re}$. With no starting net worth, the debt contract a lá Bernanke et al. (1999) implies that the firm would not be able to receive a loan. Hence, to avoid such a situation, it is assumed that the newly born and the surviving firm each receives an initial transfer (or subsidy) from the households. As a result, the high-risk firm´s net worth is given by

$$N_{t+1}^{re} = \gamma^{re}V_t^{re} + W^{e,re} \tag{33}$$

3.3 Optimism and Shadow Bank's Equity

In this section we enrich the model with a mechanism that produces waves of optimism and pessimism in the banking sector, which we treat as variation in confidence. Unlike Verona et al. (2013) we assume optimism is present among both types of financial intermediaries. As far as the commercial bank is concerned, it becomes more optimistic if a low-risk firm pledges collateral that exceeds the steady state level. As a result the bank lowers the lending rate, which gives the firm still more incentives to borrow. The following AR(1) process describes how optimism evolves over time:

$$\chi_t^{rb} = \rho_\chi^{rb}\chi_{t-1}^{rb} + (1-\rho_\chi^{rb})\alpha^{rb}(N_{t+1}^{se}-\bar{N}^{se}) \tag{34}$$

where χ_t is the level of optimism at time *t*, ρ^{rb} is the autoregressive parameter, $\bar{N}^{se}$ is the steady-state value of the net worth of the low-risk firm and α^{rb} is the weight of the deviation of the net worth in period *t*+1 from its steady state level. To make it clear, our functional form for the dynamics of optimism is assumed rather than derived from first principles.[10] Equation (34) embeds the idea that the interest elasticity of credit demand depends upon the level of optimism.

$$\varepsilon_{t+1}^{l,op} = \varepsilon^{l}(1+\chi_{t}^{rb}) \tag{35}$$

In a nutshell, equations (34) and (35) reveal that the higher the level of optimism, the smaller the mark-up and the lower the lending rate charged by commercial banks. Before discussing the law of motion for optimism in the shadow banking sector, we need first to examine bank equity. In line with Zhang (2010) and Suh (2012), the shadow bank sets its lending rate based on the expected return of the high-risk firm rather than on the realization after the idiosyncratic shock has already been observed. As a result, in period *t*+1 the bank might incur profits or losses. Furthermore, the shadow bank's probability of default ϕ_t is assumed to be log-normally distributed with mean equal to the intermediary's capital ratio and standard deviation equal to σ^{sb}:

$$\kappa_t^{sb} = \frac{N_t^{sb}}{L_t^{re}} \tag{36}$$

where N_t^{sb} is the shadow bank´s net worth and L_t^{re} is the loan amount provided to the high-risk firm. The model assumes that the shadow bank´s probability of default is represented by

$$\phi_t = cdf(\kappa_t^{sb}, \sigma^{sb}) \tag{37}$$

where, as mentioned, κ_t^{sb} and σ^{sb} are the mean and standard deviation of ϕ_t. Equation (37) implies that the higher the capital ratio, the lower the probability of default and vice versa. Consistent with the average of the capital-to-assets under management (AUM) ratio over the 2010-2014 period, we set the capital ratio (below which the shadow bank is deemed insolvent) at 3%. The shadow bank´s law of motion for net worth is given by

$$\begin{aligned} N_t^{sb} = {} & (1-\phi_{t-1})N_{t-1}^{sb} + [1 - F_t(\bar{\omega}_t{}^{b})]R_t^{sb}L_t^{re} + \\ & (1-\mu)\int_0^{\bar{\omega}_t^{b}} \omega dF(\omega)(1+R_t^{k,re})\mathrm{Q}_{\widetilde{K},t-1}\widetilde{K}_t^{re} - (1+r_t^{E})L_t^{re} + W^{sb} \end{aligned} \tag{38}$$

In words, current-period net worth is equal to previous period net worth excluding defaulted banks and including the profits from lending activity and the initial transfer from households for the business start-up. By endogenizing the default probability of shadow banks the model aims to capture the cyclical movement of risks in the economy. That is, if trust companies' net worth declines, the probability of

[10] The modelling choice strikes a balance between the desire to enrich the dynamics of optimism and pessimism and the need for tractability of the model. We believe that the waves of optimism and pessimism reflect the time-varying uncertainty that confronts banks. We abstract from the deeper causes for the economic outlook for the sake of making progress in understanding its consequences.

default increases and thus investors demand a higher premium over the risk-free rate. This is expected to have a negative impact on lending, as firms' funding costs rise. The possibility of profits in the shadow banking sector despite perfect competition results from the difference between the ex-ante ($\overline{\omega}_{t+1}^{a}$) and ex-post ($\overline{\omega}_{t}^{b}$) default threshold levels. That is, equation (27) now becomes

$$\overline{\omega}^{a}{}_{t+1}(1+\mathbb{E}_t R_{t+1}^{k,re})\mathrm{Q}_{\widetilde{K},t}\widetilde{K}_{t+1}^{re} = R_{t+1}^{sb}L_{t+1}^{re} \tag{39}$$

where ω_{t+1}^{a} denotes the ex-ante default threshold level. In other words, the lending rate is no longer state-contingent but is based on the expectation of the return to capital. Hence, in period t+1 it is fixed and due to the idiosyncratic shocks to the high-risk firm's capital productivity, the shadow bank could incur profits or losses. The ex-post threshold value (above which the risky firm remains solvent) is given by

$$\overline{\omega}^{b}{}_{t+1}(1+R_{t+1}^{k,re})\mathrm{Q}_{\widetilde{K},t}\widetilde{K}_{t+1}^{re} = R_{t+1}^{sb}L_{t+1}^{re} \tag{40}$$

which leads to the following expression for $\overline{\omega}^{b}{}_{t+1}$:

$$\overline{\omega}_{t+1}^{b} = \overline{\omega}_{t+1}^{a}\frac{(1+\mathbb{E}_t R_{t+1}^{k,re})}{(1+R_{t+1}^{k,re})} \tag{41}$$

Equation (41) is the main engine by Zhang (2010) introduces the possibility of banks' profits. Nonetheless, neither Zhang (2010) nor Suh (2012) offers a possible explanation for the existence of the forecast error. The present setup aims at filling this gap and considers optimism in the shadow banking sector as a plausible cause of the discrepancy between the forecasted and realized returns to capital. As is the case for the retail bank, in times of optimism (or pessimism) the trust company is prone to biased expectations regarding the productivity of the high-risk firm's physical. Consequently, equation (41) now becomes

$$\overline{\omega}_{t+1}^{b} = \overline{\omega}_{t+1}^{a}\frac{(1+\mathbb{E}_t R_{t+1}^{k,re})}{(1+R_{t+1}^{k,re})} = \overline{\omega}_{t+1}^{a}(1+\chi_t^{sb}) \Rightarrow \frac{(1+\mathbb{E}_t R_{t+1}^{k,re})}{(1+R_{t+1}^{k,re})} = (1+\chi_t^{sb}) \tag{42}$$

The law of motion for optimism in the shadow banking sector resembles that of the retail bank

$$\chi_t^{sb} = \rho_\chi^{sb}\chi_{t-1}^{sb} + (1-\rho_\chi^{sb})\alpha^{sb}(N_{t+1}^{re} - \overline{N}^{re}) \tag{43}$$

where all the variables are identical to those in equation (34) except that here they refer to the level of optimism of the shadow bank and the net worth of the high-risk firm. As can be seen, a higher level of optimism leads to a higher ex-post threshold default value for the firm.

3.4 Monetary Policy

The descriptive evidence presented above indicates that PBoC currently uses a broader range of instruments than its international peers in conducting monetary policy. To build a unified theoretical framework for analysis, we incorporate the salient features of the nonstandard instruments and monetary policy transmission channels outlined above into our DSGE framework.[11] First, PBoC sets the short-term policy rate following a standard Taylor-rule:

$$R_t = \tilde{\rho}(R_{t-1}) + (1-\tilde{\rho})[\bar{R} + \alpha_\pi(\pi_t - \bar{\pi}) + \alpha_y(Y_t - \bar{Y})] + \varepsilon_t^{MP} \quad (44)$$

where $\bar{\pi}$ is the steady-state inflation rate (assumed to be 1 in the DSGE literature), $\bar{R}$ is the steady-state short-term policy rate, and $\bar{Y}$ is the steady-state level of output. α_π and α_y are the weights assigned to inflation and output, $\tilde{\rho}$ is the interest-rate smoothing parameter and finally ε_t^{MP} stands for the monetary policy shock. Furthermore, we assume that loan and deposit rates in the commercial banking sector are restricted by the guidelines of the central bank. Motivated by the pattern in Figure 1, we assume that the deposit rate ceiling in the commercial banking sector is determined as follows:

$$r_t^d = \min(r_t^{d,mr}, r_t^{d,cb}) \quad (45)$$

where $r_t^{d,mr}$ is the market-determined deposit rate, and $r_t^{d,cb}$ the deposit rate ceiling set by the PBoC. Similarly, the lending rate in the commercial banking sector is determined by

$$r_t^l = \max(r_t^{l,mr}, r_t^{l,cb}) \quad (46)$$

where $r_t^{l,mr}$ is the market-determined lending rate, and $r_t^{l,cb}$ is the respective lending rate floor set by the PBoC. Since the two benchmark rates are rarely revised, we assume $r_t^{d,cb}$ and $r_t^{l,cb}$ to be exogenously given. In the baseline model calibrations below we assume that the deposit rate ceiling in equation (45) is strictly binding. On the contrary, consistent with Table 1 commercial banks are assumed to have some leeway in setting the lending rate.

Furthermore, PBoC steers the supply of credit in the commercial banking sector via window guidance as part of its macroeconomic control policy.[12] We assume that PBoC determines the target for the total lending of commercial banks according to a Taylor-type rule

[11] The juxtaposition of various monetary policy instruments in Chen et al. (2012) and Funke and Paetz (2012) is a natural starting point for our subsequent analysis.

[12] Window guidance in China aimed at imposing lending targets is far from unique. For example, after the global recession 2008-2009 the UK government introduced lending targets for the five major UK banks to tackle the problem of reduced lending due to a weakening of bank balance sheets.

$$L_t^{cb} = \phi_l^{cb}(L_{t-1}^{CB}) + (1 - \phi_l^{cb})(\bar{L}^{se} + \phi_l^l[L_t^{se} - \bar{L}^{se}] + \phi_l^{\pi}[\pi_t - \bar{\pi}] + \phi_l^y[Y_t - \bar{Y}]) + \varepsilon_t^{wg} \quad (47)$$

To interpret equation (47), recall that PBoC sets the loan quota in order to smooth inflation and the output gap. In this connection, ϕ_l^{π} and ϕ_l^y represent the respective strengths of monetary authority reactions to inflation and output growth and ϕ_{π}^l gives the persistence of these responses. In addition, ϕ_l^{cb} ensures that PBoC does not fully eliminate the loan supply during boom times when investment is on the rise. Furthermore, our model assumes that the interest rate on required reserves passively mimics the PBoC's policy rate, i.e. $R_t^R = R_t$. Last but not least, ε_t^{wg} is the window guidance shock. This terminates the description of nonstandard monetary policy in China, which is absent from the expositions of standard DSGE models.[13]

The rest of the model is standard and does not add sufficient intuition to warrant inclusion in the main text. The interested reader is referred to Appendix A for a description of the remaining model equations. In the remaining part of section 3, we define the interest rate liberalization scenarios considered in the simulations.

3.5 Interest Rate Liberalization Scenarios

Chinese monetary policy is in a state of flux. In the baseline scenario, we model China's financial system such that the PBoC announces strictly binding benchmark deposit rates, with a certain degree binding lending rates and "window guidance" on loan quotas, as specified in section 3.4.[14] The remaining two scenarios represent the stepwise financial market liberalization. More precisely, we consider two different forms of interest rate liberalization. In the first reform scenario, the central bank ends the control of interest rates (i.e. the penalty function becomes zero) but window guidance on loan quotas remains. This can be labelled as a "partial liberalization" scenario. In the second reform scenario, interest rate control ends and window guidance is "turned off".[15] This will be termed the complete liberalization scenario. The idea underlying the complete liberalization scenario is to correct the misallocation of credit in China. The way to do this is by raising the cost of funds for banks. Raising deposit rates will narrow the banks' net interest margin and put pressure on them to raise lending rates. Higher lending rates will increase the likelihood that banks find it profitable to lend to previously excluded firms and will force improved efficiency on borrowers who have benefited from artificially low rates. The motivation for highlighting the difference between these two reform scenarios

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13 We do not consider a reaction function for the reserve requirements (v) because they are merely used to sterilize the domestic monetary consequences of foreign reserve inflows.

14 In equation notation, the lending rate is given by $1 + r_{t+1}^l = \frac{1}{\varepsilon_{t+1}^{l,op} - 1 + \kappa^l}[\varepsilon_{t+1}^{l,op}(1 + R_{t+1}^l) + \kappa^l(1 + r_{t+1}^{l,cb})]$. Hence, the parameter κ^l represents the penalty the bank incurs when the lending rate is below the benchmark rate set by the central bank. Since the lending rate is partially flexible, we set $\kappa^l = 1 * \varepsilon_{t+1}^{l,op}$. In other words, commercial banks set their rates approximately fifty-fifty determined by the market rate and the benchmark one.

15 To cite international experience, the objective of interest rate liberalization only involved removing interest rate control in most countries (e.g. in the US and Japan), but in some countries it also involved removing quantity control of credit (e.g. in Korea in early 1990s). See Liao and Tapsoba (2014) and Ito and Krueger (1996).

is that removal of quota controls seems to be under-appreciated in the current discussion on China's financial reform, as the majority view is that the only step left in interest rate liberalization is to remove the upper limit constraint on bank deposit rates.[16]

4. Calibrated Parameters

In calibrating the DSGE model presented above, we draw from a wide range of available information. Parameters are selected in order to capture specific ratios in the Chinese economy as closely as possible, with a view to the simulation properties of the model.

The discount factor β is set to 0.9925 to match a steady-state deposit rate of 3% on an annual basis whereas $\varepsilon^{l,op}$ and ε^{d} are assumed to equal 389 and -222, respectively. This is done so as to match the average interest-rate spread (lending-deposit rate) of three percentage points on an annual basis. The survival probability of the low-risk firm γ^{se} is set at 0.96. The autoregressive parameter ρ_{χ}^{rb} of the law of motion of optimism for the low-risk firm is set at 0.95 since the process is highly persistent. Similarly, optimism in the shadow banking sector is also assumed to be very persistent, albeit somewhat less than in the commercial banking industry; hence, $\rho_{\chi}^{sb} = 0.8$. We assume that the share of firms financed via the shadow (commercial) banking sector is 35% (65%). At end-2013, bank lending amounted to about 126% of GDP and shadow banking (trust/WMP/AMP) to about 46% of GDP.[17] Last but not least, in line with Zhang (2010) the model assumes the capital adequacy ratio of the shadow bank is 10% in the steady state in order to match a steady-state shadow bank default probability of 1%. The remaining parameters are set at standard values in the DSGE literature and are summarized in Appendix B.

5. Model Properties

Up to now the analysis has focused on the model description and calibration. Next we delve more deeply into the quantitative model properties. At this point it is convenient to explain our numerical solution method. We solve the nonlinear model with occasionally binding constraints employing Guerrieri and Iacoviello´s (2015) linear first-order piecewise perturbation algorithm. The idea of the approach is to handle occasionally binding constraints as different regimes of the same model. Under one regime, the occasionally binding constraint is slack. Under the other regime, the same constraint is binding. As illustrated below, the algorithm can handle the occasionally binding interest rate

[16] A caveat to be stressed here is that although the direction is set, the path of further liberalization is highly uncertain. The Chinese government has not published an official roadmap and therefore the time-scales are still uncertain.

[17] Estimates of the size of the Chinese shadow banking sector vary widely depending on how it is defined. Li (2013: Table 1) has tied together various estimates for the years 2012/2013 ranging from 28% of GDP to 57% of GDP. For all the difficulties of making a reliable calculation, one thing is apparent: its very strong growth. A sensitivity analysis for alternative η values is provided in Figure 11.

constraints in China.[18]

One way to learn about the model properties is to examine impulse response functions (IRFs). The shocks that we consider are simple and abstract. Nevertheless, they are intended to convey a rough idea of the DSGE model implications for observables. The first two shocks considered are traditional demand and supply shocks. First we explore the response of our key variables to a contractionary monetary policy shock. In this case, the deposit rate ceiling and window guidance create distortions in the baseline scenario. Then we consider a supply shock. The supply shock is represented by a positive shock to an intermediate-good firm´s productivity. Finally we trace out the effects of an expansionary window guidance shock. Below we examine each of these hypothetical experiments in turn.

5.1 Impact of a Contractionary Monetary Policy Shock

In this section we consider an unexpected monetary tightening where the PBoC raises its short-term policy rate by one percentage point. The solid black line gives the baseline scenario results. The dashed red line and the dotted green line give the partial liberalization scenario and the full liberalization scenario, respectively.[19] The responses of selected aggregate variables to a percentage point reduction of the short-term policy rate are shown in Figure 5. Qualitatively almost all variables react as expected in all scenarios. The increase in the policy rate leads to a decrease in investment. As a result output falls which drives inflation down.[20] In accordance with the notion of monetary neutrality, the temporal pattern of output is hump-shaped back to its pre-shock level. In addition, higher deposit interest rates in the shadow banking sector increase savings and reduce consumption. Furthermore, the fall in asset prices induces a decrease in net worth of both types of firms, and negative financial acceleration ensues. Noteworthy too is the horizontal segment of the deposit rate IRF. This nicely illustrates how the occasionally binding constraint is imposed on the IRFs by Guerrieri and Iacoviello's (2015) linear first-order piecewise perturbation algorithm.

How responsive is lending to high-risk firms to changing money-market conditions? In the first instance Figure 6 shows that shadow bank lending rates do react to changes in monetary policy.[21] Furthermore, Figure 6 and 7 illustrate that interest rate deregulation is an important factor in studying the behaviour of firm-specific variables. Figure 6 illustrates how high-risk firms react to a monetary contraction. It reveals the notable feature that tighter regulation of interest rates in the commercial

[18] See Appendix C for a detailed description of the model solution algorithm.

[19] Note that the horizontal axis in all IRF graphs measures quarters after the shock, and the horizontal axis are percent deviations from steady state values. For all interest rates, the absolute deviations from the steady state are given. For the remaining variables, percent deviations from the steady state are reported.

[20] An implication is that the dynamics of the macroeconomic aggregates in response to shocks can be well approximated by a representative bank model. It fits into this picture that Fernald et al. (2014) have recently shown that monetary policy shocks generate standard IRF responses in a FAVAR framework, comprising an economic activity factor, an inflation factor, and the PBoC benchmark interest rate.

[21] This confirms the finding of Qin et al. (2014) that China´s informal lending rates are responsive to monetary policy.

banking sector leads to an increase in lending by the shadow banking sector. The interpretation of this result is straightforward. Once the deposit rate ceiling becomes binding, an additional substitution effect kicks in and "trusts" begin to engage in regulatory capital arbitrage via off-balance sheet vehicles.[22] In other words, "trusts" employ the loophole that interest rate and loan restrictions do not apply to shadow banks. This allows them to offer higher deposit interest rates which induces shadow banks to expand their balance sheet and leverage.[23] As seen in Figure 6, this establishes a commercial bank-like credit intermediation channel. Relatedly, lending rates increase after a contractionary monetary policy shock across the board.

As shown in Figure 7, this induces low-risk firms to reduce the amount of capital purchased and consequently also the demand for commercial bank loans. The flip side of this is that commercial bank financing becomes more expensive relative to borrowing money from the shadow banking sector. This gives rise to off-balance sheet shadow bank lending and provides opportunities for the shadow banking sector to partially fill the gap. In a nutshell, the reactions of lending by commercial banks and by shadow banks to a contractionary monetary policy shock are in opposite directions. Whereas commercial banks retrench, shadow banks proliferate and grow.[24] This illustrates that an important measure to discourage shadow banking is further interest rate liberalization. It has to be acknowledged, however, that the additional off-balance sheet financial intermediation due to financial repression may increase the efficiency of the Chinese economy.[25] Lastly, the evidence for investment in Figure 5 indicates that window guidance has a restraining effect on the fall in investment. Comparison of the dashed red line and the dotted green line reveals that window guidance reduces capital decumulation by half.[26]

5.2 Impact of a Positive Technology Shock to Intermediate-Good Firm's Productivity

After looking at the effects of monetary policy shocks on both the real sector and financial variables, it is worthwhile considering a supply-side shock. The reason for considering a supply shock is that this creates a trade-off for PBoC, which aims at stabilizing inflation and output. Furthermore, technology shocks are one of the main drivers of growth in an emerging country like China. We assume that the

[22] For empirical evidence on the regulatory arbitrage hypothesis, see Acharya et al. (2013).

[23] One needs to keep in mind that although shadow banks have been subject to restrictions on their leverage ratios and net capital requirements since 2010, the restrictions on their operations are still much looser than those for commercial banks. The fact that additional WMP sales allow the shadow banking sector to replace some of the lost commercial bank credit is consistent with the empirical evidence in Altunbas et al. (2009).

[24] There is ample evidence that financial repression in developing countries encourages institutions to circumvent it through nonbank intermediation. See, for example, Vittas (1992).

[25] Shadow banking can be conducive to further growth but also increase to risk. Allen et al. (2005), for example, have shown that shadow banking finance has bolstered SME growth in China. One of the conclusions to emerge is that the challenge for the Chinese regulators is to maximize the benefits of shadow banking while minimizing the systemic risks. It must be emphasized again that this paper does not address the regulatory issue of how to quantify empirically the real-world benefits and costs and thus enable one to maximize efficiency while minimizing risks. Luck and Schempp (2014) have shown that a large shadow banking sector may set the stage for a financial crisis. Plantin (2015) has studied the optimal degree of regulation when regulatory arbitrage is present.

[26] One reason is that, in a model with forward-looking agents expectation effects exist, i.e. firms anticipate the window guidance reaction from the PBoC and factor it into their decision making.

technology shock to intermediate-good firm´s productivity is highly persistent with an AR(1) coefficient of 0.9. Figure 8-10 give the flavour of the monetary policy – interest rate liberalization interface.

Inspecting the results leads to the following conclusions. First, the aggregate variables in Figure 8 closely resemble the typical IRF responses to a positive supply shock. As the marginal productivity of capital rises, output exhibits positive steady state deviations from trend while inflation recedes. Second, comparison of the case of full liberalization (dotted green lines) vs. partial deregulation (dashed red lines) reveals that full liberalization results in larger feed-through of the policy rate to the lending rate in Figure 9 and 10. From a monetary policy point of view, the tighter the relationship between short-term and long-term interest rates, the more effective is PBoC's control along the yield curve. Thus, PBoC has more scope to tailor monetary policy to macroeconomic conditions. Third, the full liberalization scenario leads to a more pronounced rate shock transmission. The reason, as before, is the effectiveness of window guidance. This subtle but important observation implies that a given shock results in a larger feed-through to the loan level and investment. The underlying reason is the financial intermediary channel of window guidance as a countervailing force. The difference between the partial reform scenario (dashed red lines) and the full reform scenario (dotted green lines) reveals that leaning-against-the-wind window guidance stabilizes the financing of firms by the commercial banking sector and thus counteracts the technology shock.[27] The next subsection completes the picture by shedding further light on a window guidance shock.

5.3 Expansionary Window Guidance Shock

Next we investigate the effect of an expansionary window guidance shock, taking into account general equilibrium effects. In our framework, the window guidance shock is modelled as an unexpected 10% expansion in the loan quotas in equation (47).[28] Against the background that window guidance still is a prominent quantity-based monetary instrument of the PBoC, such an analysis is important for the effective design of future financial market liberalization policies in China. Visually, the main results of the exercise are apparent in Figure 11 and 12. The underlying assumption is the intermediate partial reform scenario with window guidance. The experiment is conducted for two alternative shares of high-risk firms in the economy and thus two different orders of magnitude of the shadow banking sector. The dashed red line represents the results for η = 0.35, and the solid blue line for η = 0.20. Analyzing the IRFs for a larger or smaller share of shadow banks illustrates, ceteris paribus, how shadow banking entities impact monetary policy transmission.

What is the impact of the window guidance stimuli? Two key lessons emerge from the IRF graphs. First, as expected, Figure 11 reveals that the expansionary window guidance shock has real effects

[27] This opens up the possibility of a vicious circle related to the co-existence of price-based monetary policy instruments and window guidance. The full effect of price-based instruments only comes into play when there is no window guidance influence involved. But as long as the interest rate instrument alone does not deliver the desired effects, the PBoC will rely on window guidance.

[28] We set the shock at 10% because this is the approximate magnitude of previous window guidance shocks [see Chen et al. (2012), Table 2, p. 6].

on the economy: both investment and output increase. Second, the IRFs convey the important message that the effectiveness of window guidance depends on the magnitude of the shadow banking sector and thus ultimately on the degree of financial market liberalization. Comparison of IRFs for η = 0.35 vs. η = 0.20 reveals that the monetary policy impact on output and inflation is weakened under the emergence of a larger shadow banking sector because shadow bank lending is exempted from loan quotas. Ultimately, this means that progress in financial market liberalization that broadens the non-financial sector's range of financing and investment options tends to weaken the transmission of monetary policy measures via commercial banks and to erode the validity of window guidance over time. In other words, shadow banking entities change the way in which monetary policy works. Altogether this makes a strong case for removal of the window guidance toolbox under the complete liberalization scenario.[29] In other words, the Chinese authorities should end the heterodox policy mix of price-based and quantity-based monetary policy instruments in the medium term. An even stronger case for the termination of window guidance in the course of further financial market liberalization can be made if we take into account the undermining of competition among commercial banks via prescribed window guidance lending shares.

5.4 Shadow Banking and Welfare

In the previous subsections we have described the functioning of monetary policy and the dynamics of the shadow banking sector via impulse response functions. Now, we test the effectiveness of both liberalization scenarios. The fundamental challenge for PBoC is how to improve the allocation of loans without inducing economic and financial instability. To evaluate the performance for the partial vs. the full reform scenario, we finally simulated the model for 100,000 periods, ignoring the first 1,000. To compare both policy scenarios, we simulated the model for each shock separately.[30] Naturally, the evaluation depends on the welfare function applied. The welfare criterion that we use to rank all scenarios, based on a standard loss function, is to minimize the weighted sum of the variances of inflation and output gap.

$$\text{Welfare Loss} = \text{Var}(\pi_t) + \alpha_l \text{Var}(Y_t) \quad (48)$$

To be sure, equation (48) is assumed rather than derived from utility maximization.[31] A micro-founded version of the loss function derived from a second-order approximation of the representative households's utility function tends to give a low value of α_l. Since the Chinese authorities care greatly

[29] This result bears some resemblance to that of Japan in the 1980s when financial liberalization and the associated expansion of various financial intermediary channels unrestrained by window guidance gradually undermined its effectiveness.

[30] The relevance of the way of proceeding results from the fact that countries have lived through a series of economic shocks, none of which were predicted by experts in real time.

[31] Note that the derivation from first principles is a futile endeavour due to the various agents and occasionally binding constraints.

about stabilizing output, a range of 1 < α_l < 2 seems appropriate.[32]

Table 2 summarizes the results obtained for each shock and each scenario. In the first instance, we compare the welfare effects for the two reform scenarios. What becomes obvious after the conduct of the analysis is that window guidance positively affects agents' welfare. That is, the larger value of the loss function as between the two reform scenarios is observed when the central bank abolishes both interest restrictions and loan quotas. This reflects the counteracting window guidance effect of leaning against the wind, whereby the PBoC opposes the accelerator and amplification mechanisms connected with the presence of financial frictions. The other side of the coin is that when China liberalizes interest rates further, greater volatility is a likely result.

The first two rows report the results of turning window guidance on or off in isolation. Finally, we show the results for a more aggressive Taylor rule. In practice it is highly unlikely that the parameters of the Taylor rule would remain constant after further liberalization.[33] Therefore, we revise the full liberalization scenario to incorporate induced changes in the Taylor rule that compensate for the elimination of window guidance when moving from the partial to the full liberalization scenario. For now, let us assume that α_π and α_Y increase to 2.0 and 0.2, respectively. The simulations show that a more aggressive Taylor rule with respect to inflation and the output gap leads to greater stability across the board. All in all, the conclusion of this exercise is that full liberalization with a "hawkish" PBoC produces an improvement in macroeconomic stabilization and is thus the preferred policy option. Furthermore, the results indicate that waiving window guidance requires a new and active monetary policy approach that entails greater responsiveness in the short-term monetary policy rate for loan targets.

6. Concluding Remarks

Ultimately, this paper is an attempt to provide some guidance on how to pursue financial market liberalization in China. To this end, we have built a present-generation nonlinear DSGE model with occasionally binding constraints that is tractable, versatile, and quantitatively potent. While not a crystal-ball, the DSGE model provides a well-organized conceptual framework for discussing the macroeconomic effects of financial market reforms in China. The model is related to some recent papers on monetary policy alignment in DSGE models with a banking sector and financial accelerator effects on both households and heterogeneous firms. Although the sequencing of financial market reforms and the optimal institutional arrangement are clearly complex matters, both analytical reasoning and the calibration results suggest some tentative policy conclusions. First, interest rate liberalization will induce a more effective monetary policy transmission. Second, in China's case,

[32] We let the weighting parameter α_l vary within the range [1,2]. The qualitative results reported below remain unaffected and are thus robust.

[33] Lucas (1976) has convincingly argued that the parameters of macroeconometric models are unlikely to remain stable when policies change. This critique helped change the focus of policy evaluation from consideration of alternative paths of the policy instrument to consideration of rule shifts under alternative policy approaches.

interest rate liberalization not only involves removing interest rate control; it also involves removing window guidance or quantity control on bank loans. In that sense, the prevailing view that China is now only one step away (removing the upper bound limit of deposit rates) from completing interest rate liberalization is not correct. Third, after interest rate liberalization, economic activity could respond with more volatility to different shocks. Fourth, liberalization of interest rates will attract deposits back into the commercial banking sector and rein in the expansion of the shadow banking sector. In short summary: The results are of interest not merely for modelling purposes. Comparative evaluation of various liberalization scenarios provides useful insights and could provide a more solid foundation for future monetary policy in China.

Although the model captures several features of shadow banking in China and leads to a novel set of economic insights, the modelling approach has some limitations and therefore a few caveats are in order. First, on the conceptual side, what some of the literature does not always recognize, is that the numerical results are obtained in a linearized unique-equilibrium model that rules out bubbles and other non-linear dynamics characterizing crisis episodes. In other words, financial market instability is precluded by construction.[34] Modelling such potential and highly nonlinear systemic risks and their evolution as well as the regulatory responses of policymakers is beyond the DSGE´s conceptual reach. Second, in the modelling framework all financial market variables reflect changes in fundamental values and are not associated with any type of irrational behaviour. Third, the calibration exercise focuses on conceptual issues in the interpretation of the DSGE model sketched above. In other words, the calibration exercise should be viewed as theory with numbers, not empirical analysis in the strict sense of the term. Fourth, the IRFs consider various shocks one at a time. Thus they defer the problem of identifying various shocks and thus the sources of business cycle fluctuations in real time. Fifth, while DSGE models provide rough orders of magnitude for shadow banking problems, recommendations for in-depth financial reforms need to take into account complex political economy issues constraining reform implementation. All these issues are important avenues for future research. In a nutshell, we consider the DSGE modelling exercise, with its virtues and limitations, as a first step in a research agenda. An interesting cross-check of various results would be to use Chinese microcensus data if such data were to become available. This would enable the shedding of more light on the mechanisms underlying the link between shadow banking and firm performance. In spite of these limitations, we believe the modelling setup explored here offers a useful tool for understanding monetary transmission as well as the dynamics of the parallel shadow banking sector in China. It also offers some useful policy implications and guidance for future financial market liberalization in China.

34 For a DSGE model with nonlinearities, see Benes et al. (2014). Olivier Blanchard has recently labelled the ignorance of the fact that small shocks could have large adverse effects the "dark corner" in macroeconomics. See http://www.imf.org/external/pubs/ft/fandd/2014/09/pdf/blanchard.pdf. Acemoglu et al. (2014) have recently characterized how the propagation and amplification of idiosyncratic microeconomic shocks can reshape the distribution of GDP and lead to deep recessions and sizable macroeconomic tail risks.

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Table 1. Share of Commercial Loans Issued at Different Rates December 2007 – December 2014

Year	Below Benchmark	At Benchmark	Above Benchmark
December 2007	28.07	27.69	44.24
December 2008	25.56	30.13	44.31
December 2009	33.19	30.26	36.55
December 2010	27.80	29.16	43.04
December 2011	7.02	26.96	66.02
December 2012	14.16	26.10	59.74
December 2013	12.48	24.12	63.40
December 2014	13.10	19.64	67.26

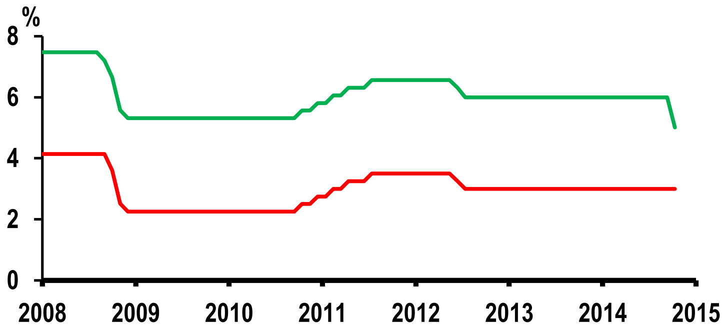
Data source: CEIC and Bloomberg.

Table 2. Welfare Loss of Alternative Policy Scenarios for $\alpha_l = 1.5$

Policy Scenario	Contractionary Monetary Policy Shock	Expansionary Technology Shock
Partial liberalization	2.9577e-04	1.8169e-05
Full liberalization	3.5006e-04	2.1096e-05
Full liberalization with a more aggressive Taylor Rule	2.6965e-04	1.8005e-05

Note: Taylor rule coefficients in the first three scenarios are α_π = 1.8 and α_Y = 0.1, respectively. In the alternative more aggressive Taylor rule the coefficients are α_π = 2.0 and α_Y = 0.2, respectively.

Figure 1. One-Year Benchmark Deposit and Lending Rates in %: January 2008 - September 2014



Note: The green (red) line is the nominal benchmark lending (deposit) rate. Data source: CEIC and Bloomberg.

Figure 2. Shadow Banking Credit Chain

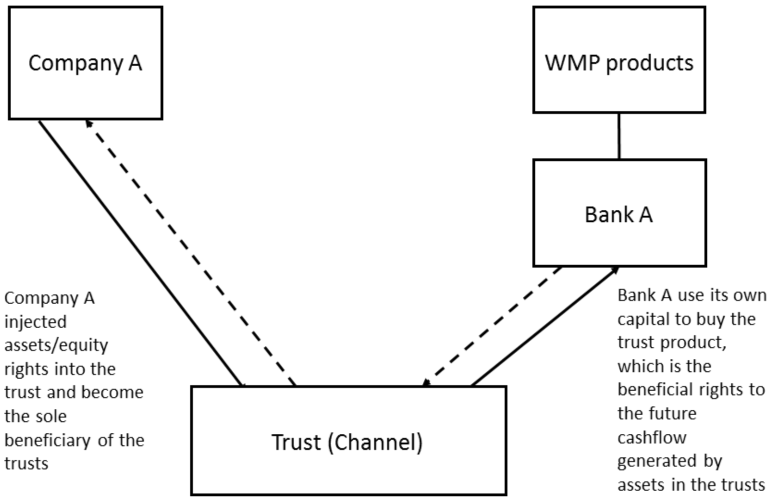
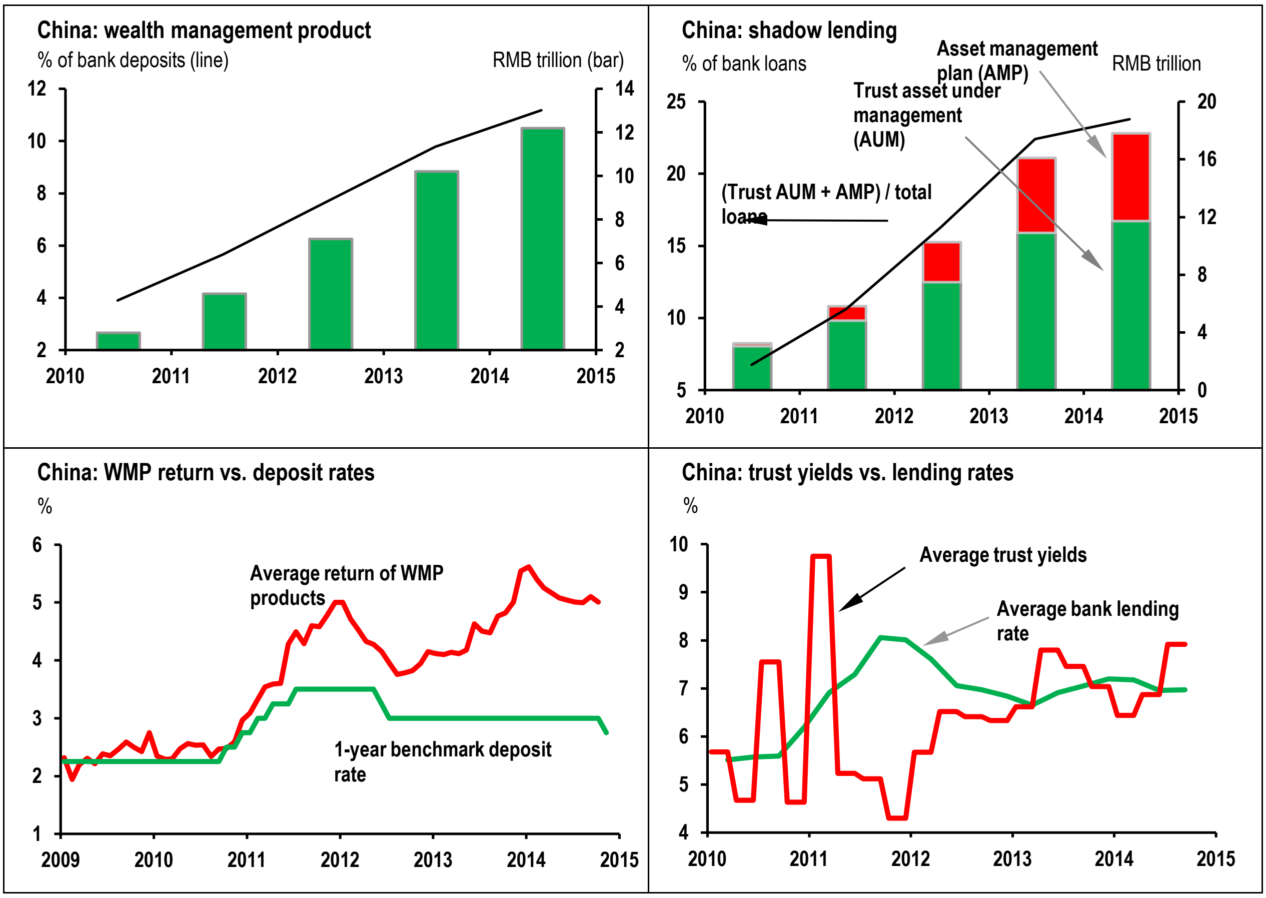


Figure 3. China's Shadow Banking Exposure



Data source: CEIC and Bloomberg.

Figure 4. Structure of the DSGE Model

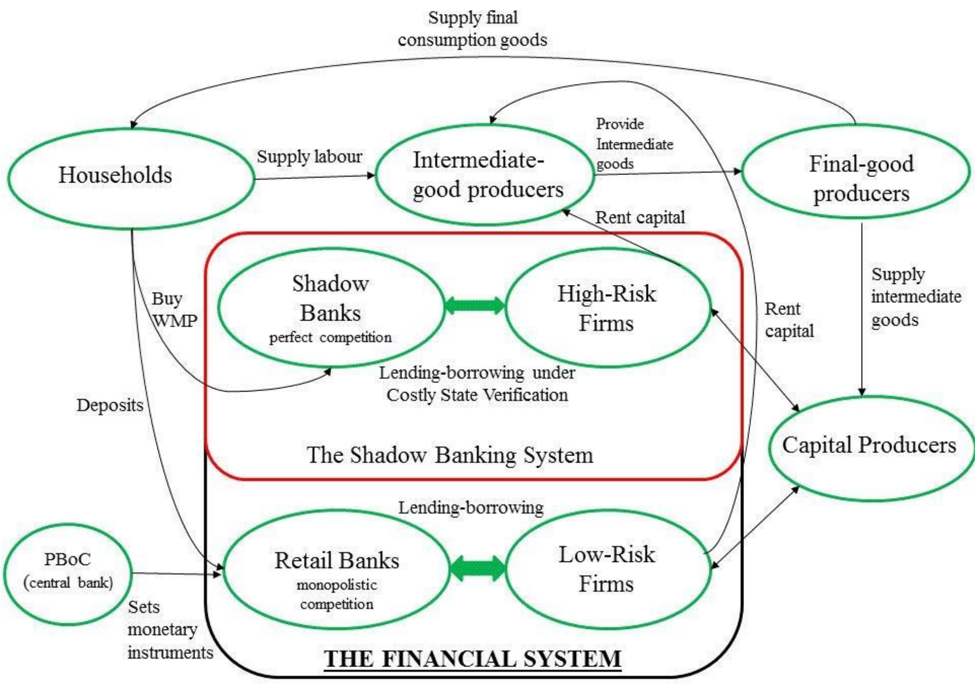


Figure 5. Impulse Responses of Selected Aggregate Variables to a 1% Increase in the Policy Rate

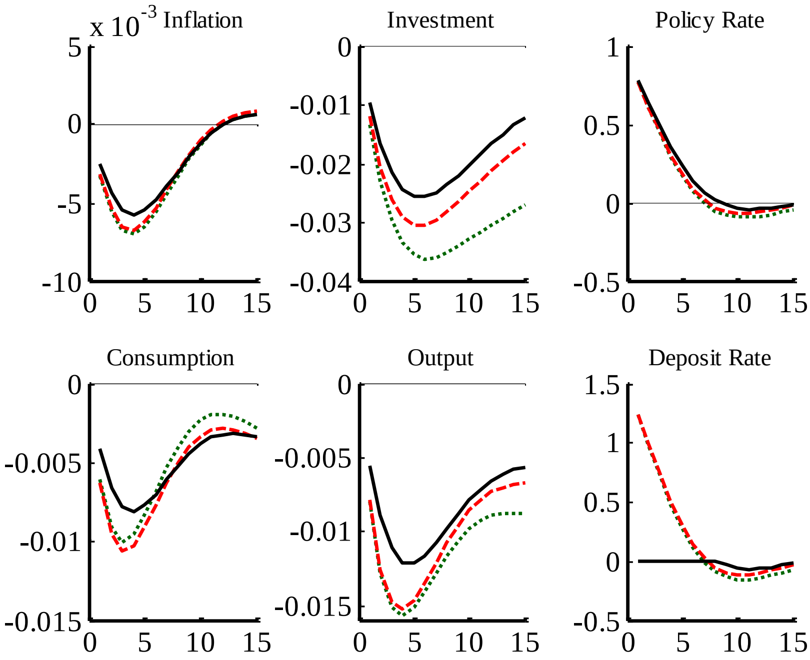


Figure 6. Impulse Responses of High-Risk Firm Variables to a 1% Increase in the Policy Rate

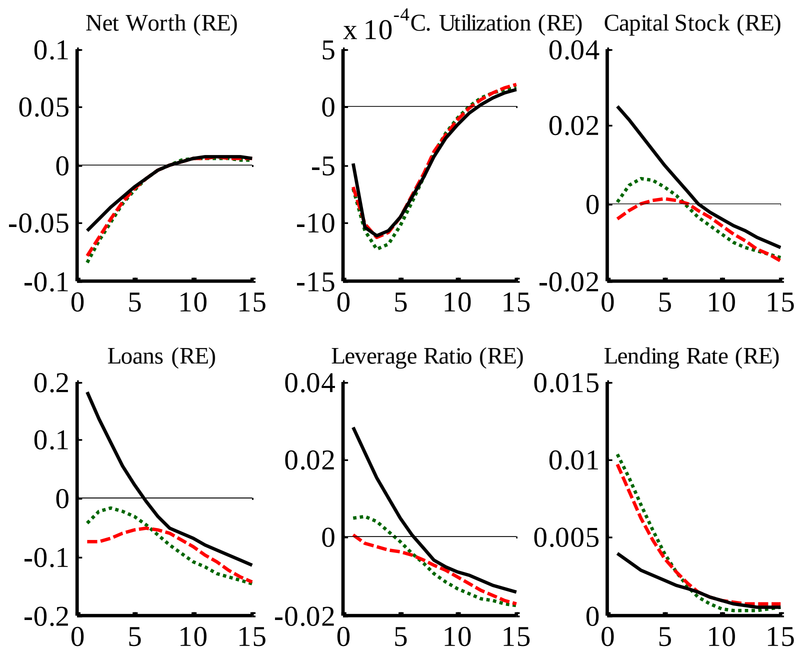


Figure 7. Impulse Responses of Low-Risk Firm Variables to a 1% Increase in the Policy Rate

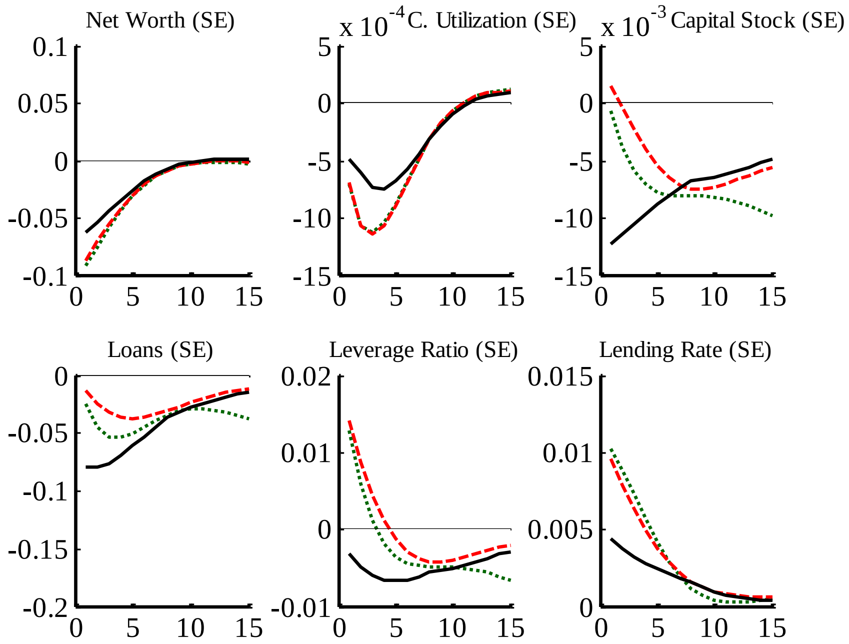


Figure 8. Impulse Responses of Selected Aggregate Variables to a Positive Technology Shock

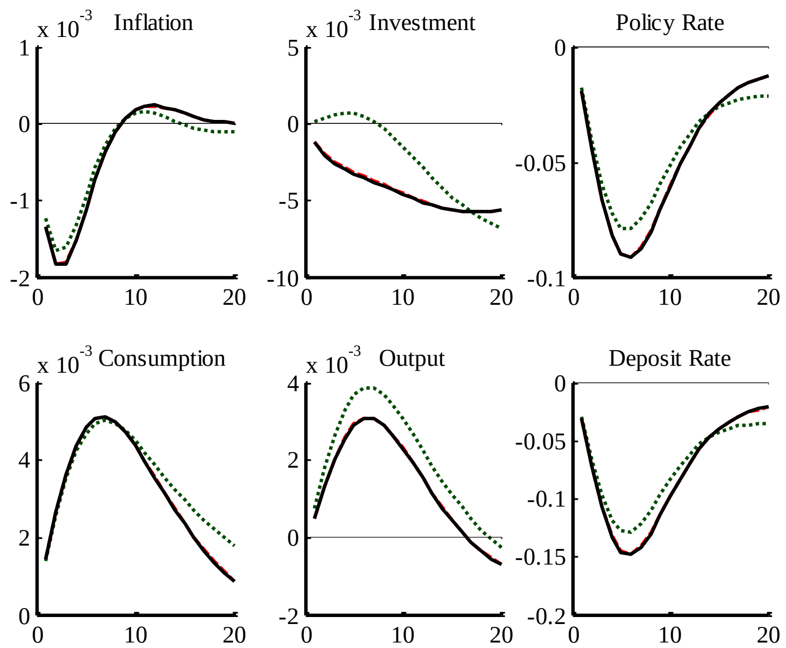


Figure 9. Impulse Responses of High-Risk Firm Variables to a Positive Technology Shock

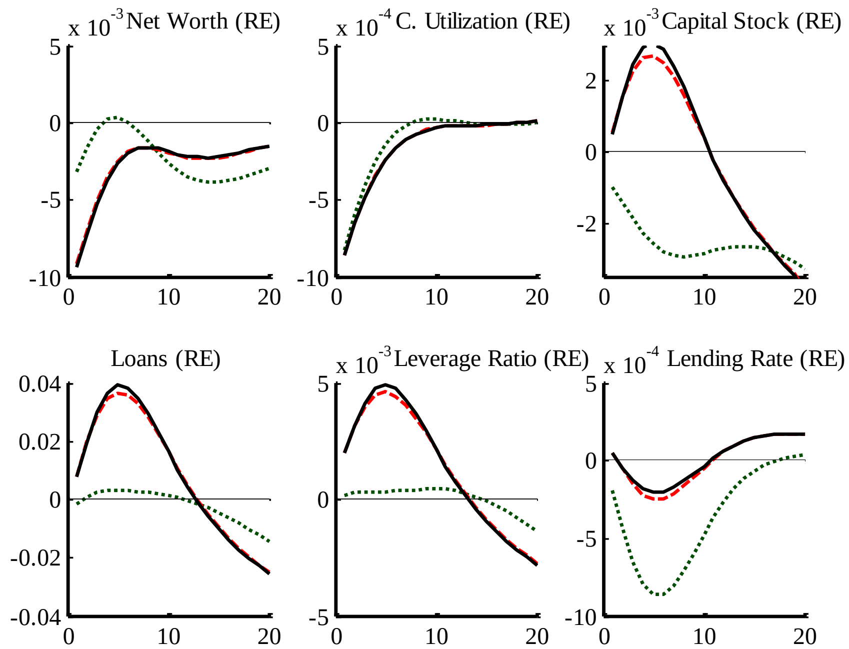


Figure 10. Impulse Responses of Low-Risk Firm Variables to a Positive Technology Shock

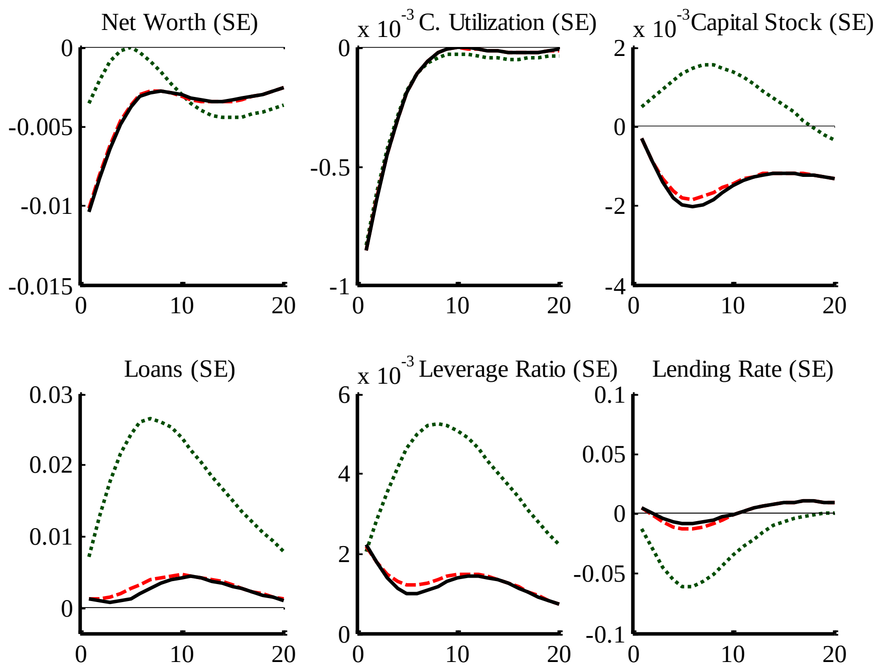
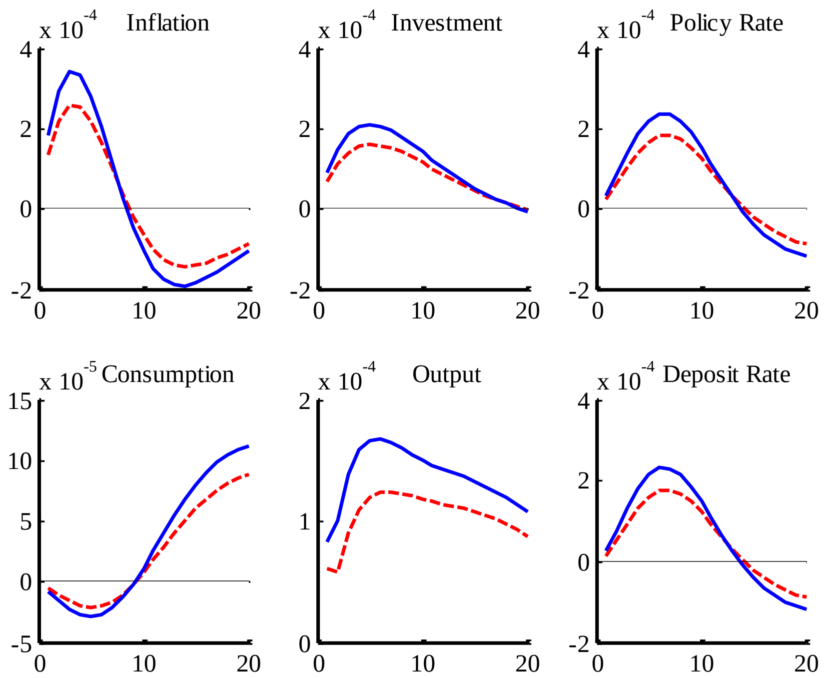


Figure 11. Impulse Responses of Aggregate Variables to the Window Guidance Shock



Appendix A. Rest of the Model

Households

The infinitely-lived households in the model consume final goods, save and supply labour services monopolistically in order to maximize the expected value of their lifetime utility. The instantaneous utility function is given by

$$U_t = log(C_t - bC_{t-1}) - \psi_l \frac{h_{j,t}^{1+\sigma_l}}{1+\sigma_l} \quad \text{(A1)}$$

where C_t represents current consumption and $h_{j,t}$ is the representative household's supply of working hours. The persistence of habit in the consumption pattern is captured by the parameter b whereas ψ_l is the preference parameter related to the disutility of labour. Finally, σ_l stands for the disutility of labour and is the inverse of the Frisch elasticity of labour supply. The key difference regarding households in the current model vs. Verona et al. (2013) is in the saving behavior. In both modelling frameworks, the households are the only investors in the financial sector of the economy. Nonetheless, the Verona et al. (2013) model has a very simplified portfolio choice problem since households receive a certain (risk-free) return on their investment. Yet modelling the Chinese shadow banking sector entails the introduction of an asset which promises a higher but risky return. Accordingly they can either deposit their money in a savings account offered by the commercial banking sector or purchase equity of the shadow bank.[35] The former is risk-free since the Chinese government guarantees the returns on deposits whereas the latter is risky.[36] The risk comes from the idiosyncratic return on capital that SMEs experience each period since part of their activity is financed by the shadow bank. Each period the household faces a budget constraint given by

$$\begin{aligned} &(1+r_t^E)(1-\phi_t)E_{t-1} + (1+r_t^d)D_{t-1} + W_{j,t}h_{j,t} + (1-\gamma^l)(1-\eta)V_t^s + \\ &\quad (1-\gamma^r)\eta V_t^r + \Pi_t^{IGF} + \Pi^{rb} + \Pi^{sb} - E_t - D_t - T_t - P_tC_t \geq 0 \end{aligned} \quad \text{(A2)}$$

In words, in period *t* households receive the return on their equity investments from the previous period $(1+r_t^E)(1-\phi_t)E_{t-1}$. ϕ_t stands for the percentage of those shadow banking institutions which declared bankruptcy in period *t*-1 and essentially it represents the return shadow banks have to pay to investors. The equation makes it clear that in the non-bankruptcy state shadow banks have to

[35] One simplification, for the sake of tractability, is that we do not explore in depth potential determinants of the high household savings in China. Given China's surplus of savings and a shortage of suitable vehicles for that thrift, it is easy to explain why investors are demanding WMP products.

[36] The argument ignores the fact that the de facto Chinese safety net for the banking sector is an ambivalent matter. On the one hand, the Chinese government announced on 30 November 2014 its plan to introduce a deposit insurance scheme which will not cover off-balance sheet vehicles. On the other hand, implicit Chinese state guarantees have recently been extended to shadow banks. When several off-balance sheet investment products were on the verge of defaulting in 2014, the authorities orchestrated bail-outs. Therefore, it remains to be seen whether the de jure safety net is a time-consistent announcement, i.e. whether the government will not stand any more behind uninsured deposits.

promise a return higher than the risk-free rate on deposits because otherwise households would prefer to invest their money in the commercial banking sector. $(1+r_t^d)D_{t-1}$ is the safe return households receive each period as a liability on their deposits whereas $W_{j,t}h_{j,t}$ represents their labour income. The next two terms, $(1-\gamma^l)(1-\eta)V_t^s$ and $(1-\gamma^r)\eta V_t^r$, stand for the lump-sum transfers received from both high-risk and low-risk firms that exit the economy. Π_t^{IGF}, Π^{rb} and Π^{sb} are the last three terms in the income part of the budget constraint and represent respectively the profits of the intermediate-good firms and the commercial and the shadow banking sectors.[37]

The expenditure side of the household's budget constraint consists of investment in new shares (equity) E_t, the newly-made deposits D_t, lump-sum taxes paid by households T_t and finally the nominal value of private consumption in period t, P_tC_t. Hence, the maximization problem of the representative household is given by

$$\max_{\{C_t,E_t,D_t\}} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log(C_t - bC_{t-1}) - \psi_l \frac{h_{j,t}^{r^{1+\sigma_l}}}{1+\sigma_l} \right\} \tag{A3}$$

subject to

$$\begin{gathered}(1+r_t^E)(1-\phi_t)E_{t-1} + (1+r_t^d)D_{t-1} + W_{j,t}h_{j,t} + (1-\gamma^l)(1-\eta)V_t^s + \\ (1-\gamma^r)\eta V_t^r + \Pi_t^{IGF} + \Pi^{rb} + \Pi^{sb} - E_t - D_t - T_t - P_tC_t \geq 0\end{gathered} \tag{A4}$$

The solution of the maximization problem leads to the following first-order conditions with respect to C_t, E_t, and D_t:

$$P_t\lambda_t = \frac{1}{C_t - bC_{t-1}} - \beta b \frac{1}{C_t - bC_{t-1}} \tag{A5}$$

$$\lambda_t = \beta(1+r_{t+1}^E)(1-\phi_{t+1})E_t(\lambda_{t+1}) \tag{A6}$$

$$\lambda_t = \beta(1+r_{t+1}^d)E_t(\lambda_{t+1}) \tag{A7}$$

where λ_t is the Lagrange multiplier (shadow price) associated with the budget constraint. As a result, we see that as long as the shadow bank's default probability is positive, $r^E > r^d$. This is the key difference between the present model and that of Verona et al. (2013), as the latter assumes that $r^E > r^d$ in each period. The return on time deposits is a function of the central bank nominal interest rate and is risk-free. The labour supply decision as well as the wage setting follows the baseline

37 The shadow banking sector operates in a perfectly competitive environment. Nonetheless, profits are realized due to the difference between the ex-ante and ex-post default threshold levels for the high-risk firms.

DSGE literature with both price and wage rigidities. In short, a labour agency bundles monopolistically provided and differentiated labour supply services of households and provides the aggregate to intermediate-good firms.

Final-good firms

The representative final-good firm competitively produces a final good Y_t using intermediate goods $Y_{i,t}$ using the Dixit-Stiglitz aggregator:

$$Y_t = [\int_0^1 Y_{i,t}^{\frac{1}{\lambda_f}} di]^{\lambda_f} \tag{A8}$$

where λ_f is the mark-up for the intermediate-good firms. The final-good firm maximizes its profits by choosing $Y_{i,t}$, taking as given the output price P_t and the input prices $P_{i,t}$

$$\max_{\{Y_{i,t}\}} \; P_t Y_t - \int_0^1 P_{i,t} Y_{i,t} di \tag{A9}$$

subject to

$$Y_t = [\int_0^1 Y_{i,t}^{\frac{1}{\lambda_f}} di]^{\lambda_f} \tag{A10}$$

The solution to the maximization problem leads to the following result:

$$Y_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{\frac{\lambda_f}{1-\lambda_f}} Y_t \tag{A11}$$

where $P_t = [\int_0^1 P_{i,t}^{\frac{1}{1-\lambda_f}} di]^{1-\lambda_f}$.

Intermediate-Good Firms

Intermediate-good firms operate in a monopolistic environment and produce differentiated intermediate goods according to the following Cobb-Douglas production function:

$$Y_{i,t} = \exp(a_t) K_{i,t}^{\alpha} H_{i,t}^{1-\alpha} \tag{A12}$$

where $Y_{i,t}$ is intermediate-good output, α denotes the share of capital, a is total factor productivity, and K_t and H_t are the capital and labour inputs.[38] The intermediate-good firm takes physical capital from both the high-risk and low-risk firms and bundles them into a single input. The CES aggregation formula is

$$K_{i,t} = [\eta(K_{i,t}^{re})^{\rho} + (1-\eta)(K_{i,t}^{se})^{\rho}]^{\frac{1}{\rho}} \tag{A13}$$

where ρ stands for the degree of substitutability between both types of capital. When $\rho = 1$, the capitals of the risky and safe entrepreneurs are perfectly substitutable. Each period the intermediate-good firm minimizes its cost function, solving the following minimization problem:

$$\min_{\{L_{i,t}, K_{i,t}^{re}, K_{i,t}^{se}\}} C_t = \frac{W_t L_{i,t}}{P_t} + r_t^{k,re} K_{i,t}^{re} + r_t^{k,se} K_{i,t}^{se} \tag{A14}$$

subject to:

$$Y_{i,t} = K_{i,t}^{\alpha} H_{i,t}^{1-\alpha} \tag{A15}$$

and

$$K_{i,t} = [\eta(K_{i,t}^{re})^{\rho} + (1-\eta)(K_{i,t}^{se})^{\rho}]^{\frac{1}{\rho}} \tag{A16}$$

As pointed out by Verona et al. (2013), since all firms face the same input prices and production technology, they also have the same marginal cost. As a result the "*t*" subscripts could be omitted. Firms' marginal cost is thus given by

$$s_t = [\frac{\widetilde{w}_t}{1-\alpha}]^{1-\frac{\alpha}{\rho+\alpha(1-\rho)}} [\frac{\alpha}{r_t^{k,se}} (K_t^{se})^{\rho-1}]^{-\frac{\alpha}{\rho+\alpha(1-\rho)}} (Y_t)^{\frac{\alpha(\rho-1)}{\rho+\alpha(1-\rho)}} \frac{\rho}{\rho+\alpha(1-\rho)} \tag{A17}$$

where $\widetilde{w}_t$ stands for the real wage in period *t*. The present model assumes intermediate-good firms set prices a lá Calvo (1983). That is, each period only $1-\xi_p$ of the firms are able to reoptimize their prices. The rest ξ_p set their prices according to the following rule:

$$P_{i,t} = P_{i,t-1}(\bar{\pi})^{\iota_1}(\pi_{t-1})^{1-\iota_1} \tag{A18}$$

[38] In line with the business cycle literature, we assume a follows an AR(1) process with an autoregressive parameter ρ_a = 0.9 and a positive innovation $\varepsilon \sim (0, \sigma_a^2)$

where π denotes the rate of inflation ($\bar{\pi}$ stands for steady-state inflation whereas $\pi_{t-1} = \frac{P_{t-1}}{P_{t-2}}$) and ι_1 represents the degree of price indexation to steady-state inflation.[39] Accordingly the intermediate-good firm chooses the price level $P_i, t = \tilde{P}_{i,t}$ that maximizes its current and expected profits:

$$\max_{\{P_{i,t}\}} \Pi_t^{IFG} = E_0 \sum_{t=0}^{\infty} (\beta\xi_p)^t \lambda_t [(P_{i,t} - S_t) Y_{i,t}] \tag{A19}$$

subject to

$$Y_{i,t} = \left(\frac{P_{i,t}}{P_t}\right) Y_t \tag{A20}$$

where S_t is the firm's nominal marginal cost and β is the standard discount factor. In the solution to the standard sticky-price maximization problem above only a symmetric equilibrium is considered where, due to facing identical marginal costs, all firms set the same price $\tilde{P}_{i,t} = \tilde{P}_t$ and the aggregate price index is given by

$$P_t = \left\{(1 - \xi_p)\tilde{P}_t^{\frac{1}{1-\lambda_f}} + \xi_p [P_{i,t-1}(\bar{\pi})^{\iota_1}(\pi_{t-1})^{1-\iota_1}]^{\frac{1}{1-\lambda_f}}\right\}^{1-\lambda_f} \tag{A21}$$

Capital Producers

Capital producers combine investment goods with undepreciated capital purchased from firms to produce new capital, which is then sold back to firms. At the end of period *t* the capital producers in the economy purchase the existing capital, $x_{K,t}$, from firms and investment goods I_t from the final-good firms. Then they combine these inputs in order to produce new capital $x'_{K,t}$ according to the following law of motion:

$$x'_{K,t} = x_{K,t} + A(I_t, I_{t-1}) \tag{A22}$$

It is assumed that there are no adjustment costs when transforming old capital into new whereas the transformation of investment goods into new capital is subject to quadratic costs represented by $A(\cdot)$. Thus, the capital producer has the following maximization problem:

[39] The derivation of staggered wages is identical to that of staggered prices. As a result we show only the latter. The interested reader may nevertheless refer to Appendix B of the earlier version of Verona et al. (2013) for all technical details related to wage stickiness.

$$\max_{\{I_t,x_{K,t}\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \lambda_t \{ \mathrm{Q}_{\widetilde{K},t}[x_{K,t} + A(I_t, I_{t-1})] - \mathrm{Q}_{\widetilde{K},t} x_{K,t} - P_t I_t \} \tag{A23}$$

The solution with respect to $x_{K,t}$ leads to the conclusion that any value is profit maximizing. Hence, $(1-\delta)\widetilde{K}_t$ satisfies this condition. The first-order condition with respect to I_t yields the following expression

$$\mathbb{E}_0[\lambda_t(\mathrm{Q}_{\widetilde{K},t}A_{1,t} - P_t) + \beta\lambda_{t+1}\mathrm{Q}_{\widetilde{K},t+1}A_{2,t}] = 0 \tag{A24}$$

which is a standard Tobin's Q equation providing a link between the market price of capital and the marginal cost of producing investment goods and where $A_{1,t} = \frac{\partial A(I_t,I_{t-1})}{\partial I_t}$ and $A_{2,t} = \frac{\partial A(I_{t+1},I_t)}{\partial I_t}$. Finally, the law of motion for the evolution of the aggregate stock of physical capital is given by

$$\widetilde{K}_{t+1} = \eta \widetilde{K}_{t+1}^{re} + (1-\eta)\widetilde{K}_{t+1}^{se} = (1-\delta)[\eta \widetilde{K}_t^{re} + (1-\eta)\widetilde{K}_t^{se}] + A(I_t, I_{t+1}) \tag{A25}$$

Aggregate Variables and Resource Constraint

The resource constraint in the economy is given by:

$$C_t + I_t + \eta\mu \int_0^{\overline{\omega}_t^b} \omega dF(\omega)(1 + R_t^{k,re}) \frac{\mathrm{Q}_{\widetilde{K},t-1}\widetilde{K}_t^{re}}{P_t} + \eta a(u_t^{re})\widetilde{K}_t^{re} + (1-\eta)a(u_t^{se})\widetilde{K}_t^{se} = Y_t \tag{A26}$$

That is, aggregate demand consists of private consumption and investment. The last three terms represent respectively shadow banks' monitoring costs and firms' costs incurred in the capital utilization process. The aggregate net worth and aggregate leverage in the present set-up are given by the following equations:

$$N_{t+1}^{ag} = \eta N_{t+1}^{se} + (1-\eta) N_{t+1}^{re} \tag{A27}$$

$$lev_{t+1}^{ag} = \eta lev_{t+1}^{se} + (1-\eta) lev_{t+1}^{re} = \eta \frac{\mathrm{Q}_{\widetilde{K},t}\widetilde{K}_{t+1}^{re}}{N_{t+1}^{re}} + (1-\eta) \frac{\mathrm{Q}_{\widetilde{K},t}\widetilde{K}_{t+1}^{se}}{N_{t+1}^{se}} \tag{A28}$$

The total amount of loans in the economy is equal to the weighted average of loans provided by both financial intermediaries:

$$L_{t+1}^{ag} = \eta L_{t+1}^{re} + (1-\eta) L_{t+1}^{se} \tag{A29}$$

The market clearing conditions for renting capital from both firms are given by:

$$\int_0^1 K_{i,t}^{re} di = \eta K_t^{re} = \eta u_t^{re} \widetilde{K}_t^{re} \tag{A30}$$

and

$$\int_0^1 K_{i,t}^{se} di = \eta K_t^{se} = \eta u_t^{se} \widetilde{K}_t^{se} \tag{A31}$$

Finally, the total transfers from households to the firms and the shadow bank must satisfy:

$$W^e = \eta W_t^{re,e} + (1-\eta) W_t^{se,e} \tag{A32}$$

Appendix B. Model Parameters

Agents	Value	Source	Description
Households			
β	0.9925	Funke & Paetz (2012)	Discount factor
σ_c	1	Funke & Paetz (2012)	Risk-aversion coefficient
σ_l	1	Funke & Paetz (2012)	Curvature of disutility of labour
λ_w	1.05	Christiano et al. (2010)	Markup, workers
ι_{w1}	0.29	Christiano et al. (2010)	Weight of wage indexation to steady-state inflation
ξ_w	0.75	Our calibration	Fraction of households that cannot reoptimize wage
b	0.8	Christiano et al. (2010)	Habit persistence in consumption
Firms			
α	0.5	Funke & Paetz (2012)	Capital share in the production function
ξ_p	0.75	Erceg et al. (2000)	Fraction of firms that cannot reoptimize
ι_1	0.16	Christiano et al. (2010)	Weight of price indexation to steady state inflation
λ_f	1.2	Christiano et al. (2010)	Markup, intermediate good firms
S''	29.3	Christiano et al. (2010)	Curvature of investment adjustment cost function
δ	0.03	Funke & Paetz (2012)	Depreciation rate of capital
ρ	0.78	Our calibration	Degree of substitutability between capital services
Entrepreneurs			
σ_a^{re}	18.9	Christiano et al. (2010)	Curvature of capital utilization cost function (high-risk firm)
σ_a^{se}	18.9	Christiano et al. (2010)	Curvature of capital utilization cost function (low-risk firm)
μ	0.3	Our calibration	Fraction of realized profits lost in bankruptcy
σ_S	$\sqrt{0.68}$	Our calibration	Steady state standard deviation of productivity shock
$W^{e,re}$	0.22	Christiano et al. (2010)	Initial transfer from households to high-risk firms
$W^{e,se}$	0.14	Christiano et al. (2010)	Initial transfer from households to low-risk firms
γ^{re}	0.95	Our calibration	Survival probability of high-risk firms
γ^{se}	0.96	Our calibration	Survival probability of low-risk firms
η	0.35	Our calibration	Share of high-risk firms
Banks			
ρ_χ^{rb}	0.9	Verona et al. (2013)	Degree of persistence of optimism (commercial banks)
ρ_χ^{cb}	0.8	Our calibration	Degree of persistence of optimism (shadow banks)
α^{rb}	40	Verona et al. (2013)	Sensitivity of optimism to low-risk firm's net worth
α^{sb}	0.9	Our calibration	Sensitivity of optimism to high-risk firm's net worth
$\bar{\chi}^{rb}$	0	Verona et al. (2013)	Steady state level of optimism (commercial banks)
$\bar{\chi}^{sb}$	0	Our calibration	Steady state level of optimism (shadow banks)
ε^l	389	Our calibration	Steady state level of lending rate elasticity
ε^d	-222	Our calibration	Time-invariant deposit rate elasticity
σ^{se}	1.5504	Our calibration	Standard deviation of shadow banks' default rate
c_d	0.01%	Our calibration	Cost parameter in the formal banking sector (deposit side)
c_l	0.0158%	Our calibration	Cost parameter in the formal banking sector (lending side)
Policy and shock processes			
ϕ_l^{cb}	0.80	Our calibration	Window guidance: sensitivity to policy rate variations
ϕ_l^l	0.30	Our calibration	Window guidance: sensitivity to loans variations
ϕ_l^π	1.80	Our calibration	Window guidance: sensitivity to inflation variations
ϕ_l^y	0.1	Our calibration	Window guidance: sensitivity to output variations
κ^l	$1 * \varepsilon^l$	Our calibration	Tightness of lending rate regulations
κ^w	0.40	Our calibration	Window guidance sensitivity parameter
ν	0.2	Actual data	Reserve ratio
$\tilde{\rho}$	0.9	Our calibration	Policy rate autoregressive parameter
ρ_a	0.9	Our calibration	Autoregressive parameter of technology shock
α_π	1.8	Our calibration	Responsiveness of the Taylor rule to inflation
α_y	0.1	Our calibration	Responsiveness of Taylor rule to the output gap

Appendix C. Model Solution Algorithm

The toolkit which solves the model with occasionally binding constraints has been proposed by Guerrieri and Iacoviello (2015). This piecewise algorithm simulates a model with occasionally binding constraints as a model with two regimes. Under the "reference" regime the constraint is slack whereas under the "alternative" regime the constraint binds. In each regime the model is linearized around the non-stochastic steady state. The implementation of the algorithm entails the consideration of two important conditions:

1. The Blanchard-Kahn conditions are fulfilled in the reference regime.

2. If the model moves away from the reference regime, it will return to the reference regime in finite time under the assumption that the agents expect no future shocks.

Having these two conditions in mind, we can move on to defining both regimes. When the occasionally binding constraint is slack, then $g(E_t X_{t+1}; X_t; X_{t-1}) \leq 0$ and as a result the linearized system of equilibrium conditions could be expressed in the following manner:

$$A E_t X_{t+1} + B X_t + C X_{t-1} + \varepsilon \epsilon_t = 0 \quad \text{(C1)}$$

where A, B and C are $n \times n$ matrices of structural parameters, ϵ is an $n \times m$ zero-mean vector of innovations whereas ε is an $n \times m$ matrix of structural parameters. In the alternative regime the constraint is binding and as a result $h(E_t X_{t+1}; X_t; X_{t-1}) > 0$ and the linearized system can be expressed as follows:

$$A^* E_t X_{t+1} + B^* X_t + C^* X_{t-1} + D^* + \varepsilon^* \epsilon_t = 0 \quad \text{(C2)}$$

where A^*, B^* and C^* are $n \times n$ are again matrices of structural parameters, D^* arises from the fact that the linearization is carried out around the steady state where the reference regime applies and again ε^* is an $n \times m$ matrix of structural parameters. The solution of the system is given by the following system of equations:

$$X_t = Pt X_{t-1} + Rt + Q_1 \epsilon_1 \quad for\ t = 1 \quad \text{(C3)}$$

$$X_t = P_t X_{t-1} + R_t \quad for\ \forall t \in [2; \infty] \quad \text{(C4)}$$

This mirrors the familiar decision rules of a linearized dynamic system where at each point in time the matrices P_t, Q_t and R_t are time varying.

The algorithm itself works in the following fashion. First and foremost, choose an initial guess about which regime (reference or alternative) in which period applies. Second, verify whether convergence is attained in the following manner:

1. Let T be the date when the current guess implies that the model returns to the reference regime. Then for any $t \geq T$ standard perturbation methods lead to a linear approximation for the decision rule for X_t as a function of X_{t-1} so that:

$$X_t = PX_{t-1} + Q\epsilon_t \tag{C5}$$

   Then, using the notation from equation (C4), for any $t \geq T$, $P_t = P$ and $R_t = 0$.

2. Using $X_t = PX_{t-1}$ and equation (C2) and conditional on the fact that agents expect no further shocks to occur, the solution for period $t-1$ is given by:

$$A^*PX_{T-1} + B^*X_{T-1} + C^*X_{T-2} + D^* = 0 \tag{C6}$$

   and thus

$$X_{T-1} = -(A^*P + B^*)^{-1}(C^*X_{T-2} + D^*) \tag{C7}$$

   and as a result $P_{T-1} = -(A^*P + B^*)^{-1}C^*$ and $R_{T-1} = -(A^*P + B^*)^{-1}D^*$.

3. Using $X_{T-1} = P_{T-1}X_{T-2} + R_{T-1}$ and either equation (1) or (2), as implied by the current guess of regime, solve for X_{T-2} given X_{T-3} in exactly the same fashion as solving for X_{T-1} given X_{T-2}.

4. Iterate backwards until you reach X_0, applying either equation (1) or (2) at each iteration, as implied by the current guess of regimes.

5. Depending on our guess on which regime (reference or alternative) applies to period 1, we obtain: $Q_1 = -(AP_2 + B)^{-1}\,\varepsilon$ or $Q_1 = -(A^*P_2 + B^*)^{-1}\,\varepsilon^*$. Quite trivially, if the constraint is always slack, $Q_1 = Q$.

6. Using the solution obtained in steps 1 to 5 to compute paths for X, verify the current guess of regimes. If the guess is verified (convergence is attained based upon a specified criterion which is normally a very small number), then stop! If no convergence is attained, then update the initial guess about which regime (reference or alternative) in which period applies and return to step 1.

Given X_0 and ϵ_1, an appropriate choice for the initial guess of regimes could be obtained by the standard linear perturbation solution to the reference regime. Thereafter, a choice for updating the initial guess that is generally regarded as robust is to use the path from the previous iteration which becomes now the new guess of regimes.