

HONG KONG INSTITUTE FOR MONETARY RESEARCH

**GOOD HOUSEKEEPING? REPUTATION,
FIXED EXCHANGE RATES, AND THE
'ORIGINAL SIN' PROBLEM**

Prasanna Gai & Kang-yong Tan

HKIMR Working Paper No.8/2004

April 2004



Hong Kong Institute for Monetary Research

(a company incorporated with limited liability)

All rights reserved.

Reproduction for educational and non-commercial purposes is permitted provided that the source is acknowledged.

Good Housekeeping? Reputation, Fixed Exchange Rates, and the ‘Original Sin’ Problem

Prasanna Gai

Australian National University

and

Hong Kong Institute for Monetary Research

and

Kang-yong Tan*

Australian National University

April 2004

Abstract

This paper examines how the choice of exchange rate regime can signal financial rectitude and, in so doing, influence a country’s ability to borrow internationally in domestic currency. We develop a model in which the constant probability of a ‘type change’ creates incentives for disciplined policymakers to fix the exchange rate in an effort to separate themselves from more opportunistic types. Because the track record of a policymaker is imperfectly observable, reputational incentives depend on the past behaviour of previous generations and there is hysteresis in the updating behaviour of creditors. ‘Original sin’ – the inflationary track record of one’s predecessors – can reverberate over time leading creditors to be wary about extending sovereign loans in domestic currency terms. Our findings seem consistent with the pattern of the currency composition of debt in Japan and Russia at the turn of the nineteenth century.

JEL Classification: F33, F34

* We thank the Hong Kong Institute of Monetary Research for its gracious hospitality during the preparation of this paper. The helpful comments of Gavin Cameron, Michael Chui, Warwick McKibbin, and Hyun Shin are also gratefully acknowledged. The usual caveat applies. Corresponding author: Prasanna Gai, Division of Economics, Research School of Pacific & Asian Studies, Australian National University, ACT 0200, Australia. Email: prasanna.gai@anu.edu.au.

1. Introduction

In a recent paper, Eichengreen and Hausmann (1999) highlight a striking characteristic of the sovereign bonds issued by emerging market countries on global capital markets. Foreign lenders seem generally unwilling to lend to emerging market countries in the domestic currency of these countries or, equivalently, to stand on the other side of a hedge contract. The inability to borrow abroad, and long term, in domestic currency exposes emerging market countries to currency mismatches that exacerbate financial instability. Table 1 shows that present shares of emerging market external debt denominated in own currency are extremely small, especially when compared with industrialised economies. Indeed, very few countries have been able to issue bonds in local currency terms since the start of the twentieth century.¹

Despite being an important facet of financial stability, there are relatively few explanations as to why some countries have traditionally been able to borrow abroad in their own currency, while emerging market countries have not.² Eichengreen and Hausmann advance “*original sin*” as one possible reason. They note that some countries (e.g. Australia) were able to develop domestic debt markets and create a constituency against opportunistic management of the exchange rate, whereas others (e.g. Argentina) found it difficult to do so.³ A history of high inflation and depreciation is held out as a key reason behind creditors’ unwillingness to lend in a unit that the borrower can manipulate. As Bordo and Rockoff (1996) and Obstfeld and Taylor (2003) emphasise, a country might therefore favour a fixed exchange rate regime because it serves as a “*good housekeeping seal of approval*” – a signal to creditors of sound financial policies.

The experiences of Japan and Russia at the turn of the nineteenth century illustrate how reputation in one sphere of policy (the monetary framework) spills over to other spheres (capital market access). In both countries, opportunistic devaluation by policymakers and monetary instability led creditors to lend in foreign currency terms, or to insist on specie and exchange rate clauses in debt contracts. These measures allowed creditors to extract payments in gold or hard currency in the event of devaluation. In an effort to build a reputation for creditworthiness, both Japan and Russia adopted the gold standard in 1897. Despite adhering to gold for almost two decades, neither country was readily able to engage in own-currency borrowing.

This paper examines how the choice of exchange rate regime can signal a reputation for financial rectitude and, in so doing, influence the currency composition of debt. Existing research (e.g. Eichengreen

¹ Bordo and Flandreau (2001) suggest that the number has increased to about twenty-five, from eight countries in 1914. To help combat the problem, several countries in the Asia-Pacific established a US\$1bn fund in June 2003 to purchase sovereign and high quality corporate bonds in the local currencies of countries in the pool. The fund is to be managed by the Bank for International Settlements, with capital from the reserves of the major regional central banks, including Japan, Australia, and Hong Kong.

² Recent attempts include Chamon and Hausmann (2002), Eichengreen et al. (2002), and Jeanne (2002).

³ For a comprehensive discussion of the nineteenth century experiences of Australia and Argentina, see Davis and Gallman (2001) and Schwartz (1989).

et al., 2002; Hausmann et al., 2001) has not considered the forces that determine country reputations and their influences on the (in)ability to borrow abroad in domestic currency.⁴ Recent advances in the game-theoretic analysis of reputation (Tirole, 1996; Mailath and Samuelson, 2001; Tadelis, 1999), however, open the door for such analysis. These models stress that the identities of key agents in the economy change over time. In an open economy it means that policymakers running a country can be replaced periodically, in contrast to the standard treatment of reputation in macroeconomics (e.g. Backus and Driffill, 1985) where government ‘type’ is treated as permanent.⁵ The constant possibility of a type change creates a desire among ‘disciplined’ policymakers to separate themselves from ‘opportunistic’ types, leading to equilibria where reputation is gradually built and maintained.

We develop a model of a small open economy that builds on these insights. Each time creditors extend loans to a country, they assign a probability to the policymaker being disciplined (as opposed to opportunistic) about maintaining the value of the exchange rate and, based on these beliefs, choose the currency composition of the debt. Creditors then receive repayments, but are unable to distinguish whether payments arise from good fortune or good economic management. Following payment, they properly observe the nature of the macroeconomic shock and update their beliefs about the type of policymaker with whom they are dealing. But since policymaker types can change over time in ways that are not transparent to lenders, there is a possibility that subsequent lending may involve a different kind of policymaker. Creditors, thus, constantly update their beliefs about the type of policymaker they face. Updating causes reputations to have value, with the premium from having a good record determined by creditors’ perceptions of the proportion of disciplined types in the population.

In such a setting, the complementarity between past and present behaviour raises the possibility of multiple equilibria. Specifically, there may be up to three steady state *Markov perfect* equilibria depending on parameter values. In the first, disciplined policymakers always maintain a fixed exchange rate regardless of their records. In the second, disciplined policymakers always act opportunistically, despite their track records. And in the third, policymakers fix only if they have a good record to maintain. Our analysis suggests that original sin – the track record of one’s predecessors – generates a persistence in creditors’ willingness to lend in foreign currency terms. Past behaviour, by shaping the way that achievements are interpreted, influences current reputational incentives. The hysteresis generated by collective reputations means that the length of time on a fixed exchange rate needed to build a reputation high enough to issue domestic currency debt may be substantial.

⁴ Jeanne (2002) is an exception. He considers the effects of monetary credibility on original sin, but does not explicitly consider how reputations are formed.

⁵ Standard models of reputation require the presence of a ‘tough’ or ‘Stackelberg’ type who can credibly commit to a particular action (e.g. zero inflation). Weak policymakers then acquire reputations by masquerading as (or pooling with) tough types. As Mailath and Samuelson (2001) observe, such models rely crucially on agents believing in the possibility of a Stackelberg type.

The approach adopted in our paper has parallels in the literature on reputation in sovereign debt.⁶ Grossman and Van Huyck (1993) analyse a model in which sovereign debt in local currency serves to shift the risk associated with the unpredictability of tax revenues from the debtor to its creditors. As in our model, they show how reputation can support a ‘risk shifting’ equilibrium, in which local currency debt is issued. In the reputational equilibrium, the amount of local currency debt is such that the short-run gains from repudiation via unexpected devaluation are smaller than the long-run costs from the loss of a trustworthy reputation. But their model lacks sufficient structure to pin down the inflation rate and does not explain how reputations are built – the analysis assumes that the length of time over which lenders remember a repudiation is an exogenous, random, variable.

In another related paper, Cole et al. (1995) develop a model in which governments attempt to regain access to international credit markets by making partial repayments on old debt. They argue that such settlements served as a signal of financial probity, and also motivate such signalling by assuming that government type changes unobservably over time. Access to the loan market is regained once a disciplined type pays enough to distinguish himself from other types. As a result, sovereign debtors are able to resume borrowing fairly quickly.⁷ But the length of time that elapses before emerging market countries are able to issue local currency debt implies that past behaviour by policymakers may play a more important role than hitherto suggested.

The paper is structured as follows. In Section 2 we motivate the model by examining the experiences of Japan and Russia in the lead-up to the adoption of the gold standard by both countries in 1897. In Section 3 we set out the model, establish conditions when a good reputation has value, and illustrate how hysteresis in the updating behaviour of creditors influences the currency composition of debt. A final section concludes.

2. Reputation and the Gold Standard: Japan and Russia at the Turn of the 19th Century

The experiences of Japan and Russia at the end of the 19th century illustrate how adherence to well understood monetary rules can influence investors’ perceptions of a country. Both countries joined the gold standard in the same year (1897), but whereas Japan was a relative newcomer to international capital markets, Russia was a seasoned borrower already able to borrow in domestic currency terms. In each case, policymakers sought membership of the gold standard to build a reputation for financial probity. The financial history of both countries has been well chronicled (e.g. Adams, 1964; Sussman and Yafeh, 2000 for Japan; and Crisp, 1953; Bloomfield, 1963 for Russia). Our treatment will, therefore, be brief and highlight links between the exchange rate regime and investor perceptions of the two countries.

⁶ Ball (1995) develops a related model of reputation in monetary policy to explore inflation persistence, but follows the Backus-Driffill approach in assuming the presence of a ‘Stackelberg’ type. Drazen and Masson (1994) also take a similar approach and, moreover, model the persistent effects of policy via the *structure* of the economy rather than the track records of previous generations. A more explicit treatment of collective reputation in monetary policy is offered by Sibert (2002). Her focus, however, is on the design and voting intentions of monetary policy committees.

⁷ For example, Cole et al. cite the case of Uruguay which, after defaulting in 1878, was able to borrow anew by 1888 after reaching a settlement with old bondholders.

2.1 Japan

Following an extended period of isolation, Japan experienced rapid changes in its financial arrangements during the Meiji Restoration (1868-1912). The government devalued the silver standard and, in 1869, reneged on all silver contracts by rewriting them in terms of gold. Japan attempted to fix to the gold standard in 1871, but the simultaneous issue of inconvertible paper money and silver coin meant that, following a decline in the world price of silver, foreign payments were made in the more valuable metal. The large-scale outflow of gold coin forced Japanese policymakers back to a silver standard in 1882, but the secular deterioration in the world price of silver and expansionary monetary policy to finance government expenditure contributed to a de facto depreciation of the currency from the mid-1880s until 1897.

Japan began to access the international capital market in 1870, issuing a bond of £1 million in London. The issue called for a (high) interest rate of 9%, had a maturity of 13 years, and required customs revenue as security. After issuing a further £2.4 million of bonds in 1873, the government withdrew from capital markets due to the weakness in the yen and fear of creditor sanctions in the event of payments difficulties (Patrick, 1967; Sussman and Yafeh, 2000). Lockwood (1954) also discusses how foreign investors were discouraged by currency instability as policymakers experimented with exchange rate policy before 1897.

The desire to finance armaments led the government to once again tap international capital markets at the end of the nineteenth century. The depreciation of silver meant a rising cost of military imports from gold standard countries. Following the Sino-Japanese War in 1895, from which Japan exacted a convertible-sterling indemnity worth some 30% of national income to bolster reserves, Japan adopted the gold standard in 1897. But exchange rate policy was also a central plank in a more comprehensive approach towards financial development. A new generation of policymakers under Finance Minister Matsukata had begun to stress the importance of establishing Japanese government bonds as an international commodity (Matsukata, 1899). And special credit banks were created whose role was to encourage foreign capital inflows and provide impetus for the development of a market for domestic bonds (Patrick, 1967).

As Sussman and Yafeh (2000) note, adoption of the gold standard in 1897 improved investor perceptions. The risk premium on foreign currency debt fell from approximately 4 percentage points to 2 percentage points, maturities lengthened, unsecured issues became possible, and capital inflows increased markedly. Table 2 details Japanese foreign bond issues from 1870-1914. As can be seen, bond issues were in foreign currency terms (sterling or francs).

Foreign investors continued to lend to Japan during the 1904-5 war with Russia, though spreads rose sharply and customs revenues were once again sought as collateral. Following the victory, the government continued to access foreign currency debt, including via the issue of debentures by the special credit banks. Sussman and Yafeh observe that Japan's reputation for sound finances during the war period was such that it was able to withstand subsequent investor concerns regarding the deterioration of the fiscal position. Although Japan was able to adhere to the gold standard for 17 years and continued to

do so after World War I, debt continued to be denominated in foreign currency terms.⁸ Exchange rate risk seems to have remained an issue in the minds of investors, despite the prolonged maintenance of exchange rate stability.

2.2 Russia

In contrast to Japan, Russia was a seasoned borrower in international capital markets by the end of the 19th century and able to issue bonds in foreign and local currency terms from the early 1800s. Her bonds had exchange rate or metallic clauses in some cases but no clauses in others. Table 3 provides details of selected bond issues for the period 1864-1909.

A feature of Table 3 is the pattern of the currency composition of debt. Between 1864 and 1887, there were a number of bond issues in paper roubles. From 1887 onwards, however, the majority of issues was in foreign currency or metallic terms. In part, this pattern reflects Bismarck's 1887 ban on German purchases of Russian government bonds, but it also reflects an increase in exchange rate risk as loans issued and payable in roubles became objects of speculation by foreign investors on the Berlin bourse.⁹ Speculation was heightened by the appointment of Vyshnegradsky, a finance minister noted for opportunistic intervention in the foreign exchange market. Koppl and Yeager (1996) estimate the persistence (or long memory) implicit in the rouble. They find that measured persistence between 1887-92 was markedly greater than in the early 1880s, suggesting currency instability was due to policy opportunism. Gregory (1979) also notes that the standard deviation of the exchange rate during the 1886-90 period rose sharply compared with the period 1881-85. To increase investor confidence, the authorities sought (under Finance Minister Witte) to actively stabilise the currency in 1894 and began to make payments in gold, rather than roubles.

Like Japan, Russia formally joined the gold standard in 1897. Crisp (1953) and Drummond (1976) describe how the desire for improved capital market access was paramount in the minds of policymakers. Adherence to the gold standard had an immediate effect. Capital inflows increased sharply, and the cost of borrowing on public debt fell from around 4.2% in 1891 to 3.9% in 1903. Nonetheless, although Russia was also on the gold standard for a further seventeen years, investors continued to question the commitment to gold. Bloomfield (1963) documents how, in 1905, foreign investors worried that Russia would devalue or return to floating exchange rates. Although the gold commitment regained some credibility in 1906 with an emergency loan from a consortium of European private banks, foreign lenders were unwilling to remove exchange rate clauses from debt contracts, suggesting that Russia had begun to suffer from 'original sin'.

⁸ Moulton (1944) suggests that some yen-denominated debt was sold to foreign investors after the adoption of the gold standard, though it is unclear if these had associated exchange rate clauses. Nonetheless, these amounts were dwarfed by the size of foreign currency-denominated debt.

⁹ Berlin speculators offered German bonds as collateral for loans in paper roubles from the Russian government or state-owned entities. These 'credit' loans were then used to purchase more German bonds to repeat the process. Roubles would thus accumulate in Berlin and, following a price fall, speculators would redeem their mark-denominated securities for more roubles than had been loaned, making a profit in the process.

3. The Model

3.1 Building Blocks

Consider a small open economy that must borrow to produce output. Time is discrete and has an infinite horizon. The economy is run by a group of policymakers of unit mass who are matched, at each interval $t = 0, 1, 2, \dots, \infty$, with a corresponding mass of atomistic creditors. Policymakers differ in their behavioural preferences and belong to two indistinguishable types disciplined or *D-types* in proportion Δ , and opportunistic or *O-types* in proportion $1 - \Delta$. *D-types* face a lower cost of maintaining fixed exchange rates than *O-types*, but incur higher costs if they renege on their commitment to the peg. As noted in the introduction, this may reflect different attitudes to the presence of currency mismatches in the economy. The distribution of types is assumed to be constant over time.

The tenure of a policymaker follows an exponential distribution, i.e. a policymaker alive at time t remains in office up to at least $t + 1$ with an exogenous probability $1 - \lambda \in (0, 1)$. If a policymaker loses office, he is replaced by a successor so that only a single generation is in control during any one period. Creditors cannot observe the exit or replacement of the policymaker and, at the start of each date, are unsure whether a policymaker has been ‘reincarnated’ as another type. The idea is that while a change in government is usually observable, shifts in internal politics and lobbying activity are less so. For example, a government may replace the central bank governor or a finance minister without any outward signs of a shift in policy. But creditors know that such replacements are possible and take this into account when forming expectations and making decisions.

We suppose that the policymaker minimises a loss function of the form

$$W_t = (y_t - \tilde{y}) + \frac{1}{2}\pi_t^2 + C(\pi_t), \quad (1)$$

where y_t and \tilde{y} are real and target output, π_t is the rate of inflation, and $C(\pi_t)$ reflects the fixed costs of maintaining (or abandoning) a commitment to a fixed exchange rate regime. Following Backus and Driffill (1985), we make the simplifying assumption that the loss function is linear in output. If PPP holds, and with suitable normalisation of the foreign price level, the inflation rate corresponds to the realised rate of currency depreciation so that $\pi_t = 0$ for a fixed exchange rate regime. The function $C(\pi_t)$ is of the form:

$$C(\pi_t) = \begin{cases} (1 - \theta_i)\bar{c} & \pi_t > 0 \\ \theta_i & \text{if } \pi_t = 0, \text{ where } i = D, O. \\ (1 - \theta_i)\underline{c} & \pi_t < 0 \end{cases} \quad (2)$$

In what follows $0 < \theta_D < 1$, $\theta_O = 1$, $\bar{c} > 0$, and $\underline{c} = 0$. The assumption that $\underline{c} = 0$ is made for analytical tractability and does not entail any loss of generality.

The per-period output of the economy is influenced by the amount of the loan, L_t , that the policymaker is able to borrow from his creditors. To highlight the role of reputation, we consider only short-term debt and exclude the possibility that output can be stored or invested. So a country borrows for a project, the loan becomes due, and then further borrowing is needed for subsequent output. We therefore suppose

$$y_t = L_t - \varepsilon_t, \quad (3)$$

where ε_t is a conditional i.i.d. supply shock with zero mean that cannot be observed by creditors until the end of the period.¹⁰ In keeping with the time inconsistency literature, there is a wedge between desired output and the ‘natural’ output made possible by borrowing, so that $\tilde{y} - L_t = k$.

When extending loans to the country, creditors must decide whether to lend in domestic currency or foreign currency terms. Under the assumption of uncovered interest parity and normalising real foreign interest rates to be zero ($r_t^* = 0$), we can express the real burden of debt as:¹¹

$$L_t [m(1 + \pi_t) + (1 - m)(1 - (\pi_t - \pi_t^e))], \quad (4)$$

where m is an indicator function such that

$$m = \begin{cases} 1 & \text{if foreign currency debt} \\ 0 & \text{if local currency debt} \end{cases}.$$

Notice that an unexpected depreciation lowers the real burden of domestic currency debt, whereas an anticipated depreciation has no effect. By contrast, depreciation (whether unanticipated or anticipated) raises the real burden of foreign currency debt. Clearly if the policymaker was committed to maintaining a fixed exchange rate regime, $\pi_t = \pi_t^e = 0$, then the real burden of the debt would be L_t regardless of the currency composition. Thus, by lending in foreign currency the creditor is less exposed to policymaker opportunism – he receives L_t if the *D-type* commits to the peg, compared with $L_t(1 + \pi_t)$ if the *D-type* floats. By choosing to lend in domestic currency, the creditor receives $L_t - L_t(\pi_t - \pi_t^e)$ if the *D-type* reneges on his commitment to fix the exchange rate.

Net output in each period is therefore

$$y_t = L_t - L_t [m(1 + \pi_t) + (1 - m)(1 - \pi_t + \pi_t^e)] - \varepsilon_t. \quad (5)$$

In order to service debt at the end of period t , output must be sufficient to meet the real debt burden, so

$$\varepsilon_t^* = L_t [1 - m(1 + \pi_t) - (1 - m)(1 - \pi_t + \pi_t^e)] \quad (6)$$

¹⁰ We abstract from competitiveness effects on output in order to simplify the algebra and focus attention on reputational forces.

¹¹ See Falcetti and Missale (2002) for a similar approach.

is the realisation of the supply shock that exhausts the debtor's surplus. We abstract from the problem of a sovereign's willingness to pay (e.g. Eaton and Gersovitz, 1981) and assume that creditors are able to make the country pay all it can. Debt is repaid in full if $\varepsilon_t \leq \varepsilon_t^*$, whereas partial payments are made if $\varepsilon_t > \varepsilon_t^*$. Accordingly, the critical value of L_t associated with ε_t^* is

$$L_t^* = \frac{\varepsilon_t^*}{[1 - m(1 + \pi_t) - (1 - m)(1 - \pi_t + \pi_t^e)]}. \quad (7)$$

If ε_t is uniformly distributed with sufficiently wide support, $\varepsilon_t \sim U[-Z, Z]$, then the probability of a *good* payments outcome for the creditor is $\Pr[G] = \Pr[\varepsilon_t \leq \varepsilon_t^*] = \frac{Z + \varepsilon_t^*}{2Z}$ and, conversely, the probability of a *bad* payments outcome is $\Pr[B] = \Pr[\varepsilon_t > \varepsilon_t^*] = \frac{Z - \varepsilon_t^*}{2Z}$. Let $h \in \{G, B\}$ denote the payments track record of the policymaker.

If we ignore the fixed cost term $C(\pi_t)$, the first-order condition to the minimisation problem implied by equations (1) and (5) balances the net output gain from unexpected inflation against the cost of an extra unit of inflation at the margin.

So the policymaker chooses

$$\pi_t = (2m - 1)L_t, \quad (8)$$

allowing the *ex post* policy losses under the flexible and fixed exchange rate regimes to be characterised as

$$\begin{aligned} W_{t,flex} &= -L_t [m(1 + (2m - 1)L_t) + (1 - m)(1 - (2m - 1)L_t + \pi_t^e)] \\ &\quad - \varepsilon_t - k + \frac{1}{2}L_t^2(2m - 1)^2, \end{aligned}$$

and

$$W_{t,fix} = -L_t [m + (1 - m)(1 + \pi_t^e)] - \varepsilon_t - k.$$

A policymaker will choose to devalue if

$$W_{t,fix} - W_{t,flex} > (1 - \theta_i)\bar{c} \quad (9)$$

Since $W_{t,fix} > W_{t,flex}$, for the *O-type*, he always prefers to opportunistically manipulate the currency. In contrast, *D-types* face a choice between rules and discretion. The *D-type* prefers to maintain a fixed exchange rate regime if the size of the debt (and hence the output shock, ε_t) is not too large. In particular, the peg is maintained if $L_t \in [0, \bar{L}]$, where

$$\bar{L} = \sqrt{\frac{(1 - \theta_D)\bar{c}}{(2m - 1)(m - 0.5)}}. \quad (10)$$

Notice that $\bar{L} = \sqrt{2(1 - \theta_D)\bar{c}}$ when $m = 0$ and also when $m = 1$, i.e. the thresh-old point at which the *D-type* devalues is the same *regardless* of the currency composition of the debt.

Creditors are thus faced with both adverse selection and moral hazard. As they cannot observe the replacement of policymakers, they cannot recognise the *type* they are dealing with. And since creditors cannot see the supply shock, they are unsure if their repayments reflect a poor outturn of nature or wilful devaluation by the policymaker. Moreover, since there is a continuum of myopic creditors, no single creditor is able to individually affect the play of the policymaker or the future play of the game. The only concern for the creditor is the probability he assigns to the policymaker delivering a *good* payments outcome in each period. So whenever a creditor is matched with a policymaker, he forms a conjecture about the composition of the policymaking group and their past and present behaviour based on the observed track record of debt repayment.

Events in each period unfold as follows. At the beginning of period t , creditors are matched with policymakers and extend loans. They assign a probability, ϕ_t , to the policymaker being disciplined and, based on these beliefs, choose the currency composition of the debt, $m(\phi_t)$. The output shock is observed by policymakers, who make their exchange rate choices. Creditors then receive their repayments from the output that is produced. At this stage, they are able to observe the realised value of the output shock and update their beliefs about the type of policymaker they are facing. At the end of the period, with probability λ , the policymaker leaves office and is replaced by a successor. The sequence of events is illustrated in Figure 1 .

3.2 The Value of a Good Reputation

We follow Tirole (1996) and analyse steady states of the model developed in Section 3.1. Since the *O-type* always sets exchange rate policy opportunistically, our attention is on the value to the *D-type* from reputation building. A key feature of the framework is the possibility that a *D-type* may always be replaced by an *O-type* at the end of each period. This provides the *D-type* with incentives to fix the exchange rate so as to separate himself from *O-types*. In so doing, the *D-type* gradually builds and develops a reputation for creditworthiness and a commitment to low inflation.

Let $\phi_{t,h} \equiv \Pr[D|h]$ be the probability that the creditor assigns to the policymaker being disciplined, given that he observes payments record, h . Upon observing a good record, the creditor's expectation of inflation is

$$\pi_{t,G}^e = \phi_{t,G} L_t (2m - 1). \quad (11)$$

Substituting $\pi_{t,G}^e$ and the expression for π_t into the loss function yields

$$W_t = -L_t [1 + \pi_{t,G}^e (1 - m)] - \frac{1}{2} L_t^2 (2m - 1)^2 - k - \varepsilon_t,$$

and taking expectations gives

$$E_{t-1}(W_t) = -L_t[1 + \pi_{t,G}^e(1 - m)] - \frac{1}{2}L_t^2(2m - 1)^2 - k.$$

So the present discounted value of losses under discretion is

$$\begin{aligned} V(\phi_{t,G}) &= -L_t[1 + \pi_{t,G}^e(1 - m)] - \frac{1}{2}L_t^2(2m - 1)^2 - k - \varepsilon_t + (1 - \theta_D)\bar{c} \\ &\quad + \frac{\delta}{1 - \delta}[-L_t[1 + \pi_{t,G}^e(1 - m)] - \frac{1}{2}L_t^2(2m - 1)^2 - k], \end{aligned}$$

which simplifies to

$$V_G = \frac{1}{1 - \delta}[-\frac{1}{2}L_t^2(2m - 1)^2 - L_t(1 + \pi_{t,G}^e(1 - m)) - k] - \varepsilon_t + (1 - \theta_D)\bar{c}. \quad (12)$$

A similar expression can be obtained for V_B . It follows that the gain from having a good record at time t is

$$V_G - V_B = \frac{L_t(1 - m)}{1 - \delta}[\pi_{t,B}^e - \pi_{t,G}^e]. \quad (13)$$

If $m = 1$ the ex post value of losses is the same, regardless of track record. Under these circumstances there are no long-term benefits to having a good track record – if creditors lend in foreign currency, the only value to a D -type from maintaining the peg is from the short-run gain from doing so.

To find $\pi_{t,h}^e$, we make use of Bayes' Rule to identify the conditional probabilities. In particular,

$$\phi_{t,G} \equiv \Pr[D | G] = \lambda\Delta + (1 - \lambda)\frac{\Delta \Pr[G | fix]}{\Delta \Pr[G | fix] + (1 - \Delta) \Pr[G | flex]}, \quad (14)$$

$$\phi_{t,B} \equiv \Pr[D | B] = \lambda\Delta + (1 - \lambda)\frac{\Delta \Pr[B | fix]}{\Delta \Pr[B | fix] + (1 - \Delta) \Pr[B | flex]}.$$

The numerator of the fraction in the second part of equation (14) represents the mass of fixing D -types with a good record, and the denominator the total mass of policymakers with good records. The term captures the creditors' perception of the proportion of disciplined 'fixers' among the population, if policymakers are known to stay in office with probability $1 - \lambda$. Better past behaviour by one's peers (reflected in a higher $\Pr[G | fix]$) raises present incentives for good behaviour. Other things equal, it raises $\pi_{t,G}^e$ and lowers $\pi_{t,B}^e$. There is thus a complementarity between past and present behaviour – when policymakers have behaved well in the past, creditors are more willing to attribute causality to past actions. A policymaker's record becomes a more informative signal of his type.

Appendix 1A derives $\pi_{t,G}^e, \pi_{t,B}^e$ in terms of the exogenous parameters of the model. Here we highlight the relationship between reputational incentives and peer group characteristics with the aid of simple

numerical examples. Group composition, Δ , influences the value of a good reputation through its effects on beliefs. Figure 2 shows that $V_G - V_B$ is concave and single-peaked as a function of Δ , reflecting the nature of the updating rules with exogenous replacements. Note $V_G - V_B = 0$ when $\Delta = 0$ and $\Delta = 1$. Intuitively, if a group becomes too homogenous then the incentives to build a reputation disappear.¹²

Changes in the likelihood of replacement, λ , have an unambiguous effect on reputational incentives (Figure 3). If replacement is certain, $\lambda = 1$, a *D-type* has no incentive to cultivate a reputation and chooses exchange rate policy in accordance with the rule specified in (9). If there are no replacements, $\lambda = 0$, policymaker type is permanent but not observable by lenders, bringing the model in line with conventional treatments of reputation (e.g. Backus and Driffill, 1985). If a *D-type* were to ever devalue, it would be regarded as an *O-type* forever. So long as the policymaker is not too impatient (i.e. fixing the exchange rate is preferred to the one-shot gain from devaluation), reputation has value and the *D-type* chooses to fix the exchange rate.¹³

3.3 Exchange Rate Choice and the Currency Composition of Debt

We now consider the role played by reputation in supporting the exchange rate choice of the *D-type* policymaker and the currency composition of debt issued by the creditors. Since the creditor's posterior probability that the policymaker is a *D-type* – the state variable, ϕ_t – completely summarises the direct effect of the past on the current environment, we focus attention on *Markov* strategies. In a Markov perfect equilibrium, policymakers minimise their loss functions, creditors' expectations are correct, and creditors use Bayes' rule to update posteriors. The posterior probability, ϕ_t , is given by

$$\begin{aligned}\phi_t &= \Pr[D | G] \times \Pr(G) + \Pr[D | B] \times \Pr(B) \\ &= [\lambda\Delta + (1 - \lambda) \frac{\Delta \Pr[G|fix]}{\Delta \Pr[G|fix] + (1-\Delta) \Pr[G|flex]}] \times \frac{Z + \varepsilon_t^*}{2Z} \\ &\quad + [\lambda\Delta + (1 - \lambda) \frac{\Delta \Pr[B|fix]}{\Delta \Pr[B|fix] + (1-\Delta) \Pr[B|flex]}] \times \frac{Z - \varepsilon_t^*}{2Z},\end{aligned}$$

and creditors' expectation of inflation is therefore

$$\pi_t^e = \phi_t L_t (2m - 1). \quad (15)$$

The complementarity between past and present behaviour, coupled with the fact that π_t^e influences the probability of good and bad states via its effects on ε_t^* , suggests the possibility of multiple equilibrium expected inflation rates in the model. Specifically, depending on parameter values, there may be up to three steady state Markov perfect equilibria:

¹² The point that the persistent possibility of a type change can sustain first-best incentives was first noted by Holmstrom (1982) in the context of the market for managerial talent.

¹³ In the Mailath and Samuelson (2001) framework, the pure strategy equilibrium without re-placements calls for the **D-type** to always devalue. Their result, however, depends on a symmetry assumption, namely $\Pr[G | fix] = \Pr[B | flex]$, which does not hold here.

- a steady state where the *D-type* always adopts a fixed exchange rate;
- a steady state where the *D-type* always adopts a floating exchange rate;
- an intermediate steady state where the *D-type* fixes if he has a good record, but floats if he does not.

Appendix 1B derives π_t^e in terms of the parameters of the model. Substituting π_t^e , along with the expression for π_t (equation 8), into the policymaker's loss function yields the realised *ex post* loss at time t when the policymaker has the option of changing the exchange rate – denote this by $W_{t, flex}(\phi_t)$. Under a fixed exchange rate, $\pi_t = 0$, so the relevant loss function is $W_{t, fix}(\phi_t)$. The *D-type* always fixes when

$$V_D(\phi_t) - V_D(\phi_t; flex) \leq 0; \quad \forall \phi_t \quad (16)$$

where

$$V_D(\phi_t) = W_{t, fix}(\phi_t) + \theta_D + \frac{\delta}{1 - \delta} (1 - \lambda) [\Pr[G | fix] V_D(\phi_{t,G}) + \Pr[B | fix] V_D(\phi_{t,B})],$$

and

$$V_D(\phi_t; flex) = W_{t, flex}(\phi_t) + (1 - \theta_D) \bar{c} + \frac{\delta}{1 - \delta} (1 - \lambda) [\Pr[G | flex] V_D(\phi_{t,G}) + \Pr[B | flex] V_D(\phi_{t,B})].$$

Equation (16) compares the present discounted value of present and future payoffs when the *D-type* always fixes with the payoffs that arise when a *D-type* opts to devalue, initially and in the future. In other words, it establishes the circumstances under which the primitive parameters governing group reputation support the choice of a fixed exchange rate regime by a *D-type*.

The cubic nature of π_t^e suggests the possibility of a third, intermediate, equilibrium in addition to the two steady states of always fixing and always floating. Here the *D-type* fixes only when he has a good record to maintain. From Appendix 1B, a necessary condition for an intermediate steady state is $a > 0$, i.e.

$$1 - \Delta + L_t(\sqrt{2(1 - \theta_D)\bar{c}} - \Delta) > 0,$$

which can alternatively be expressed as

$$\Pr[G | fix] > \frac{\Delta - 1}{S(S - \Delta)} \quad (17)$$

where $S = \sqrt{2(1 - \theta_D)\bar{c}}$. Thus a policymaker's decision to fix depends on how good past behaviour has been, i.e. on the size of $\Pr[G | fix]$. The importance of a good track record for a present policymaker, i.e. the complementarity between past and present behaviour, diminishes the more costly is an opportunistic devaluation $((1 - \theta_D)\bar{c})$, and is strengthened the greater the proportion of *D-types* in the population (Δ). Equation(17) also suggests that the supply of loans is an important constraint on the policymaker's choice. The larger the quantum of lending, L_t , the greater the importance of a good track record for current behaviour.

It remains to determine when creditors will choose to issue debt in domestic currency to a *D-type*, i.e. the circumstances under which $m(\phi_t) = 0$. Creditors will lend in domestic currency terms if the expected return from local currency debt is greater than the expected return from foreign currency debt, given a good record. So $m(\phi_t) = 0$ if:

$$\Pr[D | G] \cdot L_t + [1 - \Pr[D | G]] \cdot [L_t - L_t(\pi_t - \pi_t^e)] \geq \Pr[D | G] \cdot L_t + [1 - \Pr[D | G]] \cdot [L_t(1 + \pi_t)] \quad (18)$$

In other words, the investor's choice of the currency composition of debt depends on the inflation risk premium. In particular, $m(\phi_t) = 0$ when

$$L_t[1 - (\pi_t - \pi_t^e)] \geq L_t(1 + \pi_t),$$

or

$$\pi_t^e(\phi) - 2\pi_t \geq 0. \quad (19)$$

3.4 Building Trust

How long must a *D-type* maintain a fixed exchange rate before creditors are willing to lend in domestic currency? Let T be the number of periods of exchange rate fixing which makes a policymaker just indifferent between the ex post losses from a fixed and floating exchange rate regime. Thus at T

$$\begin{aligned} & W_{t,fix}(\phi_t) + \theta_D + \frac{\delta^T}{1 - \delta}(1 - \lambda) [\Pr[G | fix]V_D(\phi_{t,G}) + \Pr[B | fix]V_D(\phi_{t,B})] \\ & - W_{t,float}(\phi_t) - (1 - \theta_D)\bar{c} - \frac{\delta^T}{1 - \delta}(1 - \lambda) [\Pr[G | flex]V_D(\phi_{t,G}) + \Pr[B | flex]V_D(\phi_{t,B})] \\ & = 0 \end{aligned} \quad (20)$$

Substituting for equation (12), and rearranging we obtain

$$\begin{aligned} T = & \frac{1}{\ln \delta} [\ln(\frac{1}{2}L_t^2 - \theta_D + (1 - \theta_D)\bar{c}) + 2 \ln(1 - \delta) - \ln L_t - \ln(1 - \lambda) \\ & - \ln(\pi_G^e(\frac{Z}{2Z + (L_t + \pi_t^e)S} - \frac{L_t}{S}) + \pi_B^e(\frac{Z + (L_t + \pi_t^e)S}{2Z + (L_t + \pi_t^e)S} - \frac{S - L_t}{S}))] \end{aligned} \quad (21)$$

Taken together, equations (21) and (19) allow us to identify \hat{T} , the minimum number of periods of successful pegging that must elapse before creditors are indifferent between the currency composition of debt. Figure 4 illustrates this situation by plotting the net payoffs of the *D-type* over time. To the left of \hat{T} , the *D-type* prefers to fix the exchange rate and creditors lend in foreign currency terms. \hat{T} is increasing in λ and decreasing in Δ . Intuitively, as *D-types* are more likely to be replaced, creditors require the policymaker to demonstrate his commitment to the peg for much longer. And fewer periods on an exchange rate peg are needed, the greater the proportion of *D-types* in the population. Clearly, the minimum number of periods of successful pegging required to build a reputation sufficient to be able to borrow in domestic currency can be substantial.

4. Conclusion

The inability of countries at the periphery of the international monetary system to borrow in domestic currency, or to hedge exchange rate risk, exposes these economies to large-scale currency mismatches that exacerbate financial instability. One explanation for this feature of the international financial landscape is that a history of high inflation and opportunistic management of the exchange rate makes creditors unwilling to lend in a currency that the borrower can manipulate.

We show that original sin – the inflationary track record of one’s predecessors – plays an important role in shaping the currency composition of sovereign debt. A policymaker’s current incentives to manage the exchange rate are affected by his past behaviour and, because his track record is imperfectly observed by other agents in the economy, by the behaviour of his predecessors as well. This generates incentives for policymakers to try to fix the exchange rate to build a reputation for financial probity and to distinguish themselves from those who would try to opportunistically manipulate the exchange rate. Countries may, therefore, try to limit exchange rate movements to acquire a ‘good housekeeping seal of approval’, notwithstanding the costs of fixing. The complementarity between past and present behaviour means that there is hysteresis in the updating behaviour of creditors, which leads them to be wary about extending credit in domestic currency.

Our findings seem consistent with the pattern of the currency composition of debt in Japan and Russia at the turn of the nineteenth century. Policymakers actively engaged in opportunistic manipulation of the exchange rate, leading investors to lend in foreign currency terms or to stipulate exchange clauses in debt contracts. The sceptical attitude of investors towards the monetary frame-work extended well beyond the adoption of the gold standard by both countries in 1897. Although opportunistic policymakers were replaced by more disciplined types who saw commitment to the gold standard as a means of promoting capital market access and developing domestic bond markets, neither Japan nor Russia was readily able to engage in own-currency borrowing for a considerable length of time.

It is important to note that reputation is not the only factor influencing the currency composition of debt. Market microstructure and limits to portfolio diversification are also likely to be key influences on the ability of a country to issue domestic currency debt. Eichengreen and Hausmann (2003) argue that the optimal portfolio for the typical investor has a limited number of currencies. Each additional currency adds costs and risks, whilst bringing opportunities for diversification. It means that investors are likely to have a declining appetite for exotic currencies. So if a country is able to convince investors to hold its currency in their portfolios, it makes it harder for other countries to do likewise. They advocate international initiatives to help develop liquid debt markets and erode perceptions of exotic currency debt.

A number of countries in the Asia-Pacific have moved in this direction by recently establishing an Asian Bond Fund (ABF). The ABF is managed by the Bank for International Settlements and backed by the capital of high credit-rated countries such as Australia, Japan, and New Zealand. The aim is to buy sovereign local currency debt issued by Asian governments on international capital markets. Currency mismatches are effectively eliminated, since the countries backing the fund are able to hedge their exposure to the local currencies. In the medium-long term, the intention of such a fund is to encourage private sector involvement in local bond markets, increasing liquidity and improving the lending terms for governments in the region. While such international solutions may be insufficient to resolve the problem of original sin on their own, they may be a useful supplement to the development of credible institutions and policy frameworks.

References

- Adams, T. (1964), *A Financial History of Modern Japan*, Research (Japan) Ltd, Tokyo.
- Backus, D. and J. Driffill (1985), "Inflation & Reputation," *American Economic Review*, 75: 530-8.
- Ball, L. (1995), "Time Consistent Inflation Policy & Persistent Changes in Inflation," *Journal of Monetary Economics*, 36: 329-50.
- Bloomfield, A. (1963), "Short-term Capital Movements Under the Pre-1914 Gold Standard," *Princeton Studies in International Finance*, No 11, Princeton N.J.
- Bordo, M. and M. Flandreau (2001), "Core, Periphery, Exchange Rate Regimes and Globalization," NBER Working Paper No 8584, Cambridge MA: National Bureau of Economic Research.
- Bordo, M., C. Meissner, and A. Redish (2002), "How Original Sin was Overcome: The Evolution of External Debt Denominated in Domestic Currencies in the United States and the British Dominions 1800-2000," paper prepared for the IADB Conference on *Currency & Maturity Matchmaking: Redeeming Debt from Original Sin*, Washington DC, November.
- Bordo, M. and H. Rockoff (1996), "The Gold Standard as a 'Good Housekeeping Seal of Approval'," *Journal of Economic History*, 56: 389-428.
- Chamon, M. and R. Hausmann (2002), "Why Do Countries Borrow the Way They Borrow?" paper prepared for the IADB Conference on *Currency & Maturity Matchmaking: Redeeming Debt from Original Sin*, Washington DC, November.
- Cole, H., J. Dow and H. English (1995), "Default, Settlement, and Signalling: Lending Resumption in a Reputational Model of Sovereign Debt," *International Economic Review*, 36: 365-85.
- Crisp, O. (1953), "Russian Financial Policy and the Gold Standard at the End of the Nineteenth Century," *The Economic History Review*, 6: 156-72.
- Crisp, O. (1976), *Studies in the Russian Economy Before 1914*, Macmillan Press, London.
- Davis, L. and R. Gallman (2001), *Evolving Financial Markets and International Capital Flows – Britain, the Americas, and Australia 1865-1914*, Cambridge University Press, Cambridge, UK.
- Drazen, A. and P. Masson (1994), "Credibility of Policies versus Credibility of Policymakers," *Quarterly Journal of Economics*, 9: 735-54.
- Drummond, I. (1976), "The Russian Gold Standard, 1897-1914," *Journal of Economic History*, 36: 663-88.

- Eaton, J. and M. Gersovitz (1981), "Debt with Potential Repudiation: Theoretical and Empirical Analysis," *Review of Economic Studies*, 48: 289-309.
- Eichengreen, B. and R. Hausmann (1999), "Exchange Rates and Financial Fragility," in *Federal Reserve Bank of Kansas City: New Challenges for Monetary Policy*, Jackson Hole, 329-68.
- Eichengreen, B. and R. Hausmann (2003), "How to Eliminate Original Financial Sin," *Financial Times*, 22 November.
- Eichengreen, B., R. Hausmann and U. Pannizza (2002), "Original Sin: The Pain, The Mystery, and The Road to Redemption," paper prepared for the IADB Conference on *Currency & Maturity Matchmaking: Redeeming Debt from Original Sin*, Washington DC, November.
- Falchetti, E. and A. Missale (2002), "Public Debt Indexation and Denomination with an Independent Central Bank," *European Economic Review*, 46: 1825-50.
- Flandreau, M. and N. Sussman (2002), "Old Sins: Exchange Clauses and European Foreign Lending in the Nineteenth Century," paper prepared for the IADB Conference on *Currency & Maturity Matchmaking: Redeeming Debt from Original Sin*, Washington DC, November.
- Gregory, P. (1979), "The Russian Balance of Payments, the Gold Standard, and Monetary Policy: A Historical Example of Foreign Capital Movements," *Journal of Economic History*, 39: 379-400.
- Grossman, H. and J. Van Huyck (1993), "Nominally Denominated Sovereign Debt, Risk Shifting, and Reputation," *Journal of Economics & Business*, 45: 341-52.
- Hausmann, R., U. Panizza and E. Stein (2001), "Why Do Countries Float the Way They Float?" *Journal of Development Economics*, 66: 387-414.
- Holmstrom, B. (1982), "Managerial Incentive Problems – A Dynamic Perspective," in *Essays in Economics and Management in Honour of Lars Wahlbeck*, Swedish School of Economics and Business Administration, Helsinki.
- Jeanne, O. (2002), "Why do Emerging Economies Borrow in Foreign Currency?" paper prepared for the IADB Conference on *Currency & Maturity Match-making: Redeeming Debt from Original Sin*, Washington DC, November.
- Koppl, R. and L. Yeager (1996), "Big Players and Herding in Asset Markets: The Case of the Russian Rouble," *Explorations in Economic History*, 33: 367-83.
- Lockwood, W. (1968), *The Economic Development of Japan*, Princeton University Press, Princeton N.J.
- Mailath G. and L. Samuelson (2001), "Who Wants a Good Reputation?" *Review of Economic Studies*, 68: 415-41.

- Matsukata, M. (1899), *Report on the Adoption of the Gold Standard in Japan*, Government Press, Tokyo.
- Moulton, H. (1944), *Japan – An Economic & Financial Appraisal*, Brookings Institution, Washington DC.
- Obstfeld M. and A. Taylor (2003), “Sovereign Risk, Credibility and the Gold Standard: 1870-1913 versus 1925-31,” *Economic Journal*, forthcoming.
- Patrick, H. (1967), “Japan 1868-1914,” in R. Cameron, O. Crisp, H. Patrick and R. Tilly, eds., *Banking in the Early Stages of Industrialisation*, Oxford University Press, New York.
- Schwartz, H. (1989), “Foreign Creditors and the Politics of Development in Australia and Argentina, 1880-1913,” *International Studies Quarterly*, 33: 281-301.
- Sibert, A. (2003), “Collective Reputations & Monetary Policy Committees,” *Review of Economic Studies*, forthcoming.
- Sussman, N. and Y. Yafeh (2000), “Institutions, Reforms, and Country Risk: Lessons from Japanese Government Debt in the Meiji Period,” *Journal of Economic History*, 60: 442-67.
- Tadelis, S. (1999), “What’s in a Name? Reputation as a Tradable Asset,” *American Economic Review*, 89: 548-63.
- Tirole, J. (1996), “A Theory of Collective Reputations (with applications to the persistence of corruption and to firm quality),” *Review of Economic Studies*, 63: 1-22.

Appendix

1A. Creditors' Expectations of Inflation, Given Track Record

Since $\varepsilon_t \sim U[-Z, Z]$, we can calculate the conditional probabilities

$$\begin{aligned}\Pr[G \mid fix] &= \frac{\varepsilon^*}{\bar{\varepsilon}}; & \Pr[G \mid flex] &= \frac{Z}{2Z - \bar{\varepsilon}}; \\ \Pr[B \mid fix] &= \frac{\bar{\varepsilon} - \varepsilon^*}{\bar{\varepsilon}}; & \Pr[B \mid flex] &= \frac{Z - \bar{\varepsilon}}{2Z - \bar{\varepsilon}}.\end{aligned}$$

Noting that $\varepsilon_t^* = L_t[1 - m(1 + \pi_t) - (1 - m)(1 - \pi_t + \pi_t^e)]$ and $\bar{L} = \sqrt{2(1 - \theta_D)\bar{c}}$ gives

$$\Pr[G \mid fix] = \frac{L_t}{\sqrt{2(1 - \theta_D)\bar{c}}} \equiv A$$

and

$$\begin{aligned}\Pr[G \mid flex] &\equiv B \\ &= \frac{Z}{2Z - [1 - m(1 + \pi_t) - (1 - m)(1 - \pi_t + \pi_t^e)]\sqrt{2(1 - \theta_D)\bar{c}}}\end{aligned}$$

Since reputations only have value when $m = 0$, and using (11) we obtain

$$\pi_{t,G}^e = -\lambda\Delta L_t - (1 - \lambda)\frac{\Delta A L_t}{\Delta A + (1 - \Delta)B} \quad (\text{A1})$$

Rearranging the above and solving for the resulting quadratic equation gives

$$\pi_{t,G}^e = \frac{-b_G \pm \sqrt{b_G^2 - 4a_G c_G}}{2a_G} \quad (\text{A2})$$

where

$$\begin{aligned}
 a_G &= \Delta L_t^2 S, \\
 b_G &= \Delta L_t^2 [Y + S(\lambda \Delta L_t + (1 - \lambda))] + SZ(1 - \Delta), \\
 c_G &= L_t \Delta [L_t Y (\lambda \Delta L_t - \lambda + 1) + \lambda SZ(1 - \Delta)], \\
 S &= \sqrt{2(1 - \theta_D) \bar{c}}, \quad Y = 2Z + L_t S.
 \end{aligned}$$

We obtain the expression for $\pi_{t,B}^e$ in similar fashion. Specifically, we have

$$\pi_{t,B}^e = \frac{-b_B \pm \sqrt{b_B^2 - 4a_B c_B}}{2a_B} \tag{A3}$$

where

$$\begin{aligned}
 a_B &= S[S - \Delta L], \\
 b_B &= \Delta Y(S - L_t) + S(1 - \Delta)(Y - Z) + \lambda \Delta L_t S^2(1 - \Delta) + [\lambda \Delta^2 + (1 - \lambda)S](S - L_t), \\
 c_B &= \Delta[\lambda L_t S(Y - Z) + (1 - \lambda)(S - L_t)L_t Y + \lambda \Delta L_t(SZ - L_t Y)].
 \end{aligned}$$

1B. Equilibrium Expected Inflation

From (15), creditors' equilibrium expectations of inflation are given by

$$\pi_t^e = \phi L_t (2m - 1).$$

Making use of the probabilities and the expressions for S and Y in Appendix 1A, together with the expressions for ϕ_G and ϕ_B , we obtain a cubic expression for π_t^e of the form:

$$a(\pi_t^e)^3 + b(\pi_t^e)^2 + c\pi_t^e + d = 0$$

where

$$a = 4Z^2L_tS\Delta[1 - \Delta + L_t(S - \Delta)];$$

$$b = 4Z^2[L_t(\Delta(Y - S) + S^2(SY + \Delta) + \Delta^2(Z - Y) + SY\Delta(L_t - S) - S^2\Delta(SY + \Delta) + L_t\Delta(S^3 - Y\Delta + S^2\Delta) + S^2Y\Delta^2) + S^2Y(\Delta - 1)^2]$$

$$-L_t^3\Delta(1 - \lambda + \lambda\Delta) - SL_t^2[L_t(S(1 - \lambda) + \Delta(\lambda - 1)) + \lambda\Delta(L_t(S - \Delta) + 1 - \Delta)];$$

$$c = 4Z^2S[L_t(\Delta(Y - Z) + \Delta^2(Z - Y) + Y^2(S + \Delta^2 - S\Delta) + L_tY\Delta(S - Y\Delta)) - Y(Z(2\Delta - 1 - \Delta^2) + Y(\Delta - 1)^2)] - L_t^2\Delta S[L_t(1 - \lambda + \lambda\Delta) + \lambda Y(1 - \Delta)]$$

$$-L_t^2\Delta(Z - L_t^2)(1 - \lambda + \lambda\Delta) - L_t^2[L_tSY(1 - \lambda) + L_tY\Delta(\lambda - 1) + \Delta\lambda(Y - Z) + L_tY\Delta\lambda(S - \Delta) + \Delta^2\lambda(Z - Y)] - SL_t(Z + L_t^2)[L_t(S - \Delta - S\lambda + \Delta\lambda) + \Delta\lambda L_t(S - \Delta) + \Delta\lambda(1 - \Delta)];$$

$$d = L_t\Delta S[L_t(1 - \lambda + \lambda\Delta) - \lambda Y(1 - \Delta)](Z - L_t^2) + L_t[L_tY(S - \Delta) + \lambda\Delta(Y - Z) + L_tY\lambda(\Delta - S) + L_tY\Delta\lambda(S - \Delta) + \Delta^2\lambda(Z - Y)](Z + L_t^2).$$

For a cubic equation of this form, there are 6 roots. Focusing attention on non-complex roots yields

$$\pi_t^e = \left[\frac{-q \pm \sqrt{q^2 + 4p^3/27}}{2} \right]^{1/3} \quad (\text{A4})$$

where

$$p = \left(\frac{1}{a}\right) \left(\frac{-b^2}{3a} + c\right), \quad q = \frac{1}{27a^3} (2b^3 - 9abc + 27a^2d).$$