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Abstract

The paper draws a link between the conditions for low pass-through when there are flexible prices, and local-currency pricing when there are sticky nominal prices. It shows that the condition under which pass-through is less than one-half when prices are flexible is the same as the condition under which exporting firms set prices in the currency of importing countries when nominal prices must be set in advance.

JEL Classification: F10, F30, F40

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1. Introduction

A large literature has produced abundant evidence that in many export industries the pass-through of exchange rates to the final import prices is less than 100 percent. That is, prices paid by importers respond less than proportionately to exchange rate changes. Two distinct literatures have evolved that offer models of this phenomenon. One literature that focuses on industrial structure, is partial equilibrium in nature, and is more oriented toward the long run, evolved in the late 1980s and seeks to explain "pass through" and "pricing to market". Krugman (1987) and Feenstra (1989) are examples. Another literature is more macroeconomic, with a focus on nominal price stickiness. Obstfeld and Rogoff (1995, 2000), Betts and Devereux (1996, 2000) and Chari, Kehoe, and McGrattan (2002) present well-known models in this genre.

The partial equilibrium, micro-based literature investigates models of imperfect competition, in which exporting firms are able to segment markets internationally. The firm has market power, and charges a price that is marked up over unit cost. Firms are able to adjust their own prices instantaneously when there are shifts in supply and demand — that is, there is no price stickiness. There is no role for monetary values or nominal prices in these models.

In the second literature, there is nominal price stickiness. An exporting firm sets its price either in its own currency (PCP) or in the currency of the importer (LCP). Nominal prices are set in advance of shocks to demand or supply. In these models, the nominal exchange rate responds instantaneously to shocks. There are then only two possibilities for pass-through: Either there is complete pass-through under PCP, or zero pass-through under LCP.

Typically these sticky-price models are general equilibrium (macroeconomic) in nature. The currency in which prices are set is exogenous. For example, Obstfeld and Rogoff (1995, 2000) are PCP models; Betts and Devereux (1996, 2000) and Chari, Kehoe, and McGrattan (2002) assume LCP. Some work has been devoted to understanding the choice of currency for pricing exports, when prices are set in advance.[1] Giovannini (1988) presents a well-known partial equilibrium model. Some recent models have endogenized the currency in which prices are set in general equilibrium macro models.[2]

This paper is a step toward linking the two literatures. It shows that the conditions under which a firm would choose PCP vs. LCP when nominal prices are set in advance are closely related to conditions for there to be high vs. low pass-through in a flexible price model of pass-through.

Before elaborating on these links, it is helpful to highlight the fundamental differences between these two literatures. Frequently the two literatures are cited simultaneously as if they model the same phenomenon. But they do not. In the sticky price literature, partial pass-through of the exchange rate into import prices is not possible. If the price is set in the producer's currency, then when the exchange

1 I avoid the term "invoicing" since the model presented below allows a prominent role for distribution, assembly of intermediate imports, etc., using factors of production from the importing country, before the good is sold to the final user. The price is set for the final good, not the unfinished good that reaches the port of the importer.

2 Bacchetta and van Wincoop (2002) and Devereux, Engel, and Storgaard (2003). Corsetti and Pesenti (2002) is related. In that model, the percentage pass-through of the exchange rate to the final goods price is set in advance.

rate changes, the import price must change proportionately with the exchange rate. Pass-through is 100 percent. If the price is set in the importer's currency, then there is 0 percent pass-through. Those are the only two possibilities.[3] The industrial organization-based, flexible price models of partial pass-through do not even consider nominal prices. These models are usually partial equilibrium in nature, but if they were embedded in a general equilibrium framework, they would not explain nominal price stickiness. They aim to explain a change in the relative price of export goods (the price the importer pays relative to the price of other goods consumed by the importer) when there are real shocks to supply or demand. But in a monetary framework, these models could exhibit neutrality, so that a nominal shock that leads to a depreciation of the importer's currency would be accompanied by an increase in import prices (and all other prices in the importing country) that is proportionate to the depreciation.

Both types of models offer an answer to the question "What is the effect of an exchange rate change on the price of imports?" In general, that is a nonsense question: the answer depends on the source of the exchange rate change — monetary, real, supply, demand, etc. But there are two limited circumstances under which the question is reasonable. One is from the partial equilibrium standpoint of a firm that takes the exchange rate as given. The pass-through literature shows how in the long run the import price will respond to any exchange rate change. The second case in which the question makes sense is the short-run sticky price literature, in which — no matter what the source of the shock — the short-run elasticity of an import price to exchange rate changes is either zero or one.

The model set out below examines the behavior of a monopolistic exporting firm facing various stochastic shocks (to demand, costs, prices of other goods, etc.) in a static setting. Two set-ups are considered: In the first, the firm has complete flexibility in setting its price. In the second, the firm must choose a nominal price in advance. It chooses either to set prices in its own currency or the importer's currency, depending on which pricing strategy delivers higher expected discounted profits.

Let P be the price of the export good in terms of the exporter's currency, and P^* be the price expressed in the importer's currency. By definition, $P = SP^*$, where S is the nominal exchange rate. Of course, in the flexible-price setting, the currency in which the price is expressed is irrelevant. Since the exporter knows S at the time the price is set, choosing a price P^* for the importer is equivalent to choosing the price P in its own currency since the exporter can simply use the relationship $P = SP^*$ to translate the price from one currency to the other. When nominal prices must be set in advance, however, the choice of currency matters. If the exporter sets a price P in its own currency (PCP pricing), then the foreign currency price will vary ex post when the exchange rate changes ($P^* = P / S$). In that circumstance, the price paid by the importer varies when the exchange rate changes, though the producer knows with certainty the price per unit sold in its own currency. If the exporter sets the price P^* in the importer's currency, then the importer will not see any fluctuation in price when the exchange rate changes. But in terms of its own currency, the producer will see the price it receives fluctuate one-for-one with the exchange rate.

3 Corsetti and Pesenti (2002) do consider an alternative in which the export price can be indexed to the exchange rate, but there seems to be little evidence for this type of indexing in practice. Alternatively, there could appear to be partial pass-through for an industry in the short run with nominal price stickiness if the industry export price index was a weighted average of prices of firms that price in their own currency and firms that price in the importer's currency.

The theorem links the flexible price and sticky price models in a simple way, using "second-order" approximations. First, a condition is developed in the flexible price model under which $Var(p) < Var(p^*)$, where lower case letters represent the natural logs of their upper-case counterparts. That condition is equivalent to the condition that the "unconditional pass-through", defined as $\beta \equiv \frac{\operatorname{cov}(p^*, s)}{\operatorname{var}(s)}$ is greater than ½ (in absolute value).[4] Then, a condition is derived in the sticky-price model under which PCP is preferred to LCP. The two conditions — for $Var(p) < Var(p^*)$, in the flexible price model and for PCP to be preferred to LCP in the sticky price model — are identical.

The set-up and the result here are related to that in Friberg (1998). That paper also compares static flexible-price and sticky-price problems for a monopolist in partial equilibrium. However, first, the model here is more general than Friberg's, who allowed only the nominal exchange rate to be stochastic. Second, and more important, the theorem here draws a much closer link between the flexible-price and sticky-price models than Friberg. Friberg shows that there is a condition that is both sufficient for firms to choose LCP under sticky prices and for pass-through to be less than 100 percent under flexible prices. By comparison, the theorem here states that the necessary and sufficient condition for LCP pricing to be optimal under sticky prices is identical to the condition for pass-through to be less than ½ under flexible prices.

The theorem here establishes a natural link between the two literatures. When the optimally chosen export price is relatively stable in the exporter's currency under flexible prices, it is optimal to set the price in the exporter's currency under sticky prices. And when the export price is more stable in the importer's currency under flexible prices, LCP is optimal when prices must be set in advance.

2. Flexible-Price Pass-Through Problem

Consider the price setting decision for a firm that sells only in the foreign market. This firm is a monopolistic producer of its product. The firm can choose the price for its product with full information about demand, costs, etc.

We will consider the problem facing the firm of setting the nominal price for its product, taking other prices as given. Of course, since prices respond fully to all information (that is, this is not a sticky-price firm), finding the optimal nominal price and optimal real price are equivalent problems. It is helpful to express prices in nominal terms, however, to facilitate comparison with the next section in which there is nominal price stickiness.

Demand for the firm's product is of the form

$$Y^D = D(P^* / Z)N. \tag{1}$$

[4] This equivalence follows immediately because $p = s + p^*$.

Here, $P*$ is the price in foreign currency terms of the firm's product. The function $D(P^*/Z)$ means that demand for the firm's product is a function of $P*$ relative to Z. Z is a price index of products that are direct competitors to our firm's product. We assume that there are a large number of such products, and our firm is too small to affect the index Z. That is, our firm takes Z as given. N represents other factors that shift demand. These might include a shift variable related to income or tastes that affect demand through channels other than the effect on P^*/Z. N might also include Z relative to prices of other goods, or relative to a price index for all other purchases. The essential assumption about N is that the firm takes it as given.

Our assumptions imply that there are no strategic interactions between our firm and other firms. The choice of $P*$ is made taking all other prices (Z and whatever prices are included in N) as given. We have defined demand in this particular way (that is, made the distinction between Z and other exogenous variables N) because demand depends on a relative price, and we define elasticities in terms of the derivatives of demand with respect to this relative price, P^*/Z.

Define:

$$\gamma\left(\frac{P^*}{Z}\right) \equiv \frac{-\partial Y^D}{\partial(P^*/Z)} \cdot \frac{(P^*/Z)}{Y^D} = \frac{-P^*}{Z} \cdot \frac{D'(P^*/Z)}{D(P^*/Z)}$$, the elasticity of demand, and

$$\varepsilon\left(\frac{P^*}{Z}\right) \equiv \frac{\gamma'(P^*/Z)}{\gamma(P^*/Z)} \cdot \frac{P^*}{Z}$$, the elasticity of the elasticity.

Note that, following standard usage, $\gamma > 0$ (since we assume $D' < 0$), but ε could be positive or negative.

We will assume that the product is produced using two variable inputs: one is a local input, and one is a foreign input. The local input might be labor used to produce the product, and the foreign input might be foreign labor used to distribute the product, or to assemble imported intermediate goods into final products. It will become apparent that generalization to many inputs is entirely straightforward. We will assume the cost function takes the form:

$$C(Y) \cdot H(W_1, W_2). \tag{2}$$

We assume $C' > 0$. C'' may be positive or negative.[5] Y refers to output for the firm. We assume $H(W_1, W_2)$ is homogeneous of degree 1. W_i, $i = 1,2$ refer to the nominal per unit factor costs for the two factors, expressed in the currency of the producer. We will think of W_2 as the cost of the foreign input.

We use the relationships:

$P = SP^*$, where P is the price of the product in the producer's currency,

5 There is a condition for the second-order conditions for a maximum to be satisfied (see below).

$W_2 = SW_2^*$, where W_2^* is the factor cost expressed in foreign currency,

and S is the nominal exchange rate, in domestic currency per unit of foreign currency.

Define:

$$\delta(Y) \equiv \frac{C''(Y)Y}{C'(Y)}$$

$\omega(W_1, W_2) \equiv \dfrac{H_1(W_1, W_2) \cdot W_1}{H(W_1, W_2)}$. Note that because of the assumption of homogeneity of degree 1, we have $1 - \omega(W_1, W_2) \equiv \dfrac{H_2(W_1, W_2) \cdot W_2}{H(W_1, W_2)}$.

The firm's objective is to maximize nominal profits. (Since this is a full-information, flexible price problem, maximizing nominal profits is equivalent to maximizing real profits. Again, we express things in nominal terms for comparability to the problem with fixed ex ante nominal prices.) Using equations (1) and (2), profits are given by

$$\Pi(P, S, Z, N, W_1, W_2^*) = P \cdot D(P/SZ)N - C(D(P/SZ)N)H(W_1, SW_2^*) \,. \tag{3}$$

The first order condition for choosing price is (where we drop the arguments of the functions for notational convenience):

$$DN + \frac{P}{SZ}D'N - \frac{C'D'NH}{SZ} = 0 \,.$$

This can be rearranged to arrive at the familiar mark-up equation:

$$P = \frac{\gamma C'H}{\gamma - 1} \,.$$

The second order condition is:

$$\frac{2D'N}{SZ} + \frac{PD''N}{(SZ)^2} - \frac{C''(D'N)^2 H^2}{(SZ)^2} - \frac{C'D''NH}{(SZ)^2} < 0 \,.$$

With some rearranging, the second-order condition can be expressed as:

$$\Delta \equiv \gamma - 1 + \varepsilon + \delta\gamma(\gamma - 1) > 0 \,.$$

Let lower case letters be the natural logs of upper case letters. That is, define

$p = \ln(P)$, $s = \ln(S)$, $n = \ln(N)$, $z = \ln(Z)$, $w_1 = \ln(W_1)$, and $w_2^* = \ln(W_2^*)$.

Then take a first-order Taylor series approximation to $P = \dfrac{\gamma C' H}{\gamma - 1}$ to get:

$$p \cong \varepsilon(p - s - z) - \frac{\gamma\varepsilon}{\gamma - 1}(p - s - z) - \delta\gamma(p - s - z) + \delta n + \omega w_1 + (1-\omega)(s + w_2^*) .$$

Solving for p, (and replacing $\cong$ with = for convenience), we get:

$$p = \frac{\varepsilon + (\delta\gamma + (1-\omega))(\gamma - 1)}{\Delta} s + \frac{\varepsilon + \delta\gamma(\gamma-1)}{\Delta} z + \frac{\delta(\gamma-1)}{\Delta} n + \frac{\omega(\gamma-1)}{\Delta} w_1 + \frac{(1-\omega)(\gamma-1)}{\Delta} w_2^* . \quad (4)$$

Or, in terms of the importer's nominal price:

$$p^* = p - s = \frac{\varepsilon + \delta\gamma(\gamma-1)}{\Delta} z + \frac{\delta(\gamma-1)}{\Delta} n + \frac{\omega(\gamma-1)}{\Delta}(w_1 - s) + \frac{(1-\omega)(\gamma-1)}{\Delta} w_2^* . \quad (5)$$

Expressing this in terms of relative prices:

$$p^* - z = \frac{\delta(\gamma-1)}{\Delta} n + \frac{\omega(\gamma-1)}{\Delta}(w_1 - s - z) + \frac{(1-\omega)(\gamma-1)}{\Delta}(w_2^* - z) . \quad (6)$$

A standard pass-through equation such as Feenstra (1989) takes n and z as constants, and sets $\omega = 1$. In that case, the (absolute value of) the pass-through coefficient from above is:

$$\frac{\gamma - 1}{\Delta} = \frac{\gamma - 1}{\gamma - 1 + \varepsilon + \delta\gamma(\gamma-1)} .$$

Of course, when the elasticity of demand is constant, so $\varepsilon = 0$, and the cost function is linear in output, so $\delta = 0$, the pass-through coefficient is 1.

Note that only changes in real variables: n, $w_1 - s - z$, and $w_2^* - z$ will change the real price, $p^* - z$. Looking at the equation for the producer's price in his own currency, p, if the real variable, n, does not change, and the foreign currency prices z and w_2^*, do not change, but a monetary shock in the producer's country causes s and w_1 to rise by one unit, then p will rise by one unit but p^* will not change. On the other hand, if a monetary shock in the importer's country causes z and w_2^* to rise by one unit, and s to fall by one unit, p will not change, but p^* will rise by one unit.

Define $\beta_{\pi s} \equiv \text{cov}(\pi, s) / Var(s)$ for $\pi = p^*, z, n, w_1, w_2^*$. We call $-\beta_{p^*s}$ the "unconditional" pass-through elasticity because it represents the estimated pass-through elasticity from a simple regression of p^* on s, without controlling for z, n, w_1, or w_2^*. Then,

$$\beta_{p^*s} = \frac{\varepsilon + \delta\gamma(\gamma-1)}{\Delta}\beta_{zs} + \frac{\delta(\gamma-1)}{\Delta}\beta_{ns} + \frac{\omega(\gamma-1)}{\Delta}\beta_{w_1 s} + \frac{(1-\omega)(\gamma-1)}{\Delta}\beta_{w_2^* s} - \frac{\omega(\gamma-1)}{\Delta} . \quad (7)$$

It follows that $\beta_{p^*s} < -\frac{1}{2}$ (that is, the "unconditional" pass-through elasticity is $>$ ½) if and only if Condition 1 is met:

Condition 1:

$$[\varepsilon+\delta\gamma(\gamma-1)]\beta_{zs}+\delta(\gamma-1)\beta_{ns}+\omega(\gamma-1)\beta_{w_1 s}+(1-\omega)(\gamma-1)\beta_{w_2^* s}<\frac{(2\omega-1)(\gamma-1)-\varepsilon-\delta\gamma(\gamma-1)}{2}.$$

Since $p^*+s=p$, then $\operatorname{var}(p^*)+\operatorname{var}(s)+2\operatorname{cov}(s,p^*)=\operatorname{var}(p)$, so the condition $\beta_{p^*s}<-\frac{1}{2}$ is equivalent to the condition $\operatorname{var}(p^*)>\operatorname{var}(p)$.

3. Nominal Prices Set in Advance

In this section, the firm must set a price of its export in advance — before the resolution of uncertainty. It sets the price to maximize expected discounted profits (to be defined below.) The firm can set a price either in its own currency or the importer's currency. If it chooses to set a price in its own currency (we label this "PCP" for "producer-currency pricing"), then the price for the importer in the importer's currency changes one-for-one with subsequent movements in the nominal exchange rate. Conversely, if the firm sets the price in the importer's currency (designated "LCP" for "local-currency pricing"), then the price the exporter receives in his currency varies one-for-one with movements in the nominal exchange rate. The firm must decide in advance to price in either its own currency or the importer's currency.

Under PCP, expected discounted profits are given by:

$$E\Pi^{PCP}(P,S,Z,N,W_1,W_2^*,X)=E\Big[X\Big[P\cdot D(P/SZ)N-C(D(P/SZ)N)H(W_1,SW_2^*)\Big]\Big]. \tag{8}$$

In this equation, X is the discount factor, which may be stochastic. The results in this paper hold under fairly general assumptions about X — we only assume that X is exogenous for the firm. For example, if the firm is simply maximizing real profits (in terms of purchasing power of the firm's owners), then X is the inverse of the consumer price index for the firm's owners. But firm owners might be risk averse, so X for example could be the marginal utility of an increment to profit denominated in the currency of the exporter. In short, this objective for the firm holds under a variety of possible assumptions about the objectives of the firm managers and the structure of asset markets and possibilities for hedging. The assumption that X is exogenous to the firm does rule out some possibilities, however. For example, suppose a single household owns the firm, and the owner-manager discounts profits by marginal utility. The outcome for the firm might directly affect the level of consumption of the owner, and thus the marginal utility. The assumption that X is exogenous to the firm would be violated. An exogenous discount factor is more sensible when, for example, many owners own the firm, and there are many other sources of income for each owner. Thus our assumption of an exogenous discount factor is violated in the models of Feenstra and Kendall (1997) and Friberg (1998), who assume in essence that firm owners' only income is from profits (so that the firm maximizes the expected utility of profits.)

All of the other variables are defined as previously. In this section, by brute force, we derive expected profits for the firm under PCP and under LCP, and then compare the expressions to determine a condition under which PCP is preferred to LCP. We approximate expected profits using a "second-order" approximation.

In the PCP case, the firm chooses P, the price of exports denominated in its currency. The other variables, S, Z, N, W_1, W_2^*, X, are exogenous to the firm, and stochastic. The derivation of expected profits is relegated to the appendix. Here we reproduce the final expression:

$$
\begin{aligned}
E(\Pi^{PCP}) \approx\ & e^{\bar{x}+\bar{n}} P \cdot D(Pe^{-\bar{s}-\bar{z}})e^{\bar{n}}) \times \{ \frac{\gamma-\eta(\gamma-1)}{\gamma} + \frac{\gamma-\eta(\gamma-1)}{2\gamma}\sigma_x^2 + \tfrac{1}{2}(\gamma-\varepsilon+\delta\gamma(\gamma-1))(\sigma_s^2+\sigma_z^2+2\sigma_{sz}) \\
& + \frac{1-\delta(\gamma-1)}{2\gamma}\sigma_n^2 + \sigma_{xs} + \sigma_{xz} + \frac{1}{\gamma}\sigma_{xn} + (1+\delta-\gamma\delta)(\sigma_{ns}+\sigma_{nz}) \\
& - \frac{\eta(\gamma-1)}{\gamma}\left[\begin{array}{l}
\tfrac{1}{2}\omega(1+e^{\bar{w}_1}\frac{H_{11}}{H_1})\sigma_{w_1}^2 + \tfrac{1}{2}(1-\omega)(1+e^{\bar{s}+\bar{w}_2}\frac{H_{22}}{H_2})(\sigma_{w_2^*}^2+\sigma_s^2+2\sigma_{sw_2^*}) \\
+e^{\bar{w}_1+\bar{s}+\bar{w}_2}\frac{H_{12}}{H}(\sigma_{w_1w_2^*}+\sigma_{sw_1}) + \omega\sigma_{xw_1} + (1-\omega)(\sigma_{xs}+\sigma_{xw_2^*}) \\
+\frac{\gamma}{\eta}\left[\omega(\sigma_{sw_1}+\sigma_{zw_1}) + (1-\omega)(\sigma_s^2+\sigma_{sw_2^*}+\sigma_{zs}+\sigma_{zw_2^*})\right] \\
+\frac{1}{\eta}[\omega\sigma_{nw_1} + (1-\omega)(\sigma_{ns}+\sigma_{nw_2^*})]
\end{array}\right] \}
\end{aligned}
$$

In the above expression, σ_i^2 is the variance of i, and σ_{ij} is the covariance of i with j. The elasticities are defined as above in the flexible-price model. In addition, we define $\eta \equiv \frac{C(Y)}{C'(Y)Y}$.

Now consider the LCP problem. The firm chooses P^* to maximize

$$
E\Pi^{LCP}(P^*, S, Z, N, W_1, W_2^*, X) = E\left[X\left[SP^* \cdot D(P^*/Z)N - C(D(P^*/Z)N)H(W_1, SW_2^*)\right]\right]. \qquad (9)
$$

The firm sets P^*, the import price, in advance. The appendix shows that under LCP, expected profits are given by:

$$
\begin{aligned}
E(\Pi^{LCP}) \approx\ & e^{\bar{x}+\bar{n}+\bar{s}} P^* \cdot D(P^* e^{-\bar{z}})e^{\bar{n}}) \times \{ \frac{\gamma-\eta(\gamma-1)}{\gamma} + \frac{\gamma-\eta(\gamma-1)}{2\gamma}\sigma_x^2 + \tfrac{1}{2}(\gamma-\varepsilon+\delta\gamma(\gamma-1))\sigma_z^2 + \tfrac{1}{2}\sigma_s^2 + \gamma\sigma_{sz} \\
& + \frac{1-\delta(\gamma-1)}{2\gamma}\sigma_n^2 + \sigma_{xs} + \sigma_{xz} + \frac{1}{\gamma}\sigma_{xn} + (1+\delta-\gamma\delta)\sigma_{nz} - \\
& \frac{\eta(\gamma-1)}{\gamma}\left[\begin{array}{l}
\tfrac{1}{2}\omega(1+e^{\bar{w}_1}\frac{H_{11}}{H_1})\sigma_{w_1}^2 + \tfrac{1}{2}(1-\omega)(1+e^{\bar{s}+\bar{w}_2}\frac{H_{22}}{H_2})(\sigma_{w_2^*}^2+\sigma_s^2+2\sigma_{sw_2^*}) \\
+e^{\bar{w}_1+\bar{s}+\bar{w}_2}\frac{H_{12}}{H}(\sigma_{w_1w_2^*}+\sigma_{sw_1}) + \omega\sigma_{xw_1} + (1-\omega)(\sigma_{xs}+\sigma_{xw_2^*}) \\
+\frac{\gamma}{\eta}\left[\omega\sigma_{zw_1} + (1-\omega)(\sigma_{zs}+\sigma_{zw_2^*})\right] \\
+\frac{1}{\eta}[\omega\sigma_{nw_1} + (1-\omega)(\sigma_{ns}+\sigma_{nw_2^*})]
\end{array}\right] \}
\end{aligned}
$$

Comparing expressions we find:

$E(\Pi^{PCP}) > E(\Pi^{LCP})$ if and only if

$$
\tfrac{1}{2}(\gamma-\varepsilon+\delta\gamma(\gamma-1))(\sigma_s^2+2\sigma_{sz}) - \delta(\gamma-1)\sigma_{ns} - (\gamma-1)(\omega\sigma_{sw_1} + (1-\omega)(\sigma_s^2+\sigma_{sw_2^*})) > \tfrac{1}{2}\sigma_s^2 + \gamma\sigma_{sz}.
$$

This condition can be rewritten as:

Condition 2:

$$[\varepsilon+\delta\gamma(\gamma-1)]\beta_{zs}+\delta(\gamma-1)\beta_{ns}+\omega(\gamma-1)\beta_{w_1 s}+(1-\omega)(\gamma-1)\beta_{w_2^* s}<\frac{(2\omega-1)(\gamma-1)-\varepsilon-\delta\gamma(\gamma-1)}{2}.$$

The theorem then follows by inspection:

Theorem: Condition 1 and Condition 2 are identical.

The condition for PCP pricing to be preferred is exactly the condition for "unconditional" pass-through to be greater than ½ in the flexible price model. (That is, it is the condition for $\beta_{p^*s}<-\frac{1}{2}$, where β_{p^*s} is the regression slope coefficient of a regression of the import price on the exchange rate, without conditioning on any other variables.)

4. Discussion

We find then that in the sticky-price model, firms choose prices in such a way that ex post pass-through of exchange rates to import prices is 100 percent (PCP pricing) under exactly the same conditions that in the flexible-price model "unconditional" pass-through to import prices is greater than ½.

Note that the condition for PCP pricing corresponds to the condition derived in Devereux, Engel, and Storgaard (2003) in the special case in which $\varepsilon=0$ (so demand is constant elasticity), $\delta=0$ (so the cost function is linear in output), and $\omega=1$ (so only local inputs are used.) This condition also corresponds to the condition derived in Giovannini (1985) in the special case in which the exchange rate is the only random variable (so the entire left-hand side of the inequality above is zero) and $\omega=1$.

Friberg (1998) also considers the case in which the exchange rate is the only random variable an $\omega=1$. In that case, we have $-\beta_{p^*s}=\dfrac{\gamma-1}{\gamma-1+\varepsilon+\delta\gamma(\gamma-1)}$. Since $\gamma>1$ and Friberg assumes $\delta>0$, then a sufficient condition for the pass-through coefficient $(-\beta_{p^*s})$ to be less than one is $\varepsilon>0$. An even stronger condition is $\varepsilon>\gamma$. Turning to the sticky-price models, under Friberg's assumption that the only random variable is the exchange rate and $\omega=1$, the condition for firms to choose LCP pricing is $0>\gamma-1-\varepsilon-\delta\gamma(\gamma-1)$. This condition is met when $\varepsilon>\gamma$. This is Friberg's theorem: that when $\delta>0$ (and in the confines of his set-up in which the exchange rate is the only random variable and $\omega=1$), then a sufficient condition for both LCP pricing and a pass-through coefficient less than one is $\varepsilon>\gamma$. Friberg does not fully characterize the conditions for LCP vs. PCP pricing (even in his set-up), and so does not arrive at our theorem: a necessary and sufficient condition for LCP pricing is that the pass-through coefficient be less than ½.

Recall that the condition for "unconditional" pass-through to be less than ½ in the flexible price model is equivalent to the condition for $\mathrm{var}(p^*)<\mathrm{var}(p)$. So, in other words, the condition for the variance of consumer prices to be less than the variance of producer prices under flexible prices is the same condition under which LCP (zero variance of consumer prices) is better than PCP (zero variance of producer prices) under sticky prices.

Finally, note the role of distribution costs incurred in the importing country. Under flexible prices, when distribution costs are a large share of total costs (when $1-\omega$ is close to one), then there will be a large elasticity of $p*$ with respect to w_2^*. If wages are very stable (have low variance and therefore a low covariance with the exchange rate) and ω is close to zero, then the import price will tend to be stabilized in the importer's currency. That is the result noted by, for example, Goldberg and Veboven (2001); Burstein, Neves, and Rebelo (2003); and, Burstein, Eichenbaum, and Rebelo (2002). That result says that if distribution costs are significant, and those costs are relatively stable in the importer's currency, then the apparent pass-through of the exchange rate to the final goods price will be low.

But Condition 2 demonstrates that under these same conditions, LCP will be optimal. That is, when distribution costs are a large share of total costs, and when the wage in the importing cost is stable, then it is optimal for the exporting firm to set the price in the consumer's currency.

To elaborate on this condition, consider the case in which the production function is CES, so that $H(W_1, SW_2^*) = (\lambda W_1^{1-\alpha} + (1-\lambda)(SW_2^*)^{1-\alpha})^{\frac{1}{1-\alpha}}$. Then we have

$$1-\omega = \frac{(1-\lambda)(SW_2^*)^{1-\alpha}}{\lambda W_1^{1-\alpha} + (1-\lambda)(SW_2^*)^{1-\alpha}}$$

In the Cobb-Douglas case ($\alpha = 1$), we have $1-\omega = 1-\lambda$. In that case, the unconditional pass-through is lower the larger the share $1-\lambda$ of foreign inputs into the production process. That is also the case, as the theorem states, in which LCP is more likely. When foreign and domestic inputs are combined in fixed proportions (as in the model of Burstein, Eichenbaum, and Rebelo, 2002 in which labor in the importing country is used to distribute the good), we find $1-\omega = \frac{(1-\lambda)SW_2^*}{\lambda W_1 + (1-\lambda)SW_2^*}$. Under flexible prices, pass-through will be low when the cost of distribution services is high and $1-\lambda$ is large, but again those are also the circumstances in which the exporter in the sticky-price setting prefers LCP.

Condition 1 is a general statement of the standard model of pass-through in the international trade literature, and suggests empirically testable propositions. Pass-through should be high when Condition 1 is met, and low otherwise. But empirical evidence of this link — that, for example, pass-through is low when the distribution share is high, or when demand becomes more elastic at higher prices ($\varepsilon > 0$) — in fact sheds no light on whether the flexible-nominal-price model or the sticky-nominal-price approach is appropriate. The theorem of this paper shows that both types of models imply high pass-through when Condition 1 is met, and low pass-through when it is not. The work cited above finding a link between high distribution shares and low pass-through, or the studies cited in Golberg and Knetter (1997) uncovering a link between variable elasticity of demand and pass-through,[6] do not help us draw inferences about the applicability of flex-price vs. sticky-price models of import pricing.

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6 For example Aw (1993), and Goldber and Knetter (1998) find such links.

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Appendix. Algebra for Sticky-Price Models

For the PCP case, write

$$X\left[P\cdot D(P/SZ)N-C(D(P/SZ)N)H(W_1,SW_2^*)\right]=e^x\left[P\cdot D(Pe^{-s-z})e^n-C(D(Pe^{-s-z})e^n)H(e^{w_1},e^{s+w_2^*})\right]$$

where lower case letters represent logs of the upper case letters. We then take a second-order Taylor series expansion, expanding around the means of the logs: $\overline{s}\equiv Es$, $\overline{z}\equiv Ez$, $\overline{n}\equiv En$, $\overline{w}_1\equiv Ew_1$, $\overline{w}_2^*\equiv Ew_2^*$ and $\overline{x}\equiv Ex$, then take the expectation. First, expand the expression for expected discounted revenues:

$$E\left[X\left(P\cdot D(P/SZ)N\right)\right]\approx e^{\overline{x}+\overline{n}}P\cdot D(Pe^{-\overline{s}-\overline{z}})\times\begin{bmatrix}1+\tfrac{1}{2}(\sigma_x^2+\sigma_n^2+2\sigma_{xn})+\gamma(\sigma_{xs}+\sigma_{xz}+\sigma_{ns}+\sigma_{nz})\\+\tfrac{1}{2}\gamma(\gamma-\varepsilon)(\sigma_s^2+\sigma_z^2+2\sigma_{sz})\end{bmatrix}$$

In the above expression, σ_i^2 is the variance of i, and σ_{ij} is the covariance of i with j. The elasticities are defined as above, evaluated at the point where $SZ=e^{\overline{s}+\overline{z}}$.

On the cost side, first define $\eta\equiv\dfrac{C(Y)}{C'(Y)Y}$. Expanding expected discounted costs:

$$E\left[XC(D(P/SZ)N)H(W_1,SW_2^*)\right]\approx e^{\overline{x}}C(D(Pe^{-\overline{s}-\overline{z}})e^{\overline{n}})H(e^{\overline{w}_1},e^{\overline{s}+\overline{w}_2^*})\times$$
$$\begin{bmatrix}1+\tfrac{1}{2}\sigma_x^2+\dfrac{\gamma}{2\eta}(\gamma-\varepsilon+\delta\gamma)(\sigma_s^2+\sigma_z^2+2\sigma_{sz})\\+\dfrac{1}{2\eta}(1+\delta)\sigma_n^2+\dfrac{\gamma}{\eta}(\sigma_{xs}+\sigma_{xz})+\dfrac{1}{\eta}\sigma_{xn}+\dfrac{\gamma}{\eta}(1+\delta)(\sigma_{ns}+\sigma_{nz})\\+\tfrac{1}{2}\omega(1+e^{\overline{w}_1}\dfrac{H_{11}}{H_1})\sigma_{w_1}^2+\tfrac{1}{2}(1-\omega)(1+e^{\overline{s}+\overline{w}_2}\dfrac{H_{22}}{H_2})(\sigma_{w_2^*}^2+\sigma_s^2+2\sigma_{sw_2^*})\\+e^{\overline{w}_1+\overline{s}+\overline{w}_2}\dfrac{H_{12}}{H}(\sigma_{w_1w_2^*}+\sigma_{sw_1})+\omega\sigma_{xw_1}+(1-\omega)(\sigma_{xs}+\sigma_{xw_2^*})\\+\dfrac{\gamma}{\eta}\left[\omega(\sigma_{sw_1}+\sigma_{zw_1})+(1-\omega)(\sigma_s^2+\sigma_{sw_2^*}+\sigma_{zs}+\sigma_{zw_2^*})\right]\\+\dfrac{1}{\eta}[\omega\sigma_{nw_1}+(1-\omega)(\sigma_{ns}+\sigma_{nw_2^*})]\end{bmatrix}.$$

In the above expression, the elasticities, γ, ε, δ, ω, and η, as well as the functions H, H_1, H_2, H_{11}, H_{22} and H_{12}, are evaluated where the log of each of the random variables is equal to its mean (i.e., $S=e^{\overline{s}}$, $X=e^{\overline{x}}$, etc.)

We want an expression for the expected discounted profits, evaluated at the optimum price level. That is, define

$$P^{opt}\equiv\arg\max_P\left(E\Pi^{PCP}(P,S,Z,N,W_1,W_2^*,X)\right).$$

Then, to approximate the maximized value discounted profits, we should evaluate our approximation for expected profits at opt $P = P^{opt}$. Define

$$\Pi^{opt} \equiv E\Pi^{PCP}(P^{opt}, S, Z, N, W_1, W_2^*, X)$$

Consider, now, an approximation for P^{opt}, given by

$$P^{app} \equiv \frac{\gamma(P^{app}e^{-\bar{s}-\bar{z}})}{1-\gamma(P^{app}e^{-\bar{s}-\bar{z}})} C'(D(P^{app}e^{-\bar{s}-\bar{z}})e^{\bar{n}})H(e^{\bar{w}_1}, e^{s+\bar{w}_2^*})$$

P^{app} is the value of P that maximizes $e^{\bar{x}}\left[P \cdot D(Pe^{-\bar{s}-\bar{z}})e^{\bar{n}} - C(D(Pe^{-\bar{s}-\bar{z}})e^{\bar{n}})H(e^{\bar{w}_1}, e^{\bar{s}+\bar{w}_2^*})\right]$. P^{app} differs from P^{opt} only by terms that involve the second moments of the logs of the exogenous random variables: $P^{app} \approx P^{opt}(1+o(\sigma^2))$, where $o(\sigma^2)$ is a linear function of the second moments of s, z, n, w_1, w_2^*, x. (This can be seen immediately, since

$e^{\bar{x}}\left[P \cdot D(Pe^{-\bar{s}-\bar{z}})e^{\bar{n}} - C(D(Pe^{-\bar{s}-\bar{z}})e^{\bar{n}})H(e^{\bar{w}_1}, e^{\bar{s}+\bar{w}_2^*})\right]$ equals

$E\left[e^{x}\left[P \cdot D(Pe^{-s-z})e^{n} - C(D(Pe^{-s-z})e^{n})H(e^{w_1}, e^{s+w_2^*})\right]\right]$ up to the second moment terms.)

But, expected profits evaluated at P^{app} are approximately equal to expected profits evaluated at P^{opt}. Define

$$\Pi^{app} \equiv E\Pi^{PCP}(P^{app}, S, Z, N, W_1, W_2^*, X) .$$

Because P^{opt} is chosen to maximize expected discounted profits, the first-order condition is $\lim_{P^{app} \to P^{opt}} \left[\frac{\Pi^{app} - \Pi^{opt}}{P^{app} - P^{opt}}\right] = 0$. So, to the order of approximation used here, $\Pi^{app} = \Pi^{opt}$.

In the above expansion of the expected discounted cost term, let

$P \equiv \frac{\gamma}{1-\gamma} C'(D(Pe^{-\bar{s}-\bar{z}})e^{\bar{n}})H(e^{\bar{w}_1}, e^{s+\bar{w}_2^*})$, or

$$C(D(Pe^{-\bar{s}-\bar{z}})e^{\bar{n}})H(e^{\bar{w}_1}, e^{s+\bar{w}_2^*}) = \frac{\gamma-1}{\gamma} PD(Pe^{-\bar{s}-\bar{z}})e^{\bar{n}}\eta.$$

With this substitution, expected profits under PCP can be written as:

$$
\begin{aligned}
E(\Pi^{PCP}) \approx\ & e^{\bar{x}+\bar{n}} P\cdot D(Pe^{-\bar{s}-\bar{z}})e^{\bar{n}})\times\{\frac{\gamma-\eta(\gamma-1)}{\gamma}+\frac{\gamma-\eta(\gamma-1)}{2\gamma}\sigma_x^2+\tfrac{1}{2}(\gamma-\varepsilon+\delta\gamma(\gamma-1))(\sigma_s^2+\sigma_z^2+2\sigma_{sz}) \\
& +\frac{1-\delta(\gamma-1)}{2\gamma}\sigma_n^2+\sigma_{xs}+\sigma_{xz}+\frac{1}{\gamma}\sigma_{xn}+(1+\delta-\gamma\delta)(\sigma_{ns}+\sigma_{nz}) \\
& -\frac{\eta(\gamma-1)}{\gamma}\begin{bmatrix}\tfrac{1}{2}\omega(1+e^{w_1}\frac{H_{11}}{H_1})\sigma_{w_1}^2+\tfrac{1}{2}(1-\omega)(1+e^{\bar{s}+\bar{w}_2}\frac{H_{22}}{H_2})(\sigma_{w_2^*}^2+\sigma_s^2+2\sigma_{sw_2^*}) \\
+e^{\bar{w}_1+\bar{s}+\bar{w}_2}\frac{H_{12}}{H}(\sigma_{w_1w_2^*}+\sigma_{sw_1})+\omega\sigma_{xw_1}+(1-\omega)(\sigma_{xs}+\sigma_{xw_2^*}) \\
+\frac{\gamma}{\eta}\left[\omega(\sigma_{sw_1}+\sigma_{zw_1})+(1-\omega)(\sigma_s^2+\sigma_{sw_2^*}+\sigma_{zs}+\sigma_{zw_2^*})\right] \\
+\frac{1}{\eta}[\omega\sigma_{nw_1}+(1-\omega)(\sigma_{ns}+\sigma_{nw_2^*})]\end{bmatrix}\}
\end{aligned}
$$

Now consider the LCP problem. The firm chooses P^* to maximize

$$
E\Pi^{LCP}(P^*,S,Z,N,W_1,W_2^*,X)=E\left[X\left[SP^*\cdot D(P^*/Z)N-C(D(P^*/Z)N)H(W_1,SW_2^*)\right]\right].
$$

We proceed as before:

$$
E\left[X\left(SP^*\cdot D(P^*/Z)N\right)\right]\approx e^{\bar{x}+\bar{n}+\bar{s}}P^*\cdot D(P^*e^{-\bar{z}})\times\begin{bmatrix}1+\tfrac{1}{2}(\sigma_x^2+\sigma_n^2+\sigma_s^2+2\sigma_{xn}+2\sigma_{xs}+2\sigma_{ns}) \\ +\gamma(\sigma_{sz}+\sigma_{xz}+\sigma_{nz})+\tfrac{1}{2}\gamma(\gamma-\varepsilon)\sigma_z^2\end{bmatrix}.
$$

$$
\begin{aligned}
E\left[XC(D(P^*/Z)N)H(W_1,SW_2^*)\right]\approx\ & e^{\bar{x}}C(D(P^*e^{-\bar{z}})e^{\bar{n}})H(e^{\bar{w}_1},e^{\bar{s}+\bar{w}_2^*})\times \\
& \begin{bmatrix}1+\tfrac{1}{2}\sigma_x^2+\frac{\gamma}{2\eta}(\gamma-\varepsilon+\delta\gamma)\sigma_z^2+\frac{1}{2\eta}(1+\delta)\sigma_n^2 \\
+\frac{\gamma}{\eta}\sigma_{xz}+\frac{1}{\eta}\sigma_{xn}+\frac{\gamma}{\eta}(1+\delta)\sigma_{nz} \\
+\tfrac{1}{2}\omega(1+e^{\bar{w}_1}\frac{H_{11}}{H_1})\sigma_{w_1}^2+\tfrac{1}{2}(1-\omega)(1+e^{\bar{s}+\bar{w}_2}\frac{H_{22}}{H_2})(\sigma_{w_2^*}^2+\sigma_s^2+2\sigma_{sw_2^*}) \\
+e^{\bar{w}_1+\bar{s}+\bar{w}_2}\frac{H_{12}}{H}(\sigma_{w_1w_2^*}+\sigma_{sw_1})+\omega\sigma_{xw_1}+(1-\omega)(\sigma_{xs}+\sigma_{xw_2^*}) \\
+\frac{\gamma}{\eta}\left[\omega\sigma_{zw_1}+(1-\omega)(\sigma_{zs}+\sigma_{zw_2^*})\right] \\
+\frac{1}{\eta}[\omega\sigma_{nw_1}+(1-\omega)(\sigma_{ns}+\sigma_{nw_2^*})]\end{bmatrix}.
\end{aligned}
$$

Then, following steps similar to the PCP case, we can write:

$$E(\Pi^{LCP}) \approx e^{\bar{x}+\bar{n}+\bar{s}} P^* \cdot D(P^* e^{-\bar{z}}) e^{\bar{n}} \times \{ \frac{\gamma-\eta(\gamma-1)}{\gamma} + \frac{\gamma-\eta(\gamma-1)}{2\gamma}\sigma_x^2 + \tfrac{1}{2}(\gamma-\varepsilon+\delta\gamma(\gamma-1))\sigma_z^2 + \tfrac{1}{2}\sigma_s^2 + \gamma\sigma_{sz}$$
$$+\frac{1-\delta(\gamma-1)}{2\gamma}\sigma_n^2 + \sigma_{xs} + \sigma_{xz} + \frac{1}{\gamma}\sigma_{xn} + (1+\delta-\gamma\delta)\sigma_{nz} -$$
$$\frac{\eta(\gamma-1)}{\gamma}\begin{bmatrix} \tfrac{1}{2}\omega(1+e^{\bar{w}_1}\frac{H_{11}}{H_1})\sigma_{w_1}^2 + \tfrac{1}{2}(1-\omega)(1+e^{\bar{s}+\bar{w}_2}\frac{H_{22}}{H_2})(\sigma_{w_2^*}^2 + \sigma_s^2 + 2\sigma_{sw_2^*}) \\ +e^{\bar{w}_1+\bar{s}+\bar{w}_2}\frac{H_{12}}{H}(\sigma_{w_1w_2^*} + \sigma_{sw_1}) + \omega\sigma_{xw_1} + (1-\omega)(\sigma_{xs} + \sigma_{xw_2^*}) \\ +\frac{\gamma}{\eta}\left[\omega\sigma_{zw_1} + (1-\omega)(\sigma_{zs} + \sigma_{zw_2^*})\right] \\ +\frac{1}{\eta}[\omega\sigma_{nw_1} + (1-\omega)(\sigma_{ns} + \sigma_{nw_2^*})] \end{bmatrix}\}$$

Comparing expressions (and recognizing that at the point of expansion, the price under PCP equals $e^{\bar{s}}$ times the price under LCP), we find:

$E(\Pi^{PCP}) > E(\Pi^{LCP})$ if and only if

$$\tfrac{1}{2}(\gamma-\varepsilon+\delta\gamma(\gamma-1))(\sigma_s^2 + 2\sigma_{sz}) - \delta(\gamma-1)\sigma_{ns} - (\gamma-1)(\omega\sigma_{sw_1} + (1-\omega)(\sigma_s^2 + \sigma_{sw_2^*})) > \tfrac{1}{2}\sigma_s^2 + \gamma\sigma_{sz}.$$

This condition can be rewritten as:

$$[\varepsilon+\delta\gamma(\gamma-1)]\beta_{zs} + \delta(\gamma-1)\beta_{ns} + \omega(\gamma-1)\beta_{w_1s} + (1-\omega)(\gamma-1)\beta_{w_2^*s} < \frac{(2\omega-1)(\gamma-1)-\varepsilon-\delta\gamma(\gamma-1)}{2}.$$