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DEVIATIONS IN ASIAN DATA

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# Application of a Modified TAR Model to CIP Deviations in Asian Data

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## Abstract

The methodology to be used in this paper is estimation of a threshold autoregressive (TAR) model. In this model deviations are random within a band defined by transactions costs and contract risk, and autoregressive towards the band outside it. The principal reference is Tchernykh (1998). These estimates can provide indicators for policy-makers of the market's expectation of crisis. They could also provide indicators for the private sector of convergence of deviations to their usual bands. The estimation methodology is a non-linear three-regime maximum likelihood procedure. The TAR model has the potential to be applied to differentials between linked pairs of financial market prices more generally. This paper modifies the classical TAR model to allow for progressive deviations from a stochastic regime, rather than simple jumps.

# 1. Introduction

This paper analyses and tests arbitrage and covered interest parity (CIP) between forward markets for foreign exchange. The traditional literature focuses on spot-forward CIP. In this paper we look at forward-forward deviations to study the term structure of deviations from CIP. The literature, beginning with Keynes and Einzig, intuitively described deviations as random within a band defined loosely by transactions costs, but occasionally breaking out of the band, then deviations were thought to regress back to the band. This has been formalized as the Threshold Autoregression (TAR) model. The TAR model for time series originated with Howell Tong in 1978. In this model deviations are random within the band and autoregressive outside.

The TAR model has been applied to spot-forward CIP deviations more recently by Mark Taylor and co-authors. Variants of TAR models are specified and estimated in Tchernykh Branson (1998 and 2002) for forward-forward deviations from CIP. We call this forward-forward arbitrage. This is the relation between the ratio of forward exchange rates and the forward interest differential for the same maturity. These previous studies found that 3-6 months forward-forward deviations most clearly reflected TAR behaviour. This paper analyses the daily data for the 3-6 month forward deviations in the Philippines peso - US dollar rates and in the Hong Kong - US dollar rates. The empirical work is done on daily data for the period 1994-2002.

The classical TAR model assumes deviations from the band occur in a single-period jump, and then switch to the autoregressive regime. In the data from the countries of Southeast (SE) Asia deviations occur in irregular steps to a maximum, and then appear to switch to the autoregressive regime. In this paper we report results for a modification of the TAR model that is estimated eliminating these steps in the deviations, and compare these estimates with the classical TAR (so-called SETAR in Tong, 1978).

This technique includes Maximum Likelihood estimation for the coefficient and its variance which determines the speed of the mean-reversion process; estimation of the AR and TAR process; and Monte Carlo simulation.

A program was written in Visual Basic to produce the input return data for regression analysis package (RATS Econometrics Software) as a standard EXCEL-table.

Several different algorithms have been tested with the actual return data to maximize the likelihood function. Among those tested are:

- Berndt, Hall, Hall and Hausman (BHHH);
- Broyden, Fletcher, Goldfarb and Shanno (BFGS);
- The simplex algorithm;
- Genetic search algorithm.

The BFGS algorithm proved to have more stable behaviour in terms of final convergence of calculations in most cases.

### 1.1 Definition of CIP

The covered interest parity (CIP) theorem for foreign exchange states that the foreign exchange forward premium equals the interest rate differential between two relevant currencies; in log-linear form the formula for CIP can be expressed as:

$$f_i - s = r_i - r_i^* \quad (1)$$

Here in logarithmic form  $f_i$  represents the  $i$ -period forward exchange rate in terms of units of home currency per unit of foreign exchange,  $s$  is the spot exchange rate,  $r_i$  is the domestic  $i$ -period interest rate, and  $r_i^*$  is the foreign  $i$ -period interest rate.

While the literature has tended to concentrate on the form of CIP expressed in equation (1), a more general form of covered interest arbitrage would involve arbitraging along the term structure. Then the following generalized CIP condition must hold:

$$f_j - f_i = r_{ij} - r_{ij}^* . \quad (2)$$

Here in log-form  $f_i$  and  $f_j$  are the forward exchange rates for period  $i$  and  $j$ ;  $r_{ij}$  is the domestic forward interest rate, and  $r_{ij}^*$  is the foreign forward interest rate between times  $i$  and  $j$ , with  $i < j$ .

From equation (2), the generalized CIP condition, we define the deviation from CIP for arbitrage between maturities  $i$  and  $j$  at time  $t$ , as

$$y_t = (f_j - f_i) - (r_{ij} - r_{ij}^*) . \quad (3)$$

Our empirical methodology involves estimating nonlinear time series models for  $y_t$ . In particular, we wish to estimate the neutral band within which arbitrage does not take place, and the speed of mean reversion of deviations from CIP outside of the band.

### 1.2 Asian Data

This section motivates the use of Threshold Models by reviewing the behaviour of CIP deviations in recent data from Asia. These data are summarized in Graphs 1 - 6. These show data for the Thai baht, the Indonesian rupiah, the Philippine peso, the Singapore dollar, the Korean won, and the Taiwan dollar. These data were provided by JP Morgan, Singapore. Each graph shows the 3-6 month forward-forward deviation.

We will not comment in detail on all of the graphs, but rather focus on similarities and major points for subsequent analysis. All of the graphs show the Asian crisis of 1997 - 1998, with large deviations from CIP. There are quiet periods before and after the crisis, and another volatile period in 2001 in some of

the graphs. TAR behaviour can be seen during non-crisis periods in most of the cases. We will focus on these non-crisis periods in estimation. Below we will demonstrate this with the data for the Philippines and Hong Kong.

We begin with the Thai data in Graph 1. Here we see a quiet period before the 1997 crisis, which may be white noise within a TAR band, then we see the deviation opening during the crisis, with several peaks. The period from October 1998 to the end of the data resembles a TAR process. The deviation jumps in early May 1997, well before the crisis, and then widens. The movement in the deviation also came in steps. This movement in steps is characteristic of the Asian data. It suggests that the TAR model should be modified for upward deviations in several steps, rather than a single jump.

Graph 2 shows the movements in deviations for the Indonesian rupiah. The movements are similar to those of the Thai baht, but lagged about 6 months. As with the Thai baht, there is a quiet period until August 1997. The first major movement in the Thai 3-6 month forward deviation came in May 1997; the first move in the Indonesian 3-6 month forward came in August, with the big jump in December. Another relatively quiet period follows the crisis, with movements that seem to follow TAR behaviour. Another crisis erupts in March 2001, followed by smaller deviations around an elevated mean.

Movements in the deviations for the Philippine peso are shown in Graph 3. Again there is a quiet period of seemingly white noise until June 1997. Then the crisis period erupts, led by the increase in the deviation at the end of May 1997. After the crisis there are two periods of seeming TAR movement in May 1999 - August 2000, and September 2000 - April 2002. The Philippine data seem to show the clearest TAR movement, so we will begin detailed estimation on these data later in the paper.

The data for the Singapore dollar are shown in Graph 4. These data show relatively quiet periods before and after the crisis of October 1997 to the end of 1998. The deviation also leads the crisis, as in the previous graphs. The quiet periods in the Singapore data show relatively large variations around a negative mean. This suggests that the problem in Singapore was failure of arbitrage into the Singapore dollar, contrasted with the previous data. This may reflect the policy of appreciation of the Singapore dollar and some form of short-term capital controls.

Graph 5 shows the data for the Korean won. There we see again the quiet periods before and after the crisis. But in the Korean case deviations widen well before the crisis at the end of 1996. The Korean data show a pattern different from Asia, with a disturbance much earlier.

Movements in deviations for the Taiwan dollar are shown in Graph 6. These data are more turbulent than the earlier data for Asia. We see a relatively quiet period from December 1994 to mid 1997. Then the crisis comes in two pulses, October 1997 and July 1998. In these periods, and afterwards, we see behaviour that may be consistent with the TAR model. Another crisis appears at the end of 2001. This is consistent with the Asian data. The Taiwan dollar data might be a good target for the next detailed examination.

Several main points emerge from the discussion of the Asian data.

1. There are large deviations before and during crises.
2. The deviations get larger, the longer the maturity of contract as was shown in Tchernykh Branson (2002). A possible implication is that liquidity dries up due to the cumulative probability of contract risk.
3. The graphs show quiet periods where simple stochastic movements within band apply, periods of crisis, and periods where the TAR model applies.
4. In some cases, deviations open in steps. Then the strict TAR model does not hold. I am currently working on a modification of the TAR model to account for progressive deviations.

## 2. Threshold Autoregression (TAR) Model

As discussed in the introduction above, a number of authors have suggested that time series for CIP deviations may be characterized by threshold effects, such that arbitrage occurs when the size of the deviation has passed a certain level. This would suggest that CIP deviations would be largely indeterminate in a certain neighbourhood, while deviations from CIP outside of this range would not be immediately returned into the neutral band but would instead show a statistical tendency to revert towards the band - in other words that deviations from CIP may behave in a highly non-linear fashion.

A parametric model which may capture this non-linear behaviour - and which nests both instantaneous and slower mean-reversion towards the band - is the threshold autoregressive (TAR) model (Tong, 1983). Extending the work of Taylor and Peel (1998) to allow for asymmetry in the neutral band, a simple TAR model allowing deviations from covered interest parity,  $y_t$ , to follow a random walk within a band with an upper threshold of  $\kappa_1$  and a lower threshold of  $\kappa_2$ , while exhibiting mean-reverting first-order autoregressive behaviour outside of the band may be written:

$$y_{t+1} = y_t + \varepsilon_{1,t+1} \quad \text{if } y_t < \kappa_1 \text{ and } y_t > \kappa_2 \quad (4.1)$$

$$y_{t+1} = \kappa_1(1 - \beta) + \beta y_t + \varepsilon_{2,t+1} \quad \text{if } y_t \geq \kappa_1, \quad (4.2)$$

$$y_{t+1} = \kappa_2(1 - \beta) + \beta y_t + \varepsilon_{2,t+1} \quad \text{if } y_t \leq \kappa_2, \quad (4.3)$$

where  $\varepsilon_{i,t+1} \sim N(0, \sigma_i^2)$   $i=1,2$ , and  $\beta \in (0,1)$ . This is the definition of the TAR model used in this paper.

In the absence of prior knowledge about the bandwidth, this model cannot be estimated by simple least square methods. The method of maximum likelihood can, however, be applied to provide estimates of all of the unknown parameters, including the bandwidth. The log-likelihood function for this model can be written:

$$L(\beta, \sigma_1^2, \sigma_2^2, \kappa_1, \kappa_2) = -\frac{1}{2} \sum_{y_t \in (\kappa_2, \kappa_1)} [\ln(\sigma_1^2) + \varepsilon_{1,t+1}^2 / \sigma_1^2] - \frac{1}{2} \sum_{y_t \notin (\kappa_2, \kappa_1)} [\ln(\sigma_2^2) + \varepsilon_{2,t+1}^2 / \sigma_2^2]. \quad (4.4)$$

If it is believed that there are insufficient data points showing evidence of having crossed the lower threshold - such as a lack of negative deviations from CIP for example, then it may not be possible to identify the asymmetric band TAR model just outlined. This is the case in most emerging markets. In this case we may still be able to estimate the upper threshold by estimating a single-threshold model of the form:

$$y_{t+1} = y_t + \varepsilon_{1,t+1} \quad \text{if } y_t < \kappa_1, \quad (5.1)$$

$$y_{t+1} = \kappa_1(1 - \beta) + \beta y_t + \varepsilon_{2,t+1} \quad \text{if } y_t \geq \kappa_1. \quad (5.2)$$

where  $\varepsilon_{i,t+1} \sim N(0, \sigma_i^2)$   $i=1,2$ , and  $\beta \in (0,1)$ .

The likelihood function for the asymmetric model then takes the form:

$$L(\beta, \sigma_1^2, \sigma_2^2, \kappa_1) = -\frac{1}{2} \sum_{y_t < \kappa_1} [\ln(\sigma_1^2) + \varepsilon_{1,t+1}^2 / \sigma_1^2] - \frac{1}{2} \sum_{y_t \geq \kappa_1} [\ln(\sigma_2^2) + \varepsilon_{2,t+1}^2 / \sigma_2^2], \quad (5.3)$$

and estimation may proceed as before.

### 3. Modified TAR (MTAR) Model

In the data from Asia in Graphs 1-6 we saw that deviations from CIP frequently occurred in irregular steps, not single jumps as assumed by the classical TAR model. The number and magnitude of these steps differ substantially across episodes. Including these data points in estimation of the classical TAR model would put an upward bias in the estimates of the  $\beta$  parameter for the speed of autoregression. These movements may reflect a Poisson process involving information lags of some sort. Estimating this process remains a topic for a future research. Thus, at this point in the research we will proceed with the estimation of a “modified” TAR model, the MTAR model, which eliminates the steps in the deviations. This modification eliminates the data points in the steps in the deviations, and proceeds with estimation as if the deviation jumps to its maximum.



The modified asymmetric model and likelihood function for the model can be written as follows.

$$y_{t+1} = y_t + \varepsilon_{0,t+1} \quad \text{if } \kappa_2 < y_t < \kappa_1, \quad (6.1)$$

$$y_{t+1} = \kappa_1(1 - \beta_1) + \beta_1 y_t + \varepsilon_{1,t+1} \quad \text{if } y_t \geq \kappa_1, \quad (6.2)$$

$$y_{t+1} = \kappa_2(1 - \beta_2) + \beta_2 y_t + \varepsilon_{2,t+1} \quad \text{if } y_t \leq \kappa_2, \quad (6.3)$$

where,  $\varepsilon_{i,t+1} \sim N(0, \sigma_i^2)$ ,  $i=0,1,2$ ,  $\beta_j \in (0,1)$ ,  $j=1,2$ .

Let us define the *jump-space*  $\Omega$ . Consider all intervals  $[a_n, b_n]$ ,  $t_n \in [a_n, b_n]$ , when  $y_t$  is above or below the band  $[\kappa_2, \kappa_1]$  with an upper threshold  $\kappa_1$  and a lower threshold  $\kappa_2$  as in (6.2) and (6.3). For each interval  $[a_n, b_n]$  we can find such  $t_n^*$  that  $y_{t_n^*}$  is the extremum for the interval. In particular, when  $y_t$  is bigger than threshold  $\kappa_1$ , we can find the maximum  $y_{t_n^*}$  such that  $y_{t_n^*} = \max_{[a_n, b_n]}(y_t)$ . Analogously, when  $y_t$  is less than  $\kappa_2$ , the minimum is  $y_{t_n^*} = \min_{[a_n, b_n]}(y_t)$ . Then the *jump-space*  $\Omega = \left\{ \bigcup_{n=1}^N [a_n, t_n^*] \right\}$  would be the aggregate of all such intervals. In the modified TAR model we eliminate intervals  $[a_n, t_n^*]$  of jumping deviations and consider intervals of deviations going back to the threshold.

Then the likelihood function for the asymmetric modified TAR model would be:

$$L(\beta_1, \beta_2, \sigma_0^2, \sigma_1^2, \sigma_2^2, \kappa_1, \kappa_2) = -\frac{1}{2} \sum_{\kappa_2 < y_t < \kappa_1} [\ln(\sigma_0^2) + \varepsilon_{0,t+1}^2 / \sigma_0^2] - \frac{1}{2} \sum_{y_t \geq \kappa_1} I(t) [\ln(\sigma_1^2) + \varepsilon_{1,t+1}^2 / \sigma_1^2] - \frac{1}{2} \sum_{y_t \leq \kappa_2} I(t) [\ln(\sigma_2^2) + \varepsilon_{2,t+1}^2 / \sigma_2^2], \quad (6.4)$$

where

$$I(t) = \begin{cases} 1, & \dots\dots\dots \text{if } t \notin \Omega \\ 0, & \dots\dots\dots \text{if } t \in \Omega \end{cases}.$$

Given the model (6.1), (6.2), (6.3), and the likelihood function of equation (6.4), estimation can proceed as described earlier. In the application below to the data for the Hong Kong 3-6 month CIP differentials we compare results of classical TAR, modified TAR, and global AR(1) using Monte Carlo likelihood ratio tests, as described below. We present the estimates of the MTAR model for the Philippines. The TAR estimates for the Philippines were presented earlier in Tchernykh Branson (2002).

## 4. Monte Carlo Simulation and Tests

Having obtained maximum-likelihood estimates of the parameters under classical TAR or modified TAR, we can test the hypotheses of global mean reversion or zero bandwidth,  $\kappa_1 = \kappa_2$ , by estimating the restricted model and applying a likelihood ratio test. In this case, the restricted model is simply a first-order autoregression, AR(1). The likelihood ratio statistic can be constructed as twice the difference between the value of the likelihood function for the AR(1) model and the maximized value of the likelihood function for the relevant TAR model. Note that the modified TAR has fewer data points than the classical TAR for any given data set, so the associated AR(1) estimates will differ.

Accordingly, we estimate the empirical marginal significance level of the likelihood ratio statistics through Monte Carlo simulation (i.e. the parametric bootstrap). The steps are as follows: estimate the restricted AR(1) model using the actual data; use the resulting parameter estimates to calibrate an artificial AR(1) data generating process with Gaussian errors and generate 5,000 artificial data sets equal in length to the actual data set plus 100, each with an initial value of zero; for each data set, discard the first 100 data points (to avoid initial value bias), estimate the relevant TAR model and the AR(1) model, construct the likelihood ratio statistic and save it; take the resulting 5,000 values of the likelihood ratio statistic as the empirical distribution of the statistic under the null hypothesis.

## 5. Empirical Results: Hong Kong and the Philippines

### 5.1 Hong Kong

The data on the 3-month – 6-month CIP deviations, defined in equation (3) above, for the Hong Kong dollar - US dollar rates are shown in Graph 7. These are daily data for the period January 1994 through September 2002, taken from Bloomberg. The data have been matched for gaps and differing holidays to provide a full and consistent data set. In the chart a positive deviation means the 6-month forward exceeded the 3-month by more than the corresponding interest differential. This implies that when the differential is positive, arbitrage could be profitable buying 3-month and selling 6-month forward. A negative differential would imply the opposite arbitrage. Thus large differentials indicate failures of arbitrage to maintain CIP.

The Hong Kong data of Graph 7 can be divided into three distinct periods. These are shown in Graphs 8, 9, and 10. The period from January 1994 to July 1997 seems to exhibit TAR behaviour. For example, in early 1995 the differential jumped to nearly 0.005, and then reverted to what appears to be a normal range of fluctuation. The data up to July 1997 show occasional short periods of noise and longer periods of what seems to be TAR behaviour.

The second period runs from July 1997 to April 2000. With the beginning of the Asian crisis, deviations grew distinctly larger. From late 1997 to late 1998, they remained in the range 0.005-0.02. Clearly both the mean and the variance of the data increased sharply during the crisis period.

The last period runs from March 2000 to August 2002, after the crisis. This is a quiet period with deviations seeming to be random around zero with occasional downward or upward spikes. We do see below that estimates of the MTAR model are confirmed for this period. For each of these three periods, we estimate the MTAR model, and compare the results with a TAR model.

The estimation results for Hong Kong are presented in a standard format. Each graph has four panels. The first panel displays the movements of the deviations for the period. The second panel gives the MTAR estimates for behaviour above the estimated threshold. This shows the estimated upper threshold for deviations and the estimated value of autoregressive parameter  $\beta$ . The TAR and MTAR models both assume that the autoregression coefficient  $\beta$  will be between 0 and 1. Therefore, two  $t$ -statistics are presented for  $\beta$ . The first,  $t$ -stat(0), is the usual test of  $\beta > 0$ . The  $t$ -stat(1) is the test of  $\beta < 1$ . This is constructed as  $(1 - \beta) / SE(\beta)$ . We will look at both  $t$ -statistics in evaluating the  $\beta$  estimates. Both of these are significant at the 95 percent confidence level in all of the estimates reported here. The half-life  $t = \ln(0.5) / \beta$  of deviations (in days) and the time for elimination of 95 percent  $t = \ln(1 - 0.95) / \beta$  of deviations are also included in the estimates. The third panel presents the same parameter estimates for the unmodified TAR model. The critical comparison of TAR and MTAR is the estimate of  $\beta$ . This should be smaller for the MTAR model.

The results for the first period in the Hong Kong data are shown in Graph 8. The data in the top panel seem to show two episodes of TAR behaviour, February - June 1994 and January - April 1995. The BFGS algorithm gives the most stable results; so all estimation is reported using it. The second panel shows the results of estimation of the MTAR model. These results show the significant estimation of only an upper threshold, here 0.0007. This means that much of the data before mid 1994 and after January 1995 are above the threshold. The estimate of  $\beta$  is 0.879, with a  $t$ -statistic for the comparison to zero of 31.7. The estimate of  $(1 - \beta)$  is 0.121, with a  $t$ -statistic of 4.3840. The  $\beta$  estimate implies a half-life of the deviations of 5.4 days, and 23 days to eliminate 95 percent of a deviation. The third panel shows the results of estimation of an unmodified TAR model for the same period. The upper threshold from the MTAR estimates in the third panel is imposed in the TAR estimates, so the  $\beta$ s from TAR and MTAR can be compared. The  $\beta$  estimate with the TAR model is 0.908, significantly different from both 0 and 1. It is also larger than the estimate from the MTAR model, as expected. The Monte Carlo simulation confirms the results of the MTAR at the 95 percent confidence level.

Graph 9 shows the results for the crisis period. The estimates are probably dominated by the two main periods of crisis. The Monte Carlo test does not confirm the estimates, that is, the AR(1) restriction was not binding. Thus, as we expected, the MTAR does not hold during the crisis period.

The results for the final period are shown in Graph 10. In the top panel we see a period of negative deviations from July 2000 - March 2001. The  $\beta$  estimate with the MTAR model is 0.91, but it has a  $t$ -statistic for  $(1 - \beta)$  of 5.04. It has a half-life of 7.3 days and a 95 percent return time of 31.6 days. The deviations in the top panel are small, but persistent. The unmodified TAR estimate of  $\beta$  is 0.958, also significantly different from both 0 and 1, and larger than the MTAR estimate. The Monte Carlo test confirms the MTAR estimates. Thus during quiet periods the deviations in the Hong Kong data are small, but they do follow an MTAR process. They are also persistent, with fairly high values for  $\beta$ .

The comparisons of TAR and MTAR for the first and third periods use the MTAR estimate for the threshold, so the  $\beta$  estimates are comparable. As expected, in the first and last periods where the MTAR is confirmed by the Monte Carlo test, it provides a lower estimate of  $\beta$  and faster convergence than the TAR model. In the crisis period the data do not follow TAR behaviour, so the comparison is irrelevant. The comparisons of the  $\beta$  estimates in the periods in which the MTAR is confirmed support the superiority of the MTAR.

## 5.2 The Philippines

The Philippine data are shown in Graph 11. These data show four distinct regimes of behaviour. The first is a relatively quiet period before the Asian crisis broke out, from August 1994 to the end of May 1997. This may be a period of TAR behaviour. The second is the period of the crisis, from June 1997 to the end of April 1999. This is not likely to follow a TAR process. The third is the period from May 1999 to the end of August 2000. This is noisier than the first period, but may also follow TAR behaviour. The fourth is the period from September 2000 to the end of March 2002. This seems to contain another crisis at the beginning, but perhaps TAR behaviour after that.

The MTAR model has been estimated for each of these sub periods, and the results are presented in Graphs 11 - 15, following a similar format as the Hong Kong presentation, but without the TAR estimates, which were presented in Tchernykh Branson (2002). We begin the discussion of the results with Graph 12 for the first period. We will discuss these in some detail, and then summarize the rest of the results, since they follow the same format. The table in the lower panel gives the results of the MTAR estimation for the period. Only an upper threshold of 0.017 percent is significant; it can be located on the graph above. The MTAR behaviour is thus dominated by two periods, March-July 1995 and October 1995 - April 1996. The estimate of  $\beta$  is a high 0.935, with a  $t$ -statistic of 40.96. The  $t$ -statistic for  $(1 - \beta)$  is 2.84, so  $\beta$  is less than unity. The high  $\beta$  gives an estimated half-life of reversion of 10.3 days, with 44.6 days required to eliminate 95 percent of the deviation above the threshold. The Monte Carlo distribution confirms the MTAR estimates. Thus the data in this period do follow an MTAR process, but with very slow reversion to the threshold. In contrast, the Monte Carlo results did not confirm the earlier estimates of a TAR model. Thus the MTAR performs better than the earlier TAR for this period.

The results for the second period, beginning with the Asian crisis, are summarized in Graph 13. There a higher threshold of 0.025 is estimated, so most of the period from November 1997 to October 1998 is above the threshold. The  $\beta$  estimate is 0.83, with a  $t$ -statistic of 38.5. The  $t$ -statistic for  $(1 - \beta)$  is 7.17. This estimate of  $\beta$  gives a half-life of 3.76 days and a 95 percent return time of 16.25 days. These seem implausible given the time the deviations were above the threshold. Indeed, the Monte Carlo test does not confirm the estimates. This was also the case for the earlier TAR estimates, again indicating that the data for crisis periods do not follow a clear TAR process.

The results for the third period are shown in Graph 14. There the threshold is 0.012, with several episodes above it. The  $\beta$  estimate is 0.87, with a  $t$ -statistic of 31.0. The  $t$ -statistic for  $(1 - \beta)$  is 4.64. The half-life is 5 days and the 95 percent return time is 21.6 days. The Monte Carlo test confirms the MTAR estimates. The earlier TAR estimates were also confirmed. The results for the fourth period in Graph 15 have a threshold of 0.024, and a much lower  $\beta$  of 0.63. The  $t$ -statistic for  $\beta$  is 35.5, and the  $t$ -statistic for  $(1 - \beta)$

is 6.55. This value for  $\beta$  gives a half-life of 1.5 days and a 95 percent return time of 6.55 days. The Monte Carlo test also confirms the MTAR estimates for this period. The earlier TAR estimates were also confirmed.

The Philippine results confirm the MTAR estimates for each period other than the second period including the Asian crisis. The  $\beta$  estimate for the first period was a high 0.93, giving long return times. But the estimates were confirmed, in contrast with the earlier TAR estimates. The  $\beta$  estimates for the third and fourth periods were 0.87 and 0.62, respectively. These give more plausible return times. Thus the estimates for the last period seem sharpest, and the MTAR probably outperforms the TAR.

## 6. Conclusion

This paper has estimated a modified TAR model on the data for 3-6 month forward-forward deviations from CIP for the Hong Kong dollar and the Philippine peso on daily data from 1994 to 2002. The modified TAR model is confirmed by a Monte Carlo test for all periods except the Asian crisis of 1997-1999.

Earlier work on CIP deviations, for example Taylor and Peel (1998) and Tchernykh (1998), used the classical TAR model that was developed by Tong. That model assumed that the CIP deviations appear as a single jump outside a threshold, and then a regression back to the threshold. In the review of the data on CIP deviations here, in Graphs 1 - 6, we observe that deviations frequently develop as a series of jumps to a peak, and then the regression begins. Inclusion of these series of jumps in the estimation biases the estimate of the autoregression coefficient  $\beta$  upward. To eliminate this bias, we developed the modified TAR (MTAR) model that eliminates the jump data from estimation. This model was used in estimation on the Hong Kong and the Philippine data. The regression coefficient was significantly between zero and one in all cases. Further research will model the jumps as separate Poisson processes and include them in the estimation.

The review of the Asian data led to two observations in addition to the deviations opening in steps: there are large deviations before and during crises and the deviations get larger, the longer the maturity of the forward contracts. Thus CIP deviations can be used as leading indicators of crises.

The estimation of the MTAR on the Hong Kong data was confirmed for the two periods before and after the Asian crisis. The estimation yields very low thresholds and fairly persistent small deviations in these relatively quiet periods. The MTAR and classical TAR were compared on the Hong Kong data, confirming the bias in the estimate of  $\beta$  using the classical TAR model.

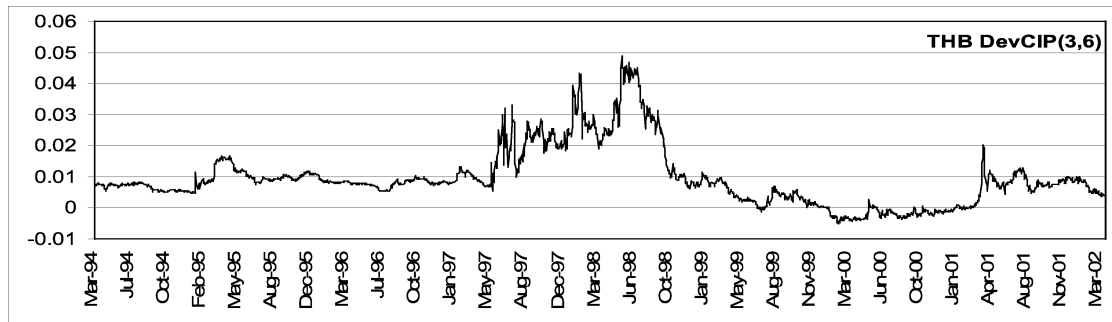
The Philippine data are much noisier, showed a wider range of regimes. The MTAR was confirmed on the three identified non-crisis regimes. There the  $\beta$  estimates ranged from 0.93 to 0.62. Thus the parameters of the confirmed MTAR models vary substantially across regimes. The MTAR for the Philippine data perform better than the earlier TAR estimates.

Thus deviations from CIP, and estimates of the MTAR model, can be useful to policy-makers and traders. Policy-makers can use these as signals of bubbles and coming crises. Traders can use them as the basis for trading rules for arbitrage.

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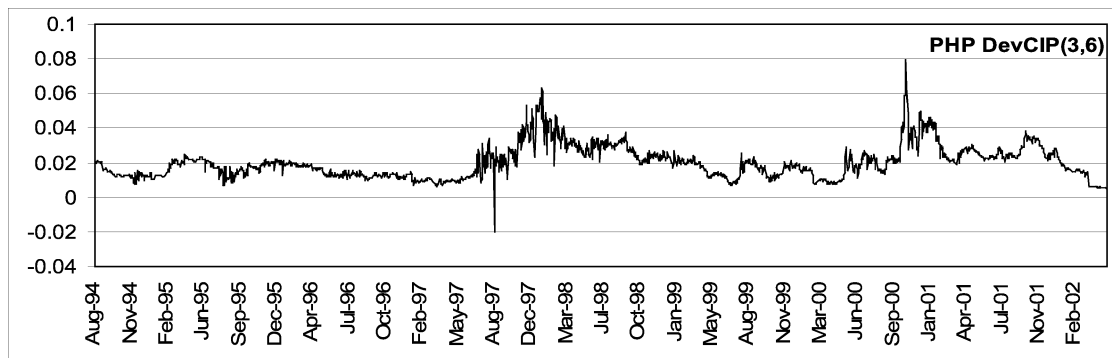
**Graph 1. Thai Baht**



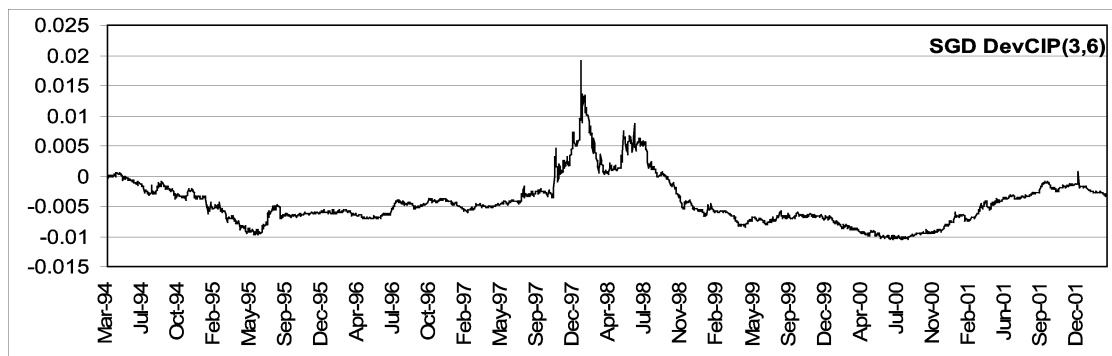
**Graph 2. Indonesia Rupiah**



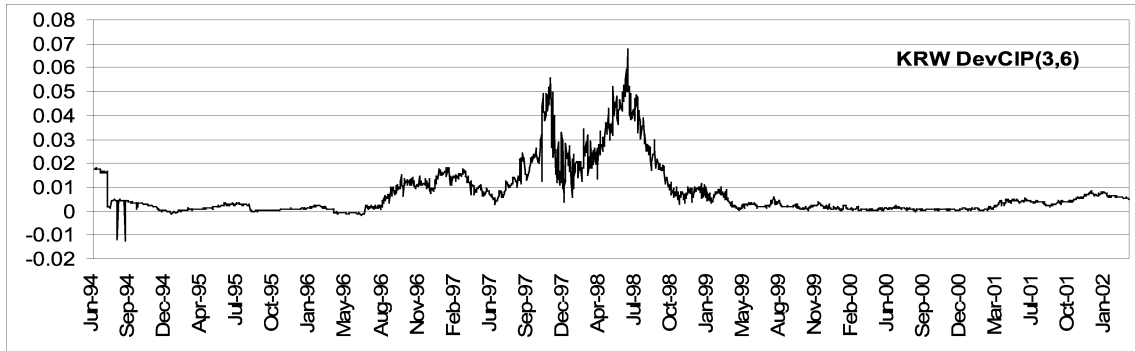
**Graph 3. Philippines Peso**



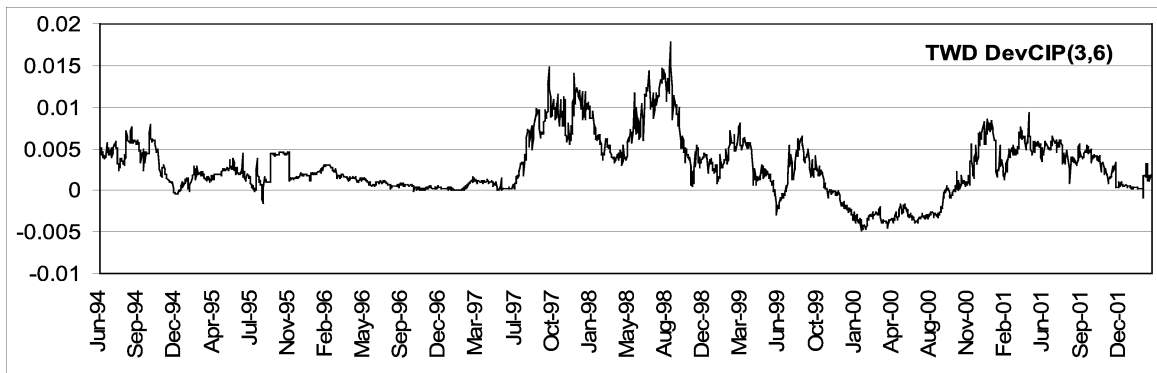
**Graph 4. Singapore Dollar**



**Graph 5. Korean Won**



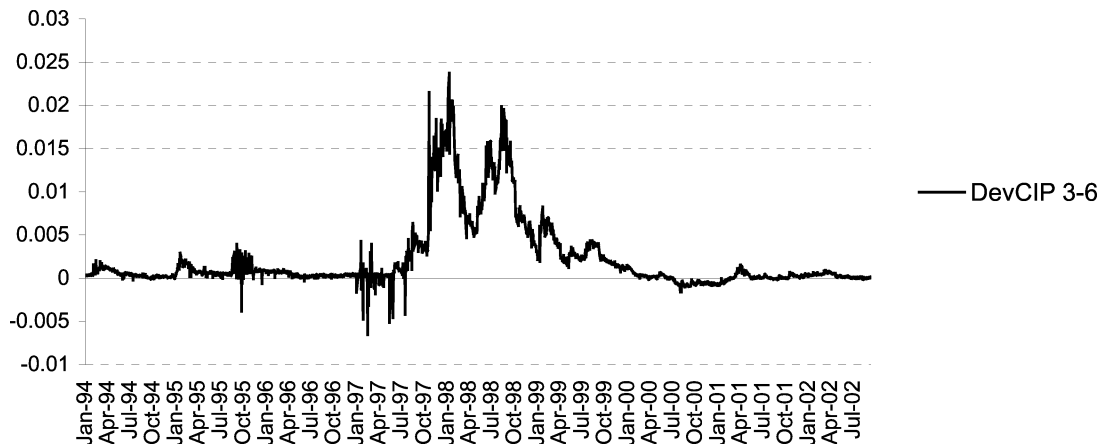
**Graph 6. Taiwan Dollar**



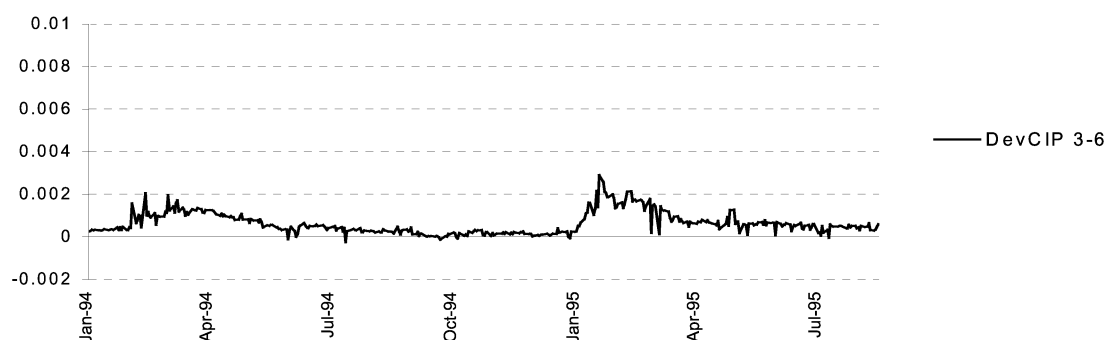
**Graph 7. Hong Kong Foreign Exchange Forward-Forward Deviations.**

These data were divided into 3 parts and the following research has been done separately for each part.

**Forex Forward-Forward Deviations**





**Graph 8. Forex Forward-Forward Deviations (Part 1 Hong Kong)****Table 1. Comparison, MTAR and TAR**

January 1994 - August 1995

Hong Kong, Modified TAR

Estimated Parameters BFGS-Method						
<i>Regimes</i>	<i>Thresholds</i>	$\beta$	<i>T-Stat(0)</i>	<i>Half-Life Time</i>	<i>95% Return Time</i>	<i>T-Stat(1)</i>
Upper	0.0007	0.8790204	31.754966	5.37543635	23.232249	4.3840
Lower	None					

Hong Kong, TAR (unmodified)

Estimated Parameters BFGS-Method						
<i>Regimes</i>	<i>Thresholds</i>	$\beta$	<i>T-Stat(0)</i>	<i>Half-Life Time</i>	<i>95% Return Time</i>	<i>T-Stat(1)</i>
Upper	0.0007	0.9082582	24.618555	7.20328179	31.132066	2.4857
Lower	None					

Graph 9. Forex Forward-Forward Deviations (Part 2 Hong Kong)

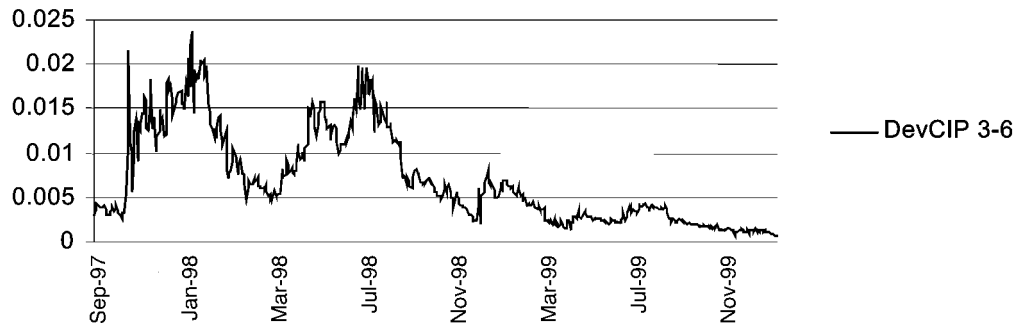


Table 2. Part 2 Comparison

September 1995 - April 2000

Hong Kong, Modified TAR

Estimated Parameters BFGS-Method						
<i>Regimes</i>	<i>Thresholds</i>	$\beta$	<i>T-Stat(0)</i>	<i>Half-Life Time</i>	<i>95% Return Time</i>	<i>T-Stat(1)</i>
Upper	0.006	0.898338	19.671941	6.46539137	27.942957	2.2260
Lower	None					

Hong Kong, TAR (unmodified)

Estimated Parameters BFGS-Method						
<i>Regimes</i>	<i>Thresholds</i>	$\beta$	<i>T-Stat(0)</i>	<i>Half-Life Time</i>	<i>95% Return Time</i>	<i>T-Stat(1)</i>
Upper	0.006	0.8841332	26.153292	5.62858981	24.32636	3.4269
Lower	None					

Graph 10. Forex Forward-Forward Deviations (Part 3 Hong Kong)

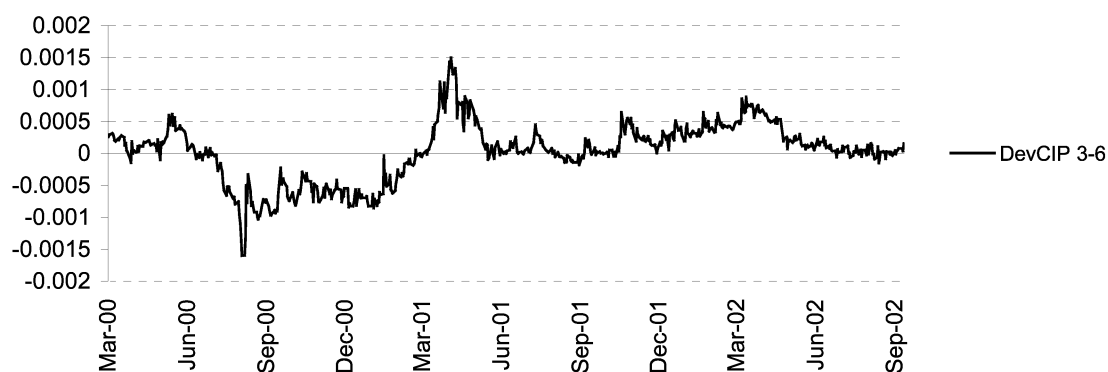


Table 3. Part 3 Comparison

May 2000 - August 2002

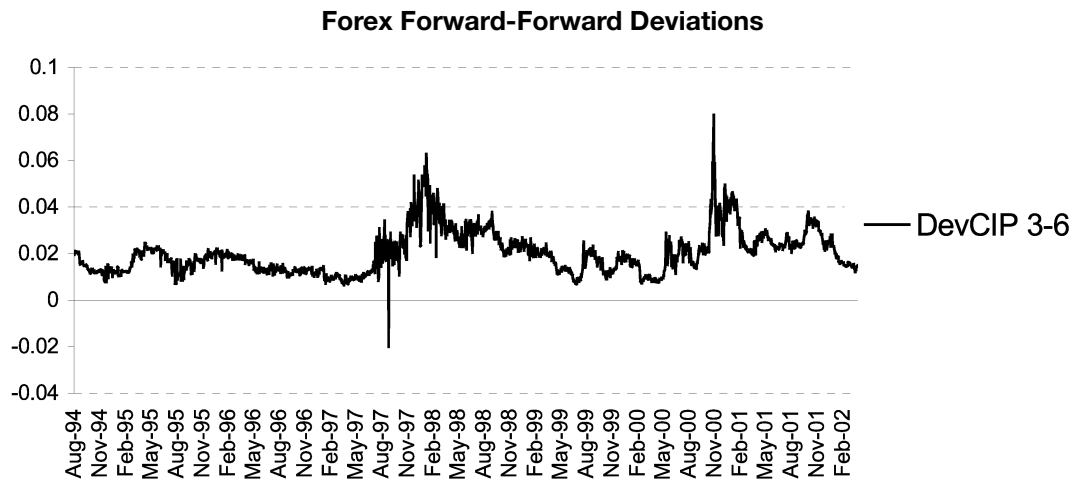
Estimated Parameters BFGS-Method						
<i>Regimes</i>	<i>Thresholds</i>	$\beta$	<i>T-Stat(0)</i>	<i>Half-Life Time</i>	<i>95% Return Time</i>	<i>T-Stat(1)</i>
Upper	0.00005	0.9096475	50.73474	7.31954561	31.63455	5.0391
Lower	None					

Hong Kong TAR (unmodified)

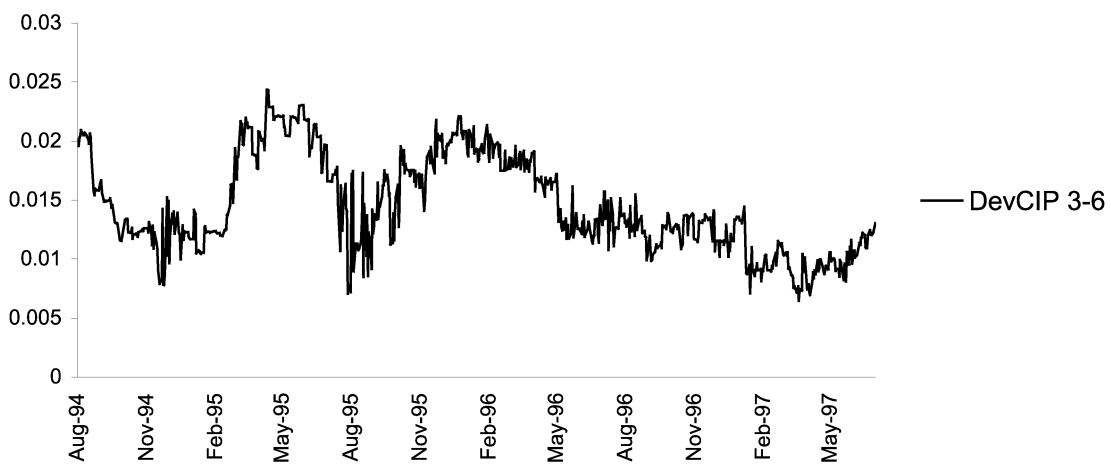
Estimated Parameters BFGS-Method						
<i>Regimes</i>	<i>Thresholds</i>	$\beta$	<i>T-Stat(0)</i>	<i>Half-Life Time</i>	<i>95% Return Time</i>	<i>T-Stat(1)</i>
Upper	0.00005	0.9581789	60.126629	16.2250424	70.123467	2.6236
Lower	None					

**Graph 11. The Philippines Foreign Exchange Forward-Forward Deviations**

These data were divided into 4 parts and the following research has been done separately for each part.



**Graph 12. Forex Forward-Forward Deviations (Part 1 The Philippines)**

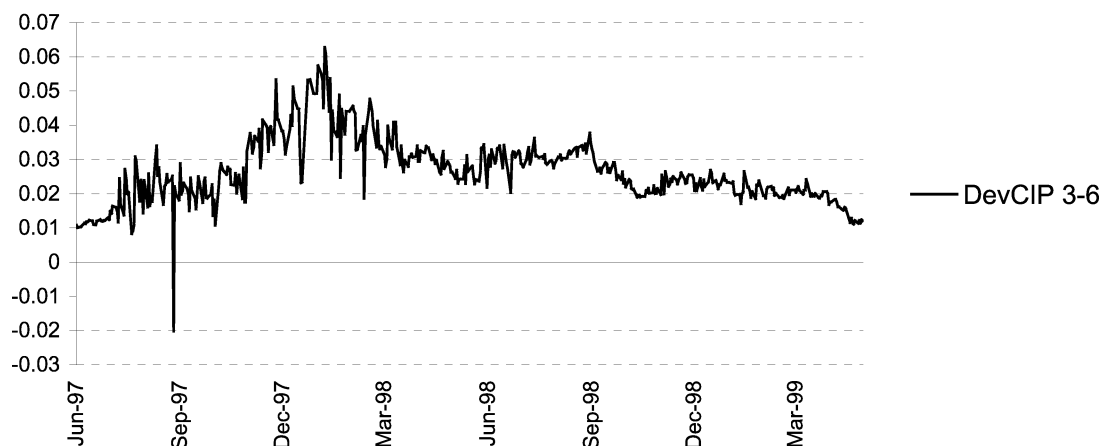


August 1994 - June 1997

MTAR

Estimated Parameters BFGS-Method						
<i>Regimes</i>	<i>Thresholds</i>	$\beta$	<i>T-Stat(0)</i>	<i>Half-Life Time</i>	<i>95% Return Time</i>	<i>T-Stat(1)</i>
Upper	0.017	0.9350798	40.956881	10.3264611	44.630222	2.8436
Lower	None					

**Graph 13. Forex Forward-Forward Deviations (Part 2 The Philippines)**

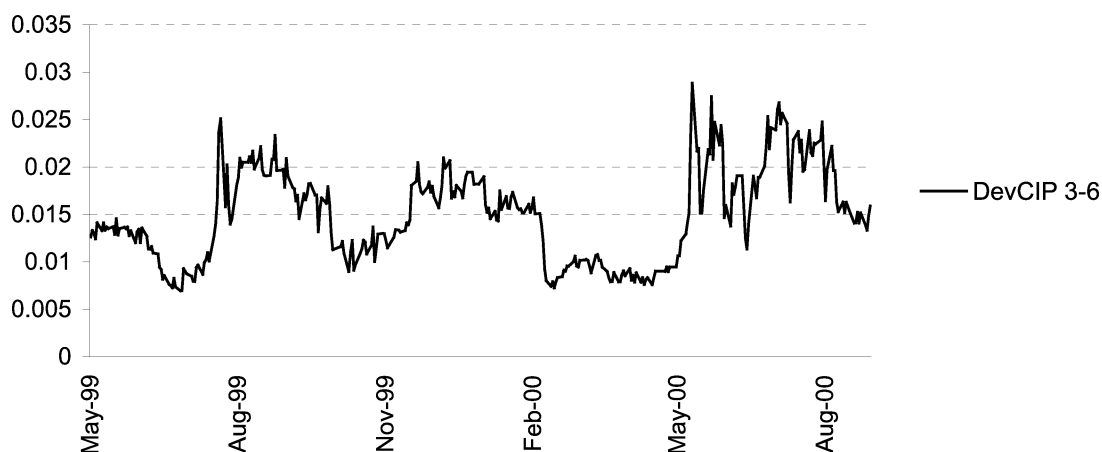


July 1997 - April 1999

MTAR

Estimated Parameters BFGS-Method						
<i>Regimes</i>	<i>Thresholds</i>	$\beta$	<i>T-Stat(0)</i>	<i>Half-Life Time</i>	<i>95% Return Time</i>	<i>T-Stat(1)</i>
Upper	0.025	0.8317062	35.485112	3.76146042	16.256761	7.1797
Lower	None					

**Graph 14. Forex Forward-Forward Deviations (Part 3 The Philippines)**

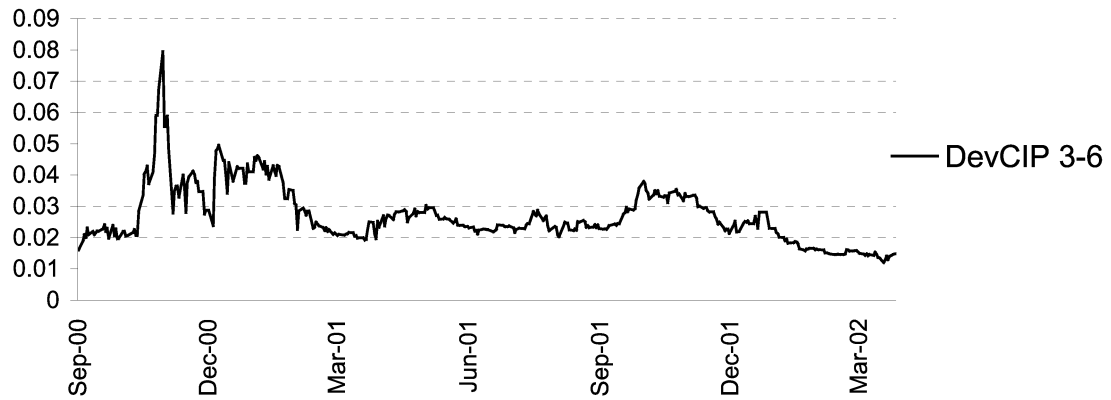


May 1999 - August 2000

MTAR

Estimated Parameters BFGS-Method						
<i>Regimes</i>	<i>Thresholds</i>	$\beta$	<i>T-Stat(0)</i>	<i>Half-Life Time</i>	<i>95% Return Time</i>	<i>T-Stat(1)</i>
Upper	0.012	0.8702267	31.004291	4.98661629	21.551797	4.6232
Lower	None					

Graph 15. Forex Forward-Forward Deviations (Part 4 The Philippines)



September 2000 - April 2002

MTAR

Estimated Parameters BFGS-Method						
<i>Regimes</i>	<i>Thresholds</i>	$\beta$	<i>T-Stat(0)</i>	<i>Half-Life Time</i>	<i>95% Return Time</i>	<i>T-Stat(1)</i>
Upper	0.024	0.6328683	11.212282	1.5150994	6.5481506	6.5048
Lower	None					