HONG KONG INSTITUTE FOR MONETARY RESEARCH

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HKIMR Working Paper No.20/2004

October 2004

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Abstract

Most theoretical analysis of flexible vs. fixed exchange rates take the degree of nominal rigidity to be independent of the exchange rate regime choice itself. But informal policy discussion often suggests that a credible exchange rate peg may increase internal price flexibility. This paper explores the relationship between exchange rate policy and price flexibility, in a model where price flexibility is an endogenous choice of profit-maximizing firms. A fixed exchange rate may increase the optimal degree of price flexibility by increasing the volatility of demand facing firms. We find that a unilateral peg, such as a Currency Board, adopted by a single country, will increase internal price flexibility, perhaps by a large amount. On the other hand, when an exchange rate peg is supported by bilateral participation of all monetary authorities (such as a monetary union), price flexibility is likely to be little affected, and may actually be less than under freely floating exchange rates.

Keywords: Monetary Policy, Collateral Constraints, Exchange Rate
JEL Classification: F0, F4

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I thank participants of the 2003 Bundesbank Spring Conference, Michael Dotsey, Fabio Ghironi, Luca Guerrieri, Roberto Perrotti, and especially three anonymous referees for very helpful comments on this paper. I thank SSHRC, Royal Bank of Canada for financial and the Bank of Canada for financial assistance.

1. Introduction

The classic argument for flexible exchange rates is that they enhance the ability of the economy to respond to shocks, in the presence of nominal rigidities (e.g. Friedman 1953). But a qualification to this is that by eliminating the use of the exchange rate as a mechanism for adjustment, an exchange rate peg may increase internal price flexibility within a country. This has been especially important in the analysis of the conditions for single (small) countries to follow unilateral 'hard peg' policies, fixing the exchange rate under a currency board or dollarization rule. Since these countries will generally not have access to compensating policy responses from the monetary authorities of the currency to which they are pegging, the need to increase internal price flexibility after a peg becomes more critical. Another area where this discussion is important is that of the impact of a monetary union on flexibility. To the extent that a single currency encourages price flexibility within the different regions of the monetary union, this will reduce the loss from the absence of exchange rate adjustment. To this extent, the economic case for a monetary union may be enhanced by the formation of the monetary union, as suggested by Frankel and Rose (1998). [1]

Is price flexibility likely to take place automatically in response to changes in monetary policy conditions, through the decisions of individual price setters? We could think of price stickiness as being determined by the trade-off between 'costs of price flexibility' (information or planning costs, for instance) and benefits of ex-post price adjustment. These benefits would be higher, the more volatile is the environment within which a price setter operates. If an exchange rate peg substantially increases the volatility of demand for their product, the elimination of the exchange rate as a policy lever may cause price setters to adjust more frequently.

This paper provides a theoretical investigation of the implications of exchange rate rules for the flexibility of nominal prices, in an economy where price flexibility is itself endogenous. In a two-country model, there are shocks to relative national demands, and country specific velocity shocks. Given this uncertainty, profit maximizing firms may choose ex-ante to incur a cost so as to have the flexibility to adjust their prices ex-post. Within this setting, we ask a) what features determine the equilibrium degree of price flexibility, and b) in what way does an exchange rate peg affect the degree of price flexibility?

The incentive for ex-post price flexibility for any one firm is higher, the greater is the variance of nominal demand it faces for its good. An increase in monetary variability will increase the variance of nominal demand. But the variance of nominal demand will also depend on the degree of price flexibility itself. This introduces a strategic interaction between the pricing decisions of firms. We find that, for standard parameter values, the incentive for flexibility is increasing in the total number of firms who choose to adjust their ex-post prices; there is a strategic complementarity in the choice of flexibility. If only a small number of price setters adjust their price, then there may be little incentive for the marginal price setter to pay the menu cost. But if all price setters choose to adjust, the volatility of prices will increase the overall volatility of demand facing a price setter, increasing the incentive for a given firm to adjust.

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1 In the discussion of EMU, the likelihood of wage and price flexibility being enhanced by the single currency was considered, e.g. OECD, (1999).

How does exchange rate policy affect the degree of price flexibility? The key feature of a fixed exchange rate is that it requires that monetary policy to adjust to internal and external shocks in lieu of exchange rate adjustment. Whether or not this enhances price flexibility depends on whether the policy increases the volatility of firm's demand. The answer to this depends on the type of shocks that occur, and the way in which the fixed exchange rate system operates.

We first focus on a one-sided peg, which describes a situation where one country fixes its exchange rate against a trading partner, and accepts sole responsibility for maintaining the peg. Our model predicts unambiguously that such an arrangement will increase internal price flexibility in the pegging country, while leaving foreign price flexibility unchanged. In a one-sided peg, the domestic monetary authority must respond to all shocks, domestic and foreign, in order to protect the peg. This must lead to an increase in the overall volatility of nominal demand facing firms. Therefore, more firms will choose to incur the costs of price flexibility.

However, a cooperative peg, involving active participation of all monetary authorities, has an ambiguous effect on price flexibility, depending on the source of shocks. If most shocks are 'real', coming from fluctuations in relative demand for one country's good relative to another, then a cooperative peg also will increase price flexibility in all countries. But if most shocks are 'monetary' coming from exogenous shocks to the velocity of money, then a cooperative peg will reduce price flexibility in all the pegging countries.

How big is the impact of an exchange rate change on price flexibility? By strongly linking the decisions of each agent with that of other agents, the presence of strategic complementarity allows for changes in the external environment to have a potentially very large effect on the equilibrium degree of price flexibility. We illustrate this point by exploring the effect of the exchange system on output and relative price volatility.

Holding the degree of price flexibility constant, a unilateral peg will lead to a substantial increase in output volatility, and a large fall in terms of trade volatility. But the peg itself increases the incentive for firms to have flexible prices. If there is substantial strategic complementarity in the choice of price flexibility, then there may be a very large increase in the share of price setters choosing the option of expost flexibility. For a standard parameterization of the model, we find that this indirect effect of the exchange rate peg on price flexibility can be of the same order of magnitude as the the direct effect of the increased volatility in nominal aggregate demand coming from the peg itself. As a result, the volatility of GDP remains essentially unchanged after a move from floating exchange rates to a unilateral peg. The volatility of the terms of trade however, is substantially reduced, because the peg tends to cause nominal prices to co-move positively across countries. The endogenous adjustment in price flexibility therefore can explain why a comparison of fixed and floating exchange rates may show little differences in the behavior of GDP, but substantial differences in relative price variability. This 'puzzle' has been discussed by Baxter and Stockman (1990), Flood and Rose (1995) and others.

How does the presence of endogenous price flexibility affect optimal monetary policy rules? In general, if monetary authorities wish to target the flexible price equilibrium allocation, they will set policy so that firms never have to adjust prices. As a result, the possibility of endogenous price flexibility will not affect the optimal monetary rule (see Dotsey King and Wolman (1999)). In our model, the flexible price allocation is inefficient due to the absence of complete international financial markets. As a result, an optimal monetary rule does not replicate the flexible price allocation. In principle, this could mean that the presence of endogenous price flexibility significantly alters optimal monetary rules, relative to an economy with exogenously sticky prices. But in our calibrated model, we find that optimal monetary rules are almost the same as those in an economy with exogenously sticky prices. An optimal monetary policy in the model allows for only a very very small degree of price flexibility.

The paper is related to a large recent literature evaluating the effects of monetary rules in sticky price equilibrium models . But our departure is in allowing for the degree of price stickiness itself to be an endogenous variable. In this respect, the paper is related to the literature on state-dependent pricing and menu-costs of price change (see Ball and Romer 1991, Dotsey, King and Wolman, 1999). The model is most closely related to Ball and Romer (1991). They show the possibility of multiple equilibrium, in an environment where price setters can choose ex-post whether to adjust prices, given a common menu cost of price change, within a one-country environment. Our analysis differs because we allow a distribution of firm specific menu costs, and we assume that price setters choose in advance whether or not to have the ex-post flexibility to adjust price. This is more in line with the view that a large change in monetary policy regime (e.g. fixing the exchange rate) may lead to structural changes in the flexibility of contracts within a monetary economy. Moreover, our focus is not primarily on multiple equilibrium, but more on the role of strategic complementarity in the choice of flexibility. Finally of course, we use a two country model.

The next section sets out the basic technology of endogenous price flexibility for a given firm. Section 3 incorporates this model into a two country general equilibrium environment. Section 4 examines the link between price flexibility and the exchange rate regime. Section 5 investigates the predictions of the model for output and relative price volatility, while section 6 discusses the optimal monetary policy under endogenous price flexibility. Some conclusions follow.

2. The Firm and the Choice of Price Flexibility

We first describe the decision faced by a single firm with respect to the choice of price flexibility. In the typical model of state *dependent pricing* [2], a firm chooses whether or not to adjust its price *ex-post*, given information on its demand and cost, by trading off the benefits of price adjustment relative to the direct (e.g. menu) costs of price change. By contrast, our assumption is that the firm must invest ex-ante in flexibility. That is, a firm must choose *ex-ante* whether to have the flexibility to adjust its price ex-post, after observing the realized state of the world. The firm incurs a fixed (labor) cost in order to have this flexibility. Roughly speaking, this decision may correspond more accurately to the way in which changes in monetary policy or other structural features of the economy would impact on the institutional characteristics of nominal price or wage setting.

[2] See, e.g. Dotsey King and Wolman (1999).

Let a firm i have the production function

$$Y_i = (H_i - D_i\Phi_i)^{\alpha} \tag{2.1}$$

where Y_i is the firms output, H_i is total employment, and Φ_i is a firm-specific fixed cost of flexibility. Assume that the firm knows Φ_i. We let D_i be an indicator variable, whereby $D_i = 1$ $(D_i = 0)$ if the firm chooses to (not to) incur the cost of ex-post price flexibility. The firm's production function indicates that it has some firm specific factor of production, together with which it combines labor to produce output for sale.

Assume that the firm faces market demand: [3]

$$X_i = (\frac{P_i}{P})^{-\lambda}X \tag{2.2}$$

where P_i is the firm's price, P is the (possibly stochastic) industry price, $\lambda > 1$ is the firm's own elasticity of demand, and X is the (stochastic) total market demand shock. Assume the the firm faces a wage W (also stochastic) . From the production technology (2.1), the firms total operating cost is

$$W\,(Y_i)^{\frac{1}{\alpha}} + WD_i\Phi_i \tag{2.3}$$

Assume that the firm evaluates its expected profits using a stochastic discount factor Γ. [4] Then discounted expected profits may be written as

$$E\Gamma\big(P_i(\frac{P_i}{P})^{-\lambda}X - W((\frac{P_i}{P})^{-\lambda}X)^{\frac{1}{\alpha}} - WD_i\Phi_i\big) \tag{2.4}$$

We further assume that the firm knows the distribution of the discount factor, the market demand and the wage.

The firm chooses P_i to maximize (2.4). If $D_i = 1$, then the firm can choose its price after observing P, X and W, and it sets the following price:

$$\tilde{P}_i = \delta[W^{\alpha}(\hat{X})^{1-\alpha}]^{\omega} \tag{2.5}$$

where $\delta = \big(\frac{\lambda}{\alpha(\lambda-1)}\big)^{\alpha\omega}$, $\omega = \frac{1}{\alpha+\lambda(1-\alpha)}$ and $\hat{X} = P^{\lambda}X$. When $\alpha = 1$, the firm's price is a constant markup over the wage. But when $\alpha < 1$, the optimal price will depend on a geometric average of the wage and market demand.

3 We use this form of demand because it is obtained from the general equilibrium model analyzed below.

4 In the next section, we determine Γ from the preferences of the firm's household-shareholders.

When $D_{it} = 0$, the firm must choose the price *ex-ante*. In that case, its optimal price is given by:

$$\bar{P}_i = \delta \frac{E[\Gamma W(\hat{X})^{\frac{1}{\alpha}}]^{\alpha\omega}}{E[\Gamma \hat{X}]^{\alpha\omega}} \quad (2.6)$$

When the wage and market demand are known ex-ante, (2.5) and (2.6) give the same answer. But in general the two prices will differ, even in expectation, as the distribution of market demand and wages will influence the mean pre-set price that the firm sets.

Now substituting (2.5) and (2.6) respectively into the expected profit function (2.4), we may evaluate the firm's expected profits (excluding fixed costs) under $D_i = 1$ and $D_i = 0$. Let $\Theta = \{\Gamma, W, \hat{X}\}$, then:

$$\tilde{V}(\Theta) = \Psi E\Gamma(W^{\alpha(1-\lambda)}\hat{X})^{\omega} \quad (2.7)$$

$$\bar{V}(\Theta) = \Psi(E\Gamma W\hat{X}^{\frac{1}{\alpha}})^{(1-\lambda)\alpha\omega}(E\Gamma\hat{X})^{\lambda\omega} \quad (2.8)$$

where $\Psi = \delta^{-\lambda}(\delta - 1)$. What determines whether the firm incurs the fixed cost of price flexibility? The firm will choose $D_i = 1$ whenever the gain in discounted expected profits exceeds the discounted expected fixed costs. That is, $D_i = 1$ whenever

$$\tilde{V}(\Theta) - \bar{V}(\Theta) \geq E\Gamma W\Phi_i$$

Since Φ_i is known to the firm ex-ante, $E\Gamma W\Phi_i = \Phi_i E\Gamma W$. We can therefore rewrite this condition as

$$\Delta(\Theta) \equiv \frac{[\tilde{V}(\Theta) - \bar{V}(\Theta)]}{E\Gamma W} \geq \Phi_i \quad (2.9)$$

where $\Delta(\Theta)$ represents the gain to price flexibility.

2.1 Approximation of 2.9

To provide analytical results in the next section, we can evaluate the gains to price flexibility by taking a second order logarithmic approximation to $\Delta(\Theta)$ around the mean value $E\ln(\Theta)$. In the Appendix, it is shown that

$$\Delta(\Theta) \approx \frac{\Omega\alpha}{2}\left[\sigma_w^2 + (\frac{(1-\alpha)}{\alpha})^2\sigma_x^2 + 2\frac{(1-\alpha)}{\alpha}\sigma_{\hat{w}x}\right] > 0 \quad (2.10)$$

where $\Omega = \frac{V(\bar{\Theta})}{\bar{\Gamma}\bar{W}}\lambda(\lambda-1)\omega^2 > 0,\ V(\bar{\Theta})$ represents profits evaluated at the unconditionalmean $E\ln\Theta$, and $\sigma_w^2,\ \sigma_x,\ \sigma_{wx}$ represents the variance of the wage, market demand, and their covariance.

From (2.10), we see that, up to a second order, the incentive for a firm to incur the costs of price flexibility depends on the variance of the wage, the variance of market demand, and their covariance. Note that if $\alpha = 1$, so that marginal cost is independent of output, then uncertainty in market demand gives no incentive for flexible prices, and the gains from flexibility depend only on uncertainty in wages. Intuitively, if $\alpha = 1$, then optimal expected profits are linear in market demand, and further, if the wage is known, then the firm's price is the same whether it is set before or after Θ is observed. In this case, there is no gain to price flexibility. More generally however, optimal profits are convex in W when prices are flexible, but linear in W under a fixed price. Hence, wage volatility raises expected profits when prices are flexible relative to expected profits with preset prices. When $\alpha < 1$, optimal profits are concave in market demand $\hat{X}$, either when prices are flexible or fixed. But, intuitively the optimized profit function is more concave in demand when prices are fixed than when they are flexible. Hence, uncertainty in market demand increases the benefits to price flexibility, for $\alpha < 1$.

Finally, we note that (2.10) does not depend on the properties of the stochastic discount factor Γ_t. Up to a second order approximation, the discount factor affects profits of fixed and flexible price firms in the same way.

2.2 Determination of Price Flexibility in the Aggregate

The left hand side of (2.9) is common to all firms. Hence, firms will differ in their choice of price flexibility solely due to differences in their specific fixed costs of flexibility. Without loss of generality, let each firm i draw from a distribution of fixed costs, $\Phi(i)$, described by; $\Phi(0) = 0, \ \Phi'(i) > 0$. Hence firms are ranked according their fixed cost of flexibility. In that case, we may describe the determine of price flexibility in the aggregate as the measure z of firms, $0 \leq z \leq 1$ who choose to incur the fixed cost of price flexibility. Then z is determined by the following condition:

$$\Delta(\Theta) = \Phi(z), \quad 0 \leq z \leq 1 \tag{2.11}$$

$$\Delta(\Theta) > \Phi(1) \quad z = 1 \tag{2.12}$$

This condition gives a link between the underlying uncertainty facing firms and the aggregate degree of flexibility in the economy. So far however, we have left Θ unexplained. In the next section we develop a two country model that identi¯es the macroeconomic sources of the uncertainty facing firms.

3. A Two Country Model

Consider a two country world economy, where countries are called 'home' and 'foreign'. Foreign variables are denoted with an asterisk. In each country, there are consumers and firms, who have a single period horizon. There is a continuum of households in each country along the unit interval, consuming both home and foreign goods. Households receive income from wages and the ownership of firms. Firms have the production technology as described by the previous section, and sort themselves into two categories; those with fixed prices, and those with flexible prices.

3.1 Households

The home country household $i,\ i \in (0,1)$, has preferences given by:

$$\ln\left(C(i)\right) + \chi \ln\left(\frac{M(i)}{P}\right) - \frac{\eta}{1+\psi} H(i)^{1+\psi} \tag{3.1}$$

where $C(i)$ is a composite of the consumption of home and foreign goods, given by:

$$C(i) = \left(\frac{C_h(i)}{\gamma}\right)^{\gamma} \left(\frac{C_f(i)}{1-\gamma}\right)^{1-\gamma} \tag{3.2}$$

and P is the price index, given by $P = (P_h)^{\gamma} \left(SP_f^*\right)^{1-\gamma}$, where P_f^* is the foreign currency price of foreign goods. γ represents the relative preference for home goods. $M(i)$ is the quantity of domestic money held. We assume χ is a random variable which will capture shocks to the consumption velocity of money. In addition, we let γ, the weight of the home good in composite consumption, also be a random variable, with mean 0.5.

Assume foreign country preferences are identical to home, except that foreign household's value foreign money, and assume that χ^* (the foreign velocity shock) and χ are i.i.d. The random foreign composite consumption weight γ is the same as that of home residents.

Consumption of home and foreign goods are differentiated, so that for household i, the home good consumption and price indices are

$$C_h(i) = \left(\int_0^1 C_h(i,j)^{1-\frac{1}{\lambda}} dj\right)^{\frac{1}{1-\frac{1}{\lambda}}}, \qquad P_h = (\int_0^1 P_h(j)^{1-\lambda} dj)^{\frac{1}{1-\lambda}} \tag{3.3}$$

where $\lambda > 1$. The indices for the foreign good are analogous.
Home household i faces the budget constraint:

$$PC(i) + M(i) = W(i)H(i) + M_0(i) + T(i) + \Pi \tag{3.4}$$

where $M_0(i)$ is initial money holdings, $T(i)$ is the transfer from the monetary authority, and Π is total profits of the final good firms.

Households choose money balances, labor supply, and consumption of each good to maximize utility, subject to their budget constraint[5] . We get the demand for each good, $C_h(i)$ and $C_f(i)$, that of money balances, and implicit labor supply as:

$$C_h(i,j) = \left(\frac{P_h(j)}{P_h}\right)^{-\lambda} C_h(i), \quad C_h(i) = \frac{\gamma PC(i)}{P_h} \quad C_f(i) = \frac{(1-\gamma)PC(i)}{P_f} \tag{3.5}$$

$$M(i) = \chi PC(i) \qquad W = \eta H^{\psi} PC(i) \tag{3.6}$$

[5] Households act after the realizations of the preference shocks are observed.

3.2 Firms

Firms in each country set prices, based on the technologies described in the previous section, and demand coming from home and foreign consumers. In the home country for instance, a measure z of firms set prices $\tilde{P}_h(j)$ after the state of the world is realized, and $(1-z)$ set prices $\bar{P}_h(j)$ in advance. The condition given by (2.11) (or 2.12) determines the size of the flexible price sector. Total profits of all firms are written as:

$$\int_0^z \tilde{P}_h(j)\tilde{Y}(j)dj + \int_z^1 \bar{P}_h(j)\bar{Y}(j)dj - \int_0^1 WH(i)di \tag{3.7}$$

3.3 Equilibrium

We focus on symmetric equilibria where all households and firms within a country are alike. Equilibrium is defined in the usual way. Given money market clearing, $M = M_0 + T$, households ex post budget constraints are given by:

$$PC = z\tilde{p}_h\tilde{Y}_h + (1-z)\bar{p}_h\bar{Y}_h \tag{3.8}$$

The goods market for each category of firm implies that

$$\tilde{Y}_h = \left(\frac{\tilde{P}_h}{P_h}\right)^{-\lambda} \gamma \left[\frac{PC}{P_h} + \frac{SP^*C^*}{P_h}\right] \tag{3.9}$$

$$\bar{Y}_h = \left(\frac{\bar{P}_h}{P_h}\right)^{-\lambda} \gamma \left[\frac{PC}{P_h} + \frac{SP^*C^*}{P_h}\right] \tag{3.10}$$

Labor market clearing implies

$$H = z\tilde{Y}^{\frac{1}{\alpha}} + (1-z)\bar{Y}^{\frac{1}{\alpha}} + \int_0^z \Phi(z)dz \tag{3.11}$$

where the last term on the right hand side denotes the fixed cost incurred by the measure z of firms that choose price flexibility.

Analogous conditions hold for the foreign economy.

We may define aggregate real GDP by aggregating over fixed and flexible price firms. Thus:

$$Y = \frac{z\tilde{P}_h\tilde{Y} + (1-z)\bar{P}_h\bar{Y}}{P_h}$$

3.4 Equilibrium for given price flexibility

For given $\mathcal{Z}$ and z^*, the equilibrium is very simple to characterize. Using the definition of aggregate GDP and the household budget constraint, that $PC = P_h Y$. Hence we may write the money market equilibrium condition as

$$M = \chi P_h Y \tag{3.12}$$

Using this in combination with the goods market equilibrium (3.9) and (3.10), and aggregating, we get solutions for both the exchange rate and GDP as:

$$S = \frac{1-\gamma}{\gamma}\frac{M\chi^*}{M^*\chi}, \qquad Y = \frac{M}{P_h \chi} \tag{3.13}$$

A home country monetary expansion causes exchange rate depreciation, while a positive home country velocity shock causes an appreciation. A shift in relative world demand towards home goods (rise in γ) causes an appreciation. Home GDP is determined by the value of home real balances, in terms of home goods, relative to the home velocity shock.

Since demand for the individual firm may be defined from (3.9) and (3.10), and wage determination is given from (3.6), we may use (2.5) and (3.12) to define the flexible price firm's price as:

$$\tilde{P}_t = \delta\left[\eta H^{\psi\alpha} P_h^{(\lambda-1)(1-\alpha)}\frac{M}{\chi}\right]^{\omega} \tag{3.14}$$

The appropriate discount factor for firms is given by $\Gamma = (PC)^{-1}$. [6] Then we can write $\bar{P}_h$ as:

$$\bar{P}_h = \delta\frac{E[\eta H^{\psi}(P_h^{\lambda-1}\frac{M}{\chi})^{\frac{1}{\alpha}}]^{\alpha\omega}}{E[P_h^{\lambda-1}]^{\alpha\omega}} \tag{3.15}$$

The domestic good price index is

$$P_h = \left[z\tilde{P}_h^{1-\lambda} + (1-z)\bar{P}_h^{1-\lambda}\right]^{\frac{1}{1-\lambda}} \tag{3.16}$$

Using this, (3.9), (3.10), and (3.11), we may write employment as

$$H = \left[z(\frac{\tilde{P}_h}{P_h})^{-\frac{\lambda}{\alpha}} + (1-z)(\frac{\bar{P}_h}{P_h})^{-\frac{\lambda}{\alpha}}\right](\frac{M}{\chi P_h})^{\frac{1}{\alpha}} + \int_0^z \Phi(z)dz \tag{3.17}$$

6 This is the households marginal utility of a dollar of home currency.

3.5 The determination of optimal price flexibility

To determine equilibrium price flexibility, we use condition (2.11) (or 2.12) from the previous section, in combination with the values of Γ, W, and $\hat{X}$ implied by the two-country general equilibrium model. From the model equilibrium, market demand and the wage written as:

$$\hat{X} = P_h^{\lambda-1}\frac{M}{\chi} \qquad W = \eta H^{\psi}\frac{M}{\chi} \tag{3.18}$$

This, in combination with equations (3.14) - (3.17), and (2.11) determine the values of W, $\hat{X}$, $\tilde{P}_h$, $\bar{P}_h$, P_h, H, and z for the home economy.

Notice that the simple structure of the model implies that the two economy's dichotomize. The home wage, demand, prices, employment, and equilibrium price flexibility are determined solely by the behavior of home nominal aggregate demand $\frac{M}{\chi}$. Conditional on the domestic money supply, the equilibrium z is independent of the distribution of foreign shocks and foreign monetary policy, as well as movements in the share parameter γ. This result arises from the unit elasticity of substitution between home and foreign goods. For given M, shocks to foreign demand are offset by movements in the exchange rate, so as to leave overall demand for the home country's good unchanged.

In general the model has no exact analytical solution. In the quantitative section below, we report results from the numerical solution of the exact (stochastic) model, for a given calibration. Here however we describe an approximate solution using the second-order approximation used in (2.10). In order to determine the gains to price flexibility using (2.10) we must obtain the variance of $ln(W)$ and $ln(\hat{X})$. We may write these as:

$$\ln(W) = \psi \ln(H) + \ln(M) - \ln(\chi) \tag{3.19}$$

$$\ln(X) = (\lambda - 1)\ln(P_h) + \ln(M) - \ln(\chi) \tag{3.20}$$

For given z, the model is log linear except for the price index equation (3.16) and the aggregate employment term (3.17). In the Appendix, it is shown that P_h and H may be approximated around the mean values $E\ln P_h$ and $E\ln H$ (where we use lower-case letters to denote deviations from means i.e. $p_h = ln(P_h) - Eln(P_h)$) as:

$$p_h = \frac{\varphi(z)\omega(1+\psi\varsigma)(m-\hat{\chi})}{1-(1-\alpha)\omega(\lambda-1)\varphi(z)+\psi\omega\varphi(z)} \tag{3.21}$$

$$h = \frac{\varsigma}{\alpha}(m - p_h - \hat{\chi}) \tag{3.22}$$

Here $\hat{\chi}$ represents the log deviation of the velocity shock from its mean value, $\varsigma < 1$ is a constant term given in the Appendix, and $\varphi(z)$ is an increasing function of z, which satisfies $\varphi(0) = 0,\ \varphi'(z) > 0,\ \varphi''(z) > 0$ and $\varphi(1) = 1$. Note that, by the definition of ω, we have $(1-\alpha)\omega(\lambda-1) < 1$.

Substituting (3.21) and (3.22) into (3.20) and (3.19), and then substituting into (2.10), we obtain the conditions

$$\frac{\Omega}{2\alpha}\left[\frac{(1+\psi\varsigma)}{1-(1-\alpha)(\lambda-1)\omega\varphi(z)+\psi\omega\varphi(z)}\right]^2(\sigma_m+\sigma_{\hat{\chi}}+2\sigma_{m\hat{\chi}})=\Phi(z), \quad 0\leq z\leq 1 \tag{3.23}$$

$$\frac{\Omega}{2\alpha}(1+\psi\varsigma)^2(\sigma_m+\sigma_{\hat{\chi}}+2\sigma_{m\hat{\chi}})\geq\Phi(1), \quad z=1 \tag{3.24}$$

Figure 1a illustrates the determination of z. The VV locus illustrates the left- hand side of condition (3.23). This represents the benefit of price flexibility to the marginal price setter as measured along the horizontal axis. This is higher, the higher is the variance of nominal aggregate demand $m-\hat{\chi}$ (which also equals $p+c$). The CC locus represents the fixed flexibility cost facing the marginal price setter. The CC locus is upwards sloping, by assumption; marginal firms have higher costs of price flexibility. The VV locus is also upward sloping, under the condition that $\lambda > 1+\frac{\psi\varsigma}{1-\alpha}$. This is explained by the link between the decisions made by all other firms and the incentive of any one firm to have flexible prices. To see this relationship, focus on the flexible price firm's optimal pricing policy, obtained from 3.14, which is written as:

$$\tilde{P}_h=\delta[\frac{H^{\psi\alpha}\frac{M}{\chi}(P_h^{\lambda-1}\frac{M}{\chi})^{1-\alpha}}{\theta}]^{\omega}$$

Say that there is a money shock which gives rise to a desire for the flexible price firm to adjust its price upwards (both because the money shock increases demand directly, and increases the wage). The extent to which the firm will adjust depends on the number of other firms z who also adjust. When other firms adjust, this gives rise to two opposing forces. First, given that other firms raise their price, market demand for any one firm rises, given that $\lambda-1>0$. This leads the firm to want to raise its price by more, so long as $(1-\alpha)>0$, since its marginal cost is rising. But counter to this, the rise in the price of other firms will reduce real balances $\frac{M}{P_h}$, reducing the home demand for labor. This reduces the real wage facing the firm, and reducing its desired price adjustment. If the first effect dominates, then the price response of the firm to a money shock is increasing in z. If the second effect dominates, the price response is declining in z. The first effect will dominate whenever $(\lambda-1)(1-\alpha)>\psi\varsigma$. An equivalent interpretation can be given for the case of χ shocks.

A given firm's ex-ante incentive for price flexibility depends on how much it would wish to adjust its price in response to a shock. If $(\lambda-1)(1-\alpha)>\psi\varsigma$, then in response to a shock, the firm will have a greater incentive to adjust its price, the greater is the measure of other firms adjusting. Hence the VV curve is both upward sloping in z. Intuitively, this is more likely, the flatter is labor supply (lower is ψ), the higher is the market elasticity of demand λ, and the more upward sloping is marginal cost (lower is α). If the VV curve is upward sloping, there is a *strategic complementarity* in the pricing decisions of firms; the greater the measure of other firms adjusting to a money shock, the greater is the incentive of any one firm to adjust its own price. Moreover, VV is also convex in z if $(\lambda-1)(1-\alpha)>\psi\varsigma$, so that the marginal incentive to change prices as an additional firm changes its price is higher, the greater the number of firms already changing prices. In the opposite case, when $(\lambda-1)(1-\alpha)<\psi$, the VV curve is actually downward sloping, and there is a strategic substitutability between the pricing decisions of firms. In the discussion below, we find that the conventional calibration suggests that $(\lambda - 1)(1 - \alpha) > \psi\varsigma$. In light of this, we focus henceforth on the case where the VV curve is upward sloping.

It is clear that there is the possibility of multiple equilibrium. While Figure 1a describes the case of a unique equilibrium, Figure 1b characterizes a situation where the VV curve intersects twice with the CC curve. There are three equilibria, corresponding to low z, $z = 1$, and an intermediate value of z (unstable based on the usual reasoning). In the low z equilibrium, a small fraction of firms choose price flexibility, weakening the incentives for other firms to have flexible prices. But when $z = 1$, the volatility of demand is so great that all firm's will willingly pay the costs for flexibility, because all others do. Therefore, multiple equilibria are generated by strategic complementarity in price setting. This strategic complementarity, and the possibility of multiple equilibrium, is greater, the lower is α, the lower is ψ, and the higher is λ.

In general, for different assumptions regarding $\Phi(i)$, there may be multiple crossing points. An equilibrium with high price flexibility is not necessarily associated with full flexibility.

We may state a condition for a unique equilibrium as:

Condition 1

If $\Phi(i)$ is uniform, so that $\Phi(i) = \bar{\Phi}i$, for $i > 0$, then there is a unique equilibrium whenever $V(\Theta)\lambda(\lambda - 1)(\sigma_m^2 + \sigma_\chi + 2 * \sigma_{m\chi}) < \bar{\Phi}$

The left hand side of this expression gives the value of the VV curve at $z = 1$, while the right and side gives the value the CC curve at $z = 1$. Since in this case CC curve is a straight line, and VV is convex, so long as VV falls below the CC curve at $z = 1$, then a unique equilibrium is assured.

4. Price Flexibility and the Exchange Rate Regime

We now focus on the impact of monetary policy and the exchange rate regime on the equilibrium degree of price flexibility. We assume henceforth that the equilibrium is unique. From (3.23), it is immediate to see that an increase in the volatility of money or velocity will increase the degree of price flexibility. To see how the exchange rate regime will affect price flexibility, we note that the exchange rate, in log deviation form, may be written as

$$s = m - m^* - (\hat{\chi} - \hat{\chi}^*) - 2\hat{\gamma} \tag{4.1}$$

There are different ways to define an exchange rate policy. This requires us to specify both the form of the monetary rules, as well as the degree to which each country participates in the monetary policy. Since our objective in this section is just to describe the link between exchange rate policy and price flexibility, we focus on a simple monetary rule where the authorities of one or both countries target the exchange rate directly. This has the advantage that it allows for variation in the importance that exchange rate stability plays in policy.[7]

With respect to the degree to which each country participates in the exchange rate policy, we describe two alternatives. A unilateral or one-sided policy is a situation where one country alone follows a monetary rule to target the exchange rate. Alternatively, a bilateral (or cooperative) exchange rate policy is one where both monetary authorities target the exchange rate [8] In a unilateral policy, the home monetary authority follows the rule $m = -\mu s$, where μ is the degree of exchange rate intervention, and the foreign country maintains a passive monetary rule, $m^* = 0$. Under a bilateral policy, both home and foreign monetary authorities target the exchange rate, using the rules $m = -\frac{\mu}{2}$ and $m^* = \frac{\mu}{2}$. In both cases, a value of $\mu = 0$ corresponds to a freely floating exchange rate, and $\mu \to \infty$ corresponds to a fixed (or pegged) exchange rate.

Under these intervention rules (whether unilateral or bilateral), the exchange rate can be described as

$$s = \frac{\hat{\chi} - \hat{\chi}^* - 2\hat{\gamma}}{1 + \mu}$$

Using this, and (3.23), we may establish:

Proposition 1

a) The degree of price flexibility z is higher under a unilateral peg than under a freely floating exchange rate. b) in the absence of velocity shocks, z is uniformly increasing in the degree of exchange rate intervention under a unilateral peg.

Proof: Under the assumptions made, z is determined by

$$\frac{\Omega}{2\alpha}(1 + \psi\varsigma)^2 \left(\frac{\mu^2 + 1}{(1 + \mu)^2} \sigma_\chi^2 + \frac{\mu^2}{(1 + \mu)^2} 4\sigma_\gamma^2 \right) = \Psi(z) \tag{4.2}$$

where $\Psi(z) = \bar{\Phi} z(1 + \varphi(z)\psi\varsigma\omega - (\lambda - 1)(1 - \alpha)\varphi(z)\omega)^2$

The first part of the proposition follows because the left hand side is higher when $\mu \to \infty$ (fixed exchange rate) than under $\mu = 0$ (floating exchange rate). Then, as long as the equilibrium is unique, the right hand side must be increasing in z.

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7 Below we compare this to a situation where monetary policy can directly target the stochastic disturbances. Note that the monetary rules here are not chosen optimally, in a welfare sense. We describe the welfare maximizing monetary rules in section 4 below.

8 These definitions were first made by Helpman (1980).

The second part of the proposition follows because, without velocity shocks (i.e. $\sigma_\chi^2 = 0$), the left hand side of the above condition is always increasing in μ.

To see the result more intuitively, note that equilibrium price flexibility will be higher, whenever the variance of $m - \hat{\chi}$ is higher. But in order to keep the exchange rate from changing in face of relative demand shocks, the variance of m must rise. Thus, in face of γ shocks, a one-sided peg tends to increase z. Without relative demand shocks, *var*$(m - \hat{\chi})$ is equal to *var*$(\hat{\chi})$, both under a floating exchange rate and under a unilateral peg. Although the peg offsets χ shocks, it must adjust the money supply to prevent from affecting the exchange rate, so as to leave *var*$(m - \hat{\chi})$ unchanged. Hence, when relative demand and velocity shocks are put together (and velocity shocks have equal variance), *var*$(m - \hat{\chi})$ must be higher in a one sided peg than under a floating exchange rate.

This suggests that a policy of pegging the exchange rate should enhance the price flexibility of the economy, if the exchange rate rule takes the form of a one-sided or unilateral peg, and velocity shocks are equally volatile across countries. The one-sided intervention rule will always increase the volatility of aggregate demand facing price setters. How does this compare to a bilateral pegged exchange rate? In this case, we have:

Proposition 2

The degree of price flexibility z may be higher or lower with a bilateral exchange rate peg than a freely floating exchange rate, depending on the size of relative demand shocks and velocity shocks. In the absence of relative demand shocks, z is lower under a bilateral peg.

Proof: In this case, z is determined by

$$\frac{\Omega}{2\alpha}(1+\psi\varsigma)^2\left(\frac{\frac{\mu^2}{2}+\mu+1}{(1+\mu)^2}\sigma_\chi^2+\frac{\mu^2}{(1+\mu)^2}\sigma_\gamma^2\right)=\Psi(z) \tag{4.3}$$

When $\mu \to \infty$, the variance terms inside the expression (4.3) become $\sigma_\gamma + \frac{1}{2}\sigma_\chi$. From this condition, we see that, without relative demand shocks, the volatility of nominal aggregate demand is strictly lower under a bilateral peg than under a freely floating exchange rate. Moreover, because each monetary authority cooperates in offsetting demand shocks, the volatility of aggregate demand specifically due to relative demand shocks is reduced, relative to that in a one-sided peg.

Intuitively, under a bilateral peg, in the case of velocity shocks alone, then *var*$(m - \chi)$ is lower, because countries cooperate in eliminating these shocks, rather than putting the onus all on the pegging country. As a result, the volatility of aggregate demand due to velocity shocks in both countries is reduced, whereas with a one-sided peg, the pegging country has to absorb the full effect of the velocity shock in the foreign country. Likewise, a bilateral peg also reduces the volatility of aggregate demand due to relative demand shocks, as compared with a one-sided peg, again because both countries respond to relative demand shocks.

Proposition 2 Corollary

There is always more price flexibility in a country that follows a one-sided peg than in a country that engages in a bilateral fixed exchange rate. Proof: this is demonstrated in the previous discussion.

The variance of nominal aggregate demand is always higher under a one sided peg. Hence, price flexibility must be greater in this case. Note also that in the cooperative peg, z and z^* are equal. The cooperative peg affects price flexibility in both countries, whereas the one-sided peg affects price flexibility in the pegging country alone.

From Propositions 1 and 2, we see that the question of whether a pegged exchange rate enhances price flexibility depends on the nature of the shocks as well as the nature of the peg. When relative demand shocks are the principal source of exchange rate fluctuations, then an exchange rate peg will enhance price flexibility, and more so in a country that adopts a one-sided peg. In order to stabilize the exchange rate following γ shock, countries must follow a pro-cyclical monetary policy, which increases the variance of nominal aggregate demand, and hence encourages more price flexibility on the part of firms. But when all exchange rate volatility is caused by velocity disturbances, an exchange rate peg will either leave price flexibility unchanged (in a one sided peg), or actually reduce overall price flexibility (in a bilateral peg).

How would these results be altered if we instead made the assumption that countries could fix the exchange rate by directly reacting to the shocks themselves, rather than by indirectly doing so by way of an exchange rate intervention rule? Essentially the same conclusions apply. In the case of a one-sided peg, the monetary rule given by $m = \hat{\chi} - \hat{\chi}^* + 2\hat{\gamma}$ keeps the exchange rate fixed. This would increase the *var*$(m - \hat{\chi})$ due to relative demand shocks, while leaving the variance due to velocity shocks un- changed, and hence would increase the degree of price flexibility. In a bilateral peg, the rules $m = \hat{\chi} - \hat{\gamma}$ and $m^* = \hat{\chi}^* + \hat{\gamma}$ ensure a fixed exchange rate. They eliminate the component of *var*$(m - \hat{\chi})$ due to velocity shocks, but increase the component of this variance due to relative demand shocks.

We have assumed that variance of velocity shocks is equal in the two countries. But imagine that $\sigma_\chi^2 > \sigma_{\chi^*}^2$. We might think of this as a case where overall monetary/financial stability is higher in the foreign country, and the home country chooses a pegged exchange rate in order to 'import' stability from abroad. This has been a common rationale for fixed exchange rates in countries with a history of monetary instability, especially in Latin America. Under this change, z is determined by the condition:

$$V(\bar{\Theta})\lambda(\lambda-1)\omega^2\left(\frac{\mu^2}{(1+\mu)^2}\sigma_{\chi^*}^2\frac{1}{(1+\mu)^2}\sigma_\chi^2+\frac{\mu^2}{(1+\mu)^2}4\sigma_\gamma^2\right)=\Psi(z) \tag{4.4}$$

Now it is no longer necessarily true that the left hand side is increasing in μ. If σ_χ^2 is sufficiently greater than $\sigma_{\chi^*}^2$, and relative demand shocks are unimportant, then even a one-sided peg can reduce overall aggregate demand volatility, and reduce the equilibrium degree of price flexibility. If a country follows a policy of pegging its exchange rate to import monetary stability from abroad, then the overall instability of nominal aggregate demand may fall rather than increase.

5. Output and Relative Price Stability

The traditional view of floating exchange rates (e.g. Friedman (1953)) argues that the exchange rate acts as a 'shock absorber'. A freely floating exchange rate helps to stabilize output in response to relative demand shocks, because it allows for a greater adjustment of relative prices. This suggests that the volatility of output should be higher under an exchange rate peg than under a float, while the volatility of the terms of trade should be lower. In our model, home country output is given by $Y = \frac{M}{P_h \chi}$. Taking a linear approximation, using the approximation for the home price index given by 3.21, we for we can write:

$$y = m - \hat{\chi} - \frac{\varphi(z)\omega(1+\psi\varsigma)(m-\hat{\chi})}{1-(1-\alpha)\omega(\lambda-1)\varphi(z)+\psi\omega\varsigma\varphi(z)}$$
$$= \frac{(m-\hat{\chi})(1-\varphi(z))}{1-(1-\alpha)\omega(\lambda-1)\varphi(z)+\psi\omega\varsigma\varphi(z)} \quad (5.1)$$

From this expression, we may establish:

Proposition 3

If an exchange rate peg increases the volatility of output for a given degree of price flexibility, then it will also increase the degree of price flexibility.

Proof: Expression (5.1) makes clear that output volatility will rise, for a given z, whenever the volatility of $m-\hat{\chi}$ rises. But this is exactly the same condition for an increase in price flexibility under Propositions 1 and 2.

Thus, holding z constant, a unilaterally pegged exchange rate will always increase output volatility, when the volatility of velocity shocks is equal across countries. More generally, output volatility is higher (for fixed z) under a fixed exchange rate when the variance of $m-\hat{\chi}$ is dominated by relative demand shocks.

But with endogenous movements in z, there is a countervailing force. As z rises, a higher fraction of firms choose to adjust prices ex-post, and this tends to stabilize output. Hence, the indirect effects of an exchange rate peg, through endogenous price flexibility, run counter to the direct effects, through increasing the volatility of aggregate demand.

A similar conclusion may be obtained by looking at the terms of trade. We define the terms of trade as $\frac{SP_f^*}{P_h}$. Denoting this in log changes as τ, we have

$$\tau = s + \frac{\varphi(z^*)\omega(1+\psi)(m^*-\hat{\chi}^*)}{1-(1-\alpha)\omega(\lambda-1)\varphi(z)+\psi\omega\varphi(z)} - \frac{\varphi(z)\omega(1+\psi)(m-\hat{\chi})}{1-(1-\alpha)\omega(\lambda-1)\varphi(z)+\psi\omega\varphi(z)} \quad (5.2)$$

When z and z^* are close to zero (when most prices are sticky), a fixed exchange rate prevents any terms of trade adjustment at all. But allowing price flexibility to respond to a peg creates terms of trade volatility through nominal price adjustment.

Is it possible that, taking both the direct effect on volatility and the indirect effect through increased price flexibility, that overall macroeconomic volatility is similar across exchange rate regimes? Although it is to be expected that endogenous price flexibility would lessen the impact of an exchange rate peg on output volatility, it would seem unlikely that this indirect response to the policy change would reverse the effects of the change itself. But in the presence of strategic complementarities in price setting behavior, even small policy changes might have substantial effects. Because both the benefits and costs of flexibility are increasing in the number of firms that choose flexibility, the indirect effect of policy changes, through movements in equilibrium price flexibility, may be of the same order as the direct effects.

Figure 2 and Table 1 provide a quantitative illustration of this result. We calibrate the model so that the standard deviation of relative demand shocks and velocity shocks is both set at 2 percent. The elasticity of substitution between categories of goods is set at 11, corresponding to the standard ten percent markup of price over marginal cost reported in Basu and Fernald (1997). The consumption constant elasticity of labor supply ψ is set to unity, following Christiano, Eichenbaum and Evans (1999). We assume that the distribution $\Phi(i)$ is uniform. In the calibration, we choose the cost function so that if all firms were choosing ex-post price flexibility, the total cost of this would be only 3 percent of GDP. This corresponds to the quantification of costs of price change as measured by Zbaracki et. al. (2000), and the calibration used in Dotsey et. al. (1999).

Table 1 illustrates the implications of alternative exchange rate regimes with and without endogenous price flexibility.[9] Under a floating exchange rate, output volatility is 1.7 percent, while terms of trade volatility is 6 percent. The fraction of firms choosing ex-post price flexibility is only 10 percent. Now if we impose a one-sided pegged exchange rate, but hold the degree of price flexibility unchanged, the second column of the Table shows that output volatility more than doubles to 4.2 percent, while terms of trade volatility falls to 0.2 percent. By contrast, allowing for endogenous price rigidity shows a dramatic rise in the fraction of firms choosing ex-post price flexibility – z rises from 10 percent to 68 percent. As a result, output volatility is stabilized – the standard deviation of output falls to 1.8 percent – effectively the same as under floating exchange rates. On the other hand, the volatility of the terms of trade is now only 2.6 percent, rather than 6 percent under the floating exchange rate. Hence, comparing floating and fixed exchange rates in the presence of endogenous price flexibility, we see that the fixed exchange rate regime leads to a large drop in terms of trade volatility, but effectively no movement in output volatility. The increased flexibility of nominal prices tends to offset the direct increase in macroeconomic volatility introduced by a fixed exchange rate.

How can fixed exchange rates reduce terms of trade volatility, while leaving output volatility essentially unchanged? Holding price flexibility constant, the peg leads output volatility to rise substantially, and eliminates almost all terms of trade volatility. As price flexibility increases, output volatility declines, as nominal prices adjust to offset relative demand shocks. The rise in price flexibility also raises terms of trade volatility, but this effect is limited, because to the extent that nominal prices are flexible, the

[9] As discussed in footnote ( ), Table 1 is constructed by exact numerical solution of the stochastic non-linear model. Details are given in the Appendix.

pegged exchange rate ensures that home and foreign prices respond identically to velocity shocks; the peg means that foreign velocity shocks become common 'world' nominal demand shocks, so that *relative* prices are not affected by these shocks. In the pegged exchange rate regime therefore, the terms of trade can be affected only by relative demand shocks – it is unaffected by either home or foreign velocity shocks. As a consequence, a unilateral exchange rate peg may significantly reduce terms of trade volatility, but leave output volatility essentially unchanged.

These results may help to throw light on the well-known puzzle raised by Mussa (1986), Baxter and Stockman (1990), and Flood and Rose(1995), concerning the relationship between exchange rate regimes and macroeconomic volatility. Standard theory implies that, holding the distribution of underlying fundamentals constant, a move from a fixed exchange rate to a floating exchange rate should have substantial implications for the volatility of both exchange rates and real GDP, as well as other macroeconomic aggregates. For instance, in a small economy facing a volatile external environment, a floating exchange rate should stabilize the domestic economy, when compared with a fixed exchange rate.

The evidence clearly shows that floating exchange rate regimes are associated with much higher real exchange rate volatility. But, as argued by Baxter and Stockman (1990), there is little evidence that other macroeconomic aggregates, such as the volatility of real GDP, changed substantially after economies changed from fixed to floating exchange rate regimes. Flood and Rose (1995) further show that there is little change in the underlying exchange rate fundamentals across fixed and floating exchange rate regimes. So floating exchange rates appear to cause a large increase in nominal and real exchange rate volatility, but have little affect on any other macroeconomic variables. [10] The quantitative results of our model are consistent with the observation that holding economic the distribution of economic fundamentals constant, following a move from fixed to floating exchange rates, a small economy may experience very little change in the volatility of GDP, but a substantial rise in relative price variability.[11]

6. Optimal Monetary Policy

So far we have simply compared arbitrary monetary rules that target the exchange rate. In this section, we discuss properties of the optimal monetary policy, taking into account the endogenous nature of price rigidity.

The typical result in this literature is that the monetary authority chooses a rule so as to replicate the flexible-price equilibrium. As shown by King and Wolman (1999) and Woodford (2003), this naturally implies that an optimal monetary rule ensures price stability, since a rule which obviates the necessity

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[10] This conclusion has recently been challenged for developing economies by Broda (2003). He shows that in response to *terms of trade shocks*, floating exchange rates tend to substantially cushion the impact on GDP, relative to fixed exchange rates. For other shocks, however, floating exchange rates tend to have little affect on the response.

[11] Note that in our model the consumption based real exchange rate is always constant, as PPP holds. But in a more general model with systematic home bias in preferences, the real exchange rate and the terms of trade would be positively correlated. Duarte (2003) and Dedola and Leduc (2001) propose a different explanation of the puzzle of small differences in macroeconomic volatility across exchange rate regimes. In their models, the presence of 'local currency pricing' causes output and consumption volatility to be similar across fixed and flexible exchange rate systems.

for firms to change prices ensures that a sticky price equilibrium coincides with the flexible price equilibrium.

In a two country setting, the environment becomes more complicated, because there are two new sources of market failure. First, there is a problem of strategic interaction between monetary authorities and the potential welfare losses from the absence of coordination (see for instance Benigno and Benigno 2003, Sutherland 2003). Secondly, markets for international risk-sharing may be incomplete, so that even the flexible price equilibrium of the world economy is inefficient. Obstfeld and Rogoff (2002) and Benigno (2002) show that in absence of full international risk sharing, an optimal monetary policy rule may not target the flexible price equilibrium allocation, and moreover, there may be gains to international policy coordination.

How does the possibility of endogenous price rigidity affect these conclusions? Dotsey King and Wolman (1999) note that allowing for state dependent pricing does not alter the main implication for optimal monetary policy in King and Wolman (1999). If the monetary authority wishes to replicate the flexible price equilibrium then a monetary policy ensuring price stability will achieve this. Firms will not wish to change their prices, even when they can do so in a state contingent manner.

When the monetary authority wishes to achieve an allocation other than the flexible price allocation however, the state dependent pricing technology may alter the optimal monetary policy problem. In the two country model of this paper, as described in section 2, a basic property is that the flexible price allocation (equivalent to an equilibrium of the model where the cost of flexibility is zero for all firms) is inefficient, due to the absence of international risk sharing – i.e. for the same reason as described in Obstfeld and Rogoff (2002) and Benigno (2002). In this case, an optimal monetary rule in an environment with sticky prices (equivalent to an equilibrium of the model where the cost of flexibility is in¯nite for all firms) will not in general target the flexible price allocation.[12]

We first characterize the optimal monetary rule in the case where all prices are set in advance. We may state the results in the following form:

Proposition 4

When $\Phi(i) \rightarrow \infty$, for all $i \in$ 0, 1:
a) the optimal monetary policy rule for the home and foreign country is written as:

$$M = \chi\gamma^{\frac{\alpha}{1+\psi}}\Gamma \qquad M^{*} = \chi^{*}(1-\gamma)^{\frac{\alpha}{1+\psi}}\Gamma \tag{6.1}$$

where Γ is a constant function of parameters.

b) there are no gains to international monetary policy coordination.
Proof: See Appendix.

[12] This point is explored in Devereux (2003).

Hence, in response to a positive relative demand shock for the country's output, the monetary authority should respond in the proportion $\frac{\alpha}{1+\psi}$. The higher is the elasticity of labor supply, and the smaller is the share of labor in production, the weaker should be the response of the monetary policy.

Note however that this policy will not sustain the flexible price equilibrium allocation, for reasons discussed above. The flexible price equilibrium allocation is achieved using the rules $M = \chi\Upsilon(\gamma)$ and $M^* = \chi^*\Gamma$.[13] But the optimal monetary policy should be expansionary in face of positive relative demand shocks for a country. Without monetary policy reaction, the domestic output response to relative demand shocks is smaller than efficient, due to the absence of markets for international risk sharing (as in Obstfeld and Rogoff (2002), and Benigno (2002)).[14] The intuition for part b) of the proposition comes from the way in which the model is dichotomized between home and foreign variables - there is no strategic interaction between monetary authorities.

Without relative demand shocks, the optimal rule would sustain the flexible price allocation, so the additional presence of endogenous costly price flexibility would have no implication for optimal monetary policy. Under the optimal rule, firms would have no incentive to invest in flexibility.[15] But when γ shocks are present, the optimal monetary rules 6.1 would add monetary volatility to the firms environment, and give them an incentive to invest in price flexibility.

How does endogenous price flexibility affect the optimal monetary rule? In general there will be no simple closed form representation of optimal policy comparable to 6.1. We can however establish:

Proposition 5

a) The monetary rule takes form $M = \chi\Upsilon(\gamma)$.

b) There are no gains to international policy coordination.

Proof: See Appendix.
The monetary authority should exactly accommodate domestic velocity shocks in the same way as before, since with only velocity shocks, the optimal rule would sustain the flexible price allocation.

In order to determine the form of $\Upsilon(\gamma)$, we can numerically compute optimal monetary rules where the monetary authorities take account of the impact of their policy rules on equilibrium price flexibility. We

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13 It is easy to see from (3.14) and (3.17) that if $M\chi^-1$ is constant, then the ex-post optimal price is constant.

14 Some more intuition for this can be given. In the flexible price economy, a relative demand shock raises a country's terms of trade and income. This generates an wealth effect which reduces optimal labor supply. On the other hand, the terms of trade increase itself tends to increase optimal labor supply, as the return to working increases. In equilibrium these effects exactly offset each-other, and output is constant. But with full international risk sharing, the first effect – the wealth effect, is mitigated, because part of it goes to foreign households. As a result, the second effect dominates, and equilibrium domestic output increases (as is efficient).

15 It is easy to extend the model to allow for country specific productivity shocks, and to show that this conclusion is unaffected by such an extension. That is, the optimal monetary rule with velocity and productivity shocks would still target the flexible price allocation, and so therefore endogenous price rigidity would not affect the optimal rule.

assume that Υ takes the form $\Upsilon = \gamma^a$, and solve for a by maximizing expected utility, for the home and foreign countries. We use the same calibrated economy as that of Table 1.

Table 2 shows the results. The first column shows the optimal policy with exogenous (and zero) price flexibility, and confirms Proposition 4 (where $\tilde{a} \equiv \frac{a\alpha}{1+\psi}$, where a is defined in the rule $\Upsilon = \gamma^a$, so $\tilde{a}$ = 1 corresponds to Proposition 4) We find that the optimal monetary rule when the monetary authorities take account of endogenous price flexibility calls for a slightly less pro-cyclical stance ($\tilde{a}$ = 0.89). That is, the authorities in each country place slightly less weight on the relative demand shock than they would in the pure sticky price economy. Intuitively, they take into account that the pro-cyclical stance of monetary policy leads more firms to incur the costs of price flexibility. This has welfare costs both directly, in terms of the fixed labor cost incurred by flexible price firms, and indirectly, because the closer that the firms get to the flexible price equilibrium, the further away will be the allocation from that desired by the monetary authorities. On both counts, monetary policy becomes less activist.[16]

Table 2 illustrates also the degree of price flexibility that would occur if the monetary authorities were 'passive' in the sense that they maintained $\tilde{a}$ = 1. In the benchmark case, this would lead to z = 1:3, only slightly different from the optimal rule. But if relative demand shocks were much more volatile (columns 5 and 6 in Table 2, where the standard deviation of relative demand shocks is set at 8 percent), then a passive policy would lead z to jump to 19 percent. By contrast, in this case, the *optimal* policy would be much less activist, setting $\tilde{a}$ equal to .58, and ensuring a lower z of 6 percent. Interestingly, the more volatile is the relative demand shock, the less activist is the optimal policy under endogenous price flexibility (and more closely targeting the flexible price equilibrium).

Quantitatively, we find that under an optimal monetary policy rule, price flexibility is very very low – z is about 0.01 in each country. Thus, effectively, the optimal monetary policy in this economy ensures that firms do not avail of the opportunity to use ex-post price flexibility. From a different perspective, the results indicate that the presence of endogenous price flexibility has little impact on optimal monetary policy rules.

What do these results imply for the monetary rules of section 3? First, we can note that in the sticky price economy of proposition 4, a fixed exchange rate (whether one-sided or cooperative) is never optimal. The exchange rate in the pure sticky price economy, under the optimal monetary policy rules, will be

$$S = \left(\frac{1-\gamma}{\gamma}\right)^{\frac{1-\alpha+\psi}{1+\psi}}$$

Hence, the exchange rate will still respond to relative demand shocks under the optimal rule. By extension, a fixed exchange rate is then not an optimal monetary policy rule in the economy with endogenous price flexibility, because as we found in that case, optimal monetary policy is even less responsive to relative demand shocks. As an additional implication, a one-sided peg is not optimal for a second reason, because it requires that the monetary authority responding to *foreign* as well as domestic velocity shocks.

16 We can also look at an environment where the optimal monetary rule is determined *after* firms have chosen the degree of price flexibility. We find that the quantitative results differ only slightly.

Hence, optimal monetary policies are likely to take on quite different characteristics that the simply fixed and floating exchange rate rules described in the previous section. Nevertheless, from a positive viewpoint, our results do emphasize the sensitivity of the standard predictions about fixed and floating exchange rates to the possibility of endogenous price flexibility.

7. Conclusions

Theoretical discussion of the merits of exchange rate flexibility almost always take the structural characteristics of wage and price determination to be independent of the exchange rate regime choice. But in policy circles, it is often emphasized that exchange rate commitments may help to affect private sector expectations and alter the institutional structure of wage contracting and price setting. For instance, many Latin American countries pursued exchange-rate-based stabilizations in the 1990s on the hope that the fixed exchange rate would feed directly into private sector actions. Likewise, economies such as Hong Kong that operate on Currency Board arrangements emphasize that a pre-requisite for success is the flexibility of internal prices (Latter 2002). In a different context, there has been speculation that the introduction of the euro might encourage price flexibility within Europe, and limit the cost of giving up on the exchange rate as a response to national shocks within the euro area.

This paper has developed a theoretical underpinning to the link between exchange rate regime choice and nominal price flexibility. We build up from the basic microeconomics of a firm's decision to invest in price flexibility, and then integrate this into a two country macroeconomic model where there are country specific monetary and real shocks. We argue that there is a significant difference between unilateral exchange rate pegs, where one country adopts a fixed exchange rate or a Currency Board, and a bilateral fixed exchange rate (or a Monetary Union). In the first case, a peg will increase price flexibility – perhaps by a large amount. In the second case, price flexibility is unlikely to be substantially affected – it might even be reduced.

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Table 1. Real Effects of an Exchange Rate Peg

Variable	St. dev. GDP	St. dev. Terms of Trade	Flexible Price Sector
Flexible	1.7	6.2	0.10
Fixed	4.2	0.2	0.10
Fixed	1.8	2.2	0.68

Table 2. Optimal Policy with Endogenous Flexibility

Regime	$\Phi \to \infty$	Endogenous		Endogenous, high volatility	
		Passive	Optimal	Passive	Optimal
$\tilde{a}$	1	1.000	0.890	1.000	0.580
z	0	0.013	0.010	0.190	0.060

Figure 1a.

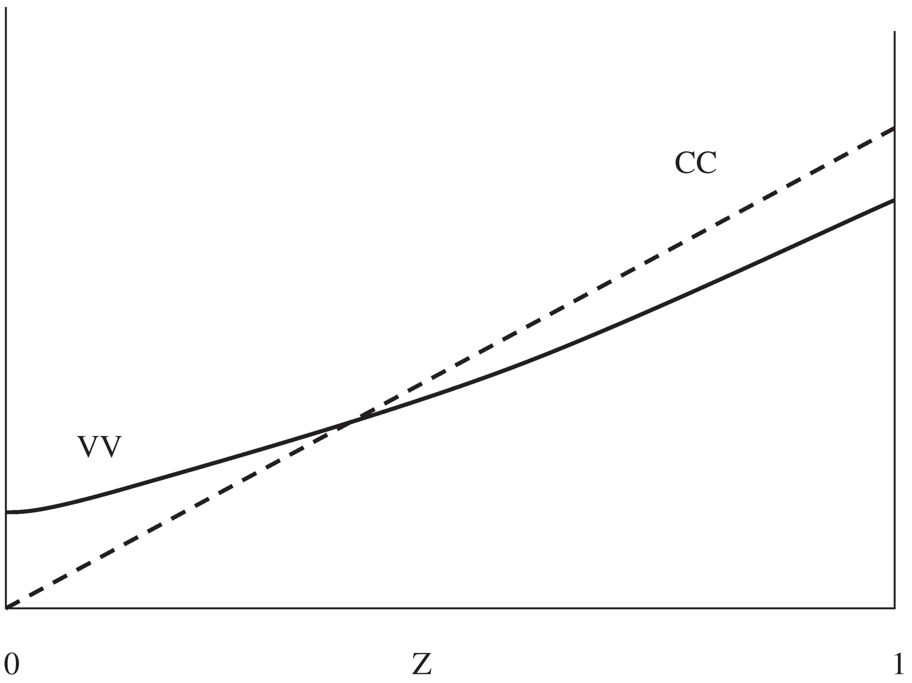
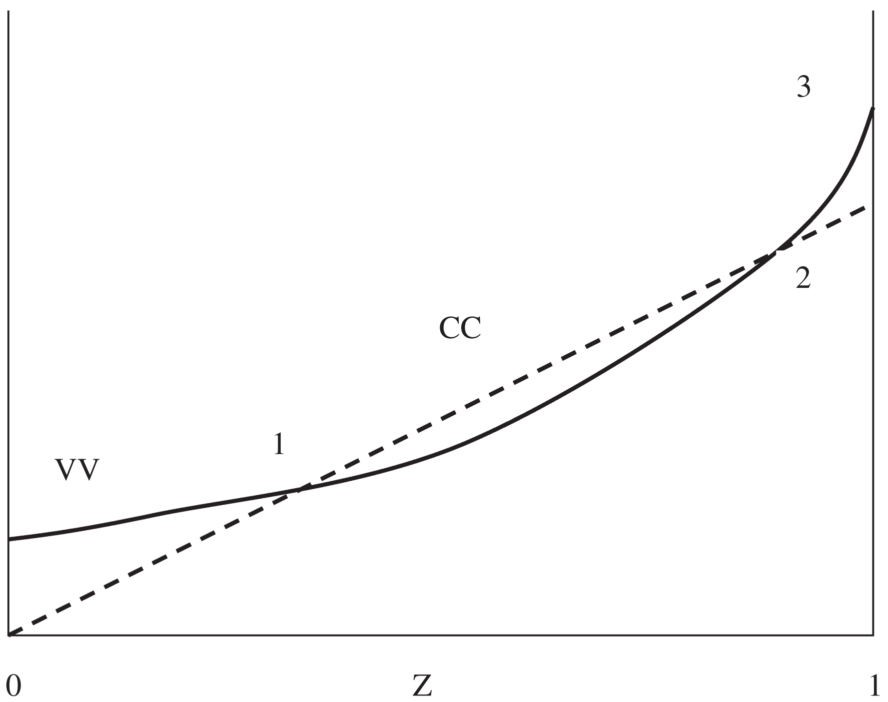


Figure 1b.



Appendix

8.1 Obtaining the approximation 2.10

We first describe how the approximation given in 2:10 is obtained. Note that:

$$\Delta(\Theta) = \frac{\Psi}{E\Gamma W}\left[E\Gamma(W^{\alpha(1-\lambda)}\hat{X})^{\omega} - (E\Gamma W\hat{X}^{\frac{1}{\alpha}})^{(1-\lambda)\alpha\omega}(E\Gamma\hat{X})^{\lambda\omega}\right] \tag{8.1}$$

This may be written in the form:

$$\Delta(\Theta) = \frac{\Psi}{E\exp(\ln\Gamma + \ln W)}\left[E\exp(\ln\Gamma + \omega(\alpha(1-\lambda)\ln(W) + \ln X))\right.$$

$$\left.-(E\exp(\ln\Gamma + \ln W + \frac{1}{\alpha}\ln\hat{X}))^{(1-\lambda)\alpha\omega}(E\exp(\ln\Gamma + \ln\hat{X}))^{\lambda\omega}\right]$$

Now take a second order logarithmic approximation of $\Delta(\Theta)$ around the mean $\bar{\Theta} = E\ln\Theta$. This gives:

$$\Delta(\Theta)) \approx \Delta(\bar{\Theta}) + \Omega E(g + \omega(\alpha(1-\lambda)w + x) \tag{8.2}$$

$$-\Omega E(((1-\lambda)\alpha + \lambda)\omega g + (1-\lambda)\alpha\omega w + ((1-\lambda)\alpha\omega + \lambda\omega)x)$$

$$+\frac{1}{2}\Omega E(g^2 + (\omega(\alpha(1-\lambda))^2 w^2 + \omega^2 x^2 + 2\omega\alpha(1-\lambda)gw + 2\omega gx + 2\omega^2\alpha(1-\lambda)wx)$$

$$-\frac{1}{2}\Omega\left[(1-\lambda)\alpha\omega E(g^2 + w^2 + \alpha^{-2}x^2 + 2gw + 2\alpha^{-1}gx + 2\alpha^{-1}wx) + \lambda\omega E(g^2 + x^2 + 2gx)\right]$$

where small case letters denote logarithmic deviations from their mean levels $g = \ln\Gamma - E\ln\Gamma$, etc. Recall that $\Omega \equiv \frac{V(\bar{\Theta})}{\exp(E\ln\Gamma + E\ln W)}$. Note that the terms in the denominator of $\Delta(\Theta)$ do not affect this approximation, because of the fact that, up to a second order, they could only be non-zero if $\Delta(\bar{\Theta})$ were non-zero, which it is not.

Using the definition of $\omega = \frac{1}{\alpha + \lambda(1-\alpha)}$, it is easy to see that the first order terms (the second and third terms in 8.2), cancel out. Then, defining Ew^2 as σ_w^2, etc, the fourth and fifth terms in expression (8.2) may be rearranged, after cancelling out terms in σ_g^2, σ_{gx}, and σ_{gw}, as:

$$\Omega(\lambda - 1)\lambda\alpha\omega^2\left[\sigma_w^2 + \frac{(1-\alpha)^2}{\alpha^2}\sigma_x^2 + 2\frac{(1-\alpha)}{\alpha}\sigma_{wx}\right] \tag{8.3}$$

which is just expression 2.10 in section 2 of the text.

8.2 Approximating P_h and H

We first approximate P_h around the mean $E \ln P_h$. Since $\bar{P}_h$ is predetermined, we have:

$$p_h = \varphi(z)\tilde{p}_h \tag{8.4}$$

where

$$\varphi(z) \equiv \frac{z\exp(E\ln \tilde{P}_h(1-\lambda))}{z\exp(E\ln \tilde{P}_h(1-\lambda)) + (1-z)\exp(E\ln \bar{P}_h(1-\lambda))}$$

is an increasing function of z, and satisfies the properties $\varphi(0) = 0, \varphi(1) = 1$. This approximation allows for the fact that the mean values $E \ln \bar{P}_h$ and $E \ln \bar{P}_h$ will not in general be the same.

From (3.14) in the paper, we may write

$$\tilde{p}_h = \omega\alpha\psi h + \omega(\lambda - 1)(1-\alpha)p_h + \omega(m - \hat{\chi}) \tag{8.5}$$

To derive (3.22) of the text, we take a linear approximation of (3.17) around the mean level of $E \ln H$. This gives

$$h = \frac{\varsigma}{\alpha}(m - p_h - \hat{\chi}) + \frac{\varsigma}{\alpha}\varkappa p_h \tag{8.6}$$

Here we define terms as:

$$\varsigma \equiv \frac{\exp(E\ln H) - \int_0^z \Phi(i)di}{\exp(E\ln H)}$$

$$\varkappa \equiv -\lambda\left[\varrho(z)(1-\varphi(z)) - (1-\varrho(z))\varphi(z)\right]\varphi(z)^{-1}$$

$$\varrho(z) \equiv \frac{z\exp(E\ln \tilde{P}_h)^{-\frac{\lambda}{\alpha}}}{z\exp(E\ln \tilde{P}_h)^{-\frac{\lambda}{\alpha}} + (1-z)\exp(E\ln \bar{P}_h)^{-\frac{\lambda}{\alpha}}}$$

The approximation 8.6 can be explained as follows. A rise in real aggregate demand $m - p_h - \hat{\chi}$ raises demand for labor by a factor of & $\frac{\varsigma}{\alpha}$, where $\varsigma < 1$ reflects that fact that employment rises less than in proportion $\frac{1}{\alpha}$ to aggregate demand due to the fixed costs of price flexibility. The second term in (8.6) re°ects the fact that changes in the flexible price index causes movements in aggregate demand between the fixed price and flexible price sectors, which do not cancel out when $E \ln \tilde{P}_h \neq E \ln \bar{P}_h$. In the exact numerical solution of the model we find the ter $\varkappa$ to be second order relative to the first terms in (8.6). Given this, and in order to simplify the exposition of the model, we ignore these compositional effects in the discussion in section. All of the propositions in section 3 would remain unchanged in the absence of this simplification. Moreover, since the quantitative results of section 5 solves the exact model, the compositional effects are automatically incorporated in Table 1.

Substituting from (8.6) (ignoring the $\varkappa$ terms as discussed) into (8.5) and then into (8.4) gives (3.21) of the text.

8.3 Optimal Monetary Rules

Here we derive the results of section 6 of the paper. Following most of the literature (e.g. Obstfeld and Rogoff 2002), assume that the optimal monetary rules maximize expected utility net of the utility of real balances.

If all prices are sticky, then the home country price may be written as

$$P_h = \delta^{\frac{\alpha}{1+\psi}} \left(E \frac{M}{\chi}^{\frac{1+\psi}{\alpha}} \right)^{\frac{\alpha}{1+\psi}} \tag{8.7}$$

It is straightforward to show that equilibrium consumption and employment in the home country is given by;

$$C = \frac{M}{P_h^{\gamma}(SP_f^*)^{1-\gamma}} = \gamma \left(\frac{M}{\chi P_h \gamma} \right)^{\gamma} \left(\frac{M^*}{\chi^* P_f^* (1-\gamma)} \right)^{(1-\gamma)} \tag{8.8}$$

$$H = \left(\frac{M}{\chi P_h} \right)^{\frac{1}{\alpha}} \tag{8.9}$$

From (8.9) and (8.7), we can show that:

$$\frac{\eta}{1+\psi} E H^{1+\psi} = \frac{\eta}{1+\psi} \frac{E(\frac{M}{\chi})^{\frac{1+\psi}{\alpha}}}{P_h^{\frac{1+\psi}{\alpha}}} = \Omega_1 \frac{E(\frac{M}{\chi})^{\frac{1+\psi}{\alpha}}}{E(\frac{M}{\chi})^{\frac{1+\psi}{\alpha}}}$$

which is a constant (Ω_1 is a constant function of parameters). Hence, for monetary policy evaluation, expected utility in this case depends only on the expected value of log composite consumption.

We may then write out the expected utility objective function for the home country monetary authority as (ignoring constants):

$$E \ln C = E\gamma(\ln \frac{M}{\chi} - \ln P_h) + E(1-\gamma)(\ln \frac{M^*}{\chi^*} - \ln P_f^*) \tag{8.10}$$

From this, it is straightforward to see that the monetary policy rule of the home country will be strategically independent of that of the foreign country – thus there are no gains to policy coordination.

Proof of Proposition 4

Without loss of generality, assume that there is a finite number of possible states of the world Σ and let any state be denoted $\epsilon \in \Sigma$. Let the money supply of each country be state contingent.

Choosing $M(\epsilon)$ to maximize (8.10) gives us the first order condition:

$$\gamma(\epsilon)\frac{1}{M(\epsilon)} - \frac{1}{M(\epsilon)}\left(\frac{M(\epsilon)}{\chi(\epsilon)}\right)^{\frac{1+\psi}{\alpha}}(E\gamma)\left(\frac{\delta^{\frac{\alpha}{1+\psi}}}{E(\frac{M}{\chi})^{\frac{1+\psi}{\alpha}}}\right) = 0$$

For a solution to exist, it is necessary to normalize the money supply in some way (since systematic money is neutral) – any normalization will do. For instance, if we fix $E(\frac{M}{\chi})^{\frac{1+\psi}{\alpha}}$, is straightforward to re-arrange this first order condition to establish the monetary rules given in Proposition 4.

Proof of Proposition 5

When some prices are endogenous, the property that the expected utility of employment is independent of the distribution of money no longer holds. But the expected utility of log consumption (8.10) is written in the same way. Thus, the objective function for the monetary authority may be described as:

$$E\gamma(\ln\frac{M}{\chi} - \ln P_h) + E(1-\gamma)(\ln\frac{M^*}{\chi^*} - \ln P_f^*) - \frac{1}{1+\psi}EH^{1+\psi} \tag{8.11}$$

The monetary authorities must choose a state contingent monetary rule to maximize (8.11) subject to the conditions on prices, employment and the determination of z from the text. These are;

$$\tilde{P}_h = \delta\left[\eta H^{\psi\alpha}P_h^{(\lambda-1)(1-\alpha)}\frac{M}{\chi}\right]^{\omega} \tag{8.12}$$

$$\bar{P}_h = \delta\frac{E[\eta H^{\psi}(P_h^{\lambda-1}\frac{M}{\chi})^{\frac{1}{\alpha}}]^{\alpha\omega}}{E[P_h^{\lambda-1}]^{\alpha\omega}} \tag{8.13}$$

$$P_h = \left[z\tilde{P}_h^{1-\lambda} + (1-z)\bar{P}_h^{1-\lambda}\right]^{\frac{1}{1-\lambda}} \tag{8.14}$$

$$H = \left[z(\frac{\tilde{P}_h}{P_h})^{-\frac{\lambda}{\alpha}} + (1-z)(\frac{\bar{P}_h}{P_h})^{-\frac{\lambda}{\alpha}}\right](\frac{M}{\chi P_h})^{\frac{1}{\alpha}} + \int_0^z \Phi(z)dz \tag{8.15}$$

$$\Delta(\Theta(z)) = \Phi(z) \tag{8.16}$$

From inspection of these equations, it is straightforward to see that there is still no strategic interaction between the home and foreign monetary authorities. Hence there are no gains to policy coordination. In addition, since in all these equations, the state contingent money supply enters in the form $\frac{M(\epsilon)}{\chi}$, it is still optimal to exactly accommodate velocity shocks. Finally, maximizing (8.11) with respect to $M(\epsilon)$, subject to (8.12)-(8.16) gives an implicit function $M(\epsilon) = \chi(\epsilon)\Upsilon(\gamma(\epsilon))$, as described in Proposition 5. Although the form of $\Upsilon(\quad)$ cannot be characterized analytically, the procedure underlying the numerical solution of Table 2 is used to describe this function.