INFLATION IN MAINLAND CHINA - MODELLING A ROLLER COASTER RIDE

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Abstract

The New Keynesian Phillips curve (NKPC) posits the dynamics of inflation as forward looking and related to marginal costs. In this paper we examine the empirical relevance of the NKPC for mainland China. The empirical results indicate that an augmented (hybrid) NKPC gives results that are consistent with the data generating process. It is in this respect that the NKPC provides useful insights into the nature of inflation dynamics in mainland China as well as useful insights for the conduct of monetary policy.

Keywords: China, inflation, New Keynesian Phillips curve

JEL Classification: C22, E31

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1. Introduction

As a result of increased integration into the global economy and continuing domestic price liberalisation, prices in China are increasingly market determined. Therefore understanding the dynamics of inflation and its cyclical interaction with real aggregates is an important question in theory and in practice, especially for central banks vis-a-vis the conduct of monetary policy. The recent experience of high GDP growth rates coupled with low inflation observed in several countries casts doubt on the traditional Phillips curve as a model of inflation dynamics. Furthermore, standard specifications of backward-looking expectations-augmented Phillips curves tend to overpredict inflation in the recent low inflation environment in many countries. This has been a major motivation for the development of a family of new forward-looking Phillips curves. A typical example is the New Keynesian Phillips Curve (NKPC), which has had success for the U.S. and euro area and which has therefore become a new consensus theory of inflation in modern monetary economics and a true branch of research. The reason mainland China makes for an interesting case study is that it is a developing country and has experienced mild deflation periods in the recent past. Another motivation is that inflation forecasting has become the dominant aspect of monetary policy in recent years. Explaining and forecasting inflation has thus never been more important.

Economic reform in China since the late 1970s can be roughly divided into five phases. In the first phase (1978-1984), farming was decentralised and agricultural prices were raised. The success of the reforms encouraged the authorities to introduce further measures in the second phase (1984-1988), including some liberalisation of enterprise pricing and wage setting. Fourteen coastal cities were also opened up to foreign trade and investment. During the third (1988-1991) and fourth (1992-1998) phases, the Chinese reform process was characterised by a lack of effective macroeconomic policy instruments. As a result, inflation increased substantially after price liberalisation until finally the authorities introduced price controls and administered sharp contractionary policies to control double-digit inflation. This was effective in stabilising prices, but it also produced a downturn in the economy and a mild deflation. When it became apparent in 1998 that the economy was slowing down markedly, the government spurred the economy with successive interest rate cuts to boost demand in the fourth phase. Finally, the fifth phase (1998 to present) can be characterised by broad-based enterprise, financial and social reforms. With the onset of SARS in early 2003, the government provided additional fiscal and monetary stimulus to offset the deleterious effects of SARS, which led to lower growth cum disinflation.

Several authors have attempted to shed light on this issue and have often come up with very different results. Feyzioglu (2004) argues that tariff cuts, monetary policy and productivity gains have contributed to the recent deflation experience. Ha et al. (2003) find evidence that the international price level, productivity growth and the nominal effective exchange rate have contributed to Chinese inflation since the beginning of the 1990s. Earlier work by Hasan (1999) focussed on monetary factors that have impacted price movements, while Oppers (1997) found that inflationary periods were generally characterised by sharp increases in aggregate demand. Brandt and Zhu (2000) explained the boom-bust inflation cycles in light of the institutional environment of the reform process. Market reforms led to inefficient credit granting to state-owned enterprises (SOEs) due to soft budget constraints. As a result, SOEs’ productivity growth lagged behind the nonstate sector and the central government was forced to
resort to money creation to finance increasing transfers to SOEs. Kamin et al. (2004) examined the prominent view that China is a source of downward pressure on global prices. Contrary to previous studies, we model key factors that have led to this inflation pattern over time using the NKPC framework. Finally, Scheibe and Vines (2005) estimated Phillips curves for mainland China using quarterly data from 1988:4 to 2002:4.

The remainder of this paper is structured as follows. In Section 2 we discuss the microeconomic foundation of the NKPC, present the empirical methodology and discuss the fit and robustness of the model. Section 3 concludes and proposes some avenues for further research. Several econometric issues and technical details are covered in an appendix.

2. Application of NKPC to Mainland China

2.1 Theoretical Background

To make this paper self-contained, we will briefly describe the NKPC model, and then explore its implications for the behaviour of inflation. The NKPC is derived from a rational-expectations-based model of optimal price-setting, with a type of price rigidity which has become customary in the monetary macroeconomics literature. The starting point is a formulation known as Calvo (1983) pricing, after the economist who first introduced it. The analytically convenient form of price rigidity faced by the Calvo (1983) firm is as follows. Each period, only a random fraction \((1 - \beta)\) of firms are able to reset their price, all other firms keep their price unchanged. When firms do get to reset their price, they must take into account that the price may be fixed for many periods. We assume they do this by choosing a log-price, \(z_t\), that minimises the loss function

\[
L(z_t) = \sum_{k=0}^{\infty} (\beta^k) E_t(\xi_t - z^*_{t+k})^2
\]

where \(\beta\) is between zero and one, \(E_t\) is the expectations operator conditional on the information available at time \(t\), \(\beta\) is the discount factor, and \(z^*_{t+k}\) is the log of the optimal price that the firm would set in period \(t+k\) if there were no price rigidity. Given equation (1), determining the optimal value of \(z_t\) is straightforward.

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1 Questions may arise as to whether this type of model is applicable to a transition economy like China. While China may not fully satisfy all the modelling assumptions, fundamental changes in the economy over the past two decades have made the model increasingly more relevant.

2 The adjustment probabilities are independent of a firm’s price history, so that the particular firms that adjust their prices in each period are randomly selected. The average time over which a price is fixed is then given by \(1/(1 - \beta)\). This modelling approach nests a wide range of assumptions about the degree of price stickiness, from perfect flexibility (\(\beta = 0\)) to complete price rigidity (the limit as \(\beta \to 0\)).

3 The fact that \(\beta < 1\) implies that the firm places less weight on future losses than on today’s losses. Future losses are actually discounted at the rate \((\beta^0)\), not just \(\beta\). This is because the firm only considers the expected future losses from the price being fixed at \(z_t\). The probability that the price will be fixed until \(t+k\) is \(\beta^k\), so the period \(t+k\) loss is weighted by this probability. The term \(E_t(\xi_t - z^*_{t+k})^2\) describes the expected loss in profits for the firm at time \(t+k\) due to the fact that it will not be able to set a frictionless optimal price that period. The quadratic function can be interpreted as an approximation of some more general profit function.
Equation (1) is differentiated with respect to the choice variable \( z_t \), and the derivative is set to zero. This means

\[
L'(z_t) = 2 \sum_{k=0}^{\infty} (\theta \beta)^k E_t(z_{t+k} - z_{t+k}^*) = 0
\]  
(2)

Separating out the \( z_t \) terms and using the geometric formula

\[
\sum_{k=0}^{\infty} (\theta \beta)^k = \frac{1}{1 - \beta \theta}
\]
(3)

we can rewrite the first-order condition for the optimal price as

\[
z_t = (1 - \beta \theta) \sum_{k=0}^{\infty} (\theta \beta)^k E_t(z_{t+k}^*)
\]  
(4)

Stated in English, this equation simply says that the optimal solution is for the firm to set its price equal to a weighted average of the prices that it would have expected to set in the future if there were no price rigidities. Unable to change prices each period, the firm chooses to try to keep close on average to the right price. And what is this optimal price, \( z_t^* \)? In the NKPC it is assumed that a firm’s optimal pricing strategy involves setting a (fixed) markup over real marginal cost \( mc \), i.e.

\[
z_t^* = \mu + mc_t
\]  
(5)

Thus, the optimal reset price can be written as

\[
z_t = (1 - \beta \theta) \sum_{k=0}^{\infty} (\theta \beta)^k E_t(\mu + mc_{t+k})
\]  
(6)

Intuitively, the theory suggests that price adjustment is based on current and expected marginal cost. Aggregating over individual behaviour then leads to a relationship where the aggregate price level in the economy \( p_t \) is just a weighted average of last period’s aggregate price level and the new optimal reset price:

\[
p_t = \theta p_{t-1} + (1 - \theta) z_t
\]  
(7)

\[
z_t = \frac{1}{1 - \theta} \left( p_t - \theta p_{t-1} \right)
\]  
(8)

Rewriting equation (6), we express the optimal reset price as\(^4\)

\[
z_t = \beta \theta E_t z_{t+1} + \left( 1 - \beta \theta \right) \left( \mu + mc_t \right)
\]  
(9)

\(^4\) Iterating forward, a first-order difference equation \( y_t = a x_t + b E_t y_{t+1} \) can be solved to give \( y_t = \alpha \sum b^k E_t x_{t+k} \) for \( k=0 \) to \( \infty \). Examining (6) reveals that \( z_t \) obeys this equation with \( y_t = z_t \), \( x_t = \mu + mc_t \), \( a = 1 - \beta \theta \) and \( b = \beta \theta \).
Substituting this expression for \( z_t \) into equation (8), we get

\[
\frac{1}{1 - \theta} (p_t - \theta p_{t-1}) = \frac{\theta \beta}{1 - \theta} (E_t p_{t+1} - \theta p_t) + (1 - \beta \theta) (\mu + mc_t)
\]  

(10)

By re-arrangement, this equation yields the pure NKPC:

\[
\pi_t = \beta E_t \pi_{t+1} + \frac{(1 - \theta)(1 - \beta \theta)}{\theta} (\mu + mc_t - p_t)
\]  

(11)

where \( \pi_t = p_t - p_{t-1} \) is the inflation rate. The pure NKPC explains current inflation by expected inflation one period ahead, conditional on information available at time \( t \) and on the forcing variable real marginal cost.\(^5\)

To implement the NKPC empirically, it is necessary to have a measure of the latent variable, real marginal cost. The exact definition of marginal cost may be a critical issue in the estimation of the NKPC. Real marginal cost may be measured in different ways, which resolve either the output gap or labour share. In the first case, a reliable measure of the output gap is necessary. The tacit assumption underlying the approximation of real marginal cost by labour share is a constant returns-to-scale production function.

One problem of the NKPC in equation (11) is it does not take into account the fact that inflation is highly persistent [see, for example, Fuhrer and Moore (1995)]. The jump behaviour implied by models with forward expected inflation is also at odds with observed inflation behaviour. This has led several authors to suggest a hybrid model that nests (11). This amounts to the empirically motivated specification

\[
\pi_t = \alpha \pi_{t-1} + \beta E_t \pi_{t+1} + \frac{(1 - \theta)(1 - \beta \theta)}{\theta} (\mu + mc_t - p_t)
\]

(12)

where \( \varepsilon_t \sim N(0, \sigma^2) \). Strictly speaking, equation (12) is an ex ante concept defined by expectations. However, due to the paucity of data on expectations in China, we employ an operational version based on realised future inflation to examine the data:\(^6\)

\[
\pi_t = \alpha \pi_{t-1} + \beta \pi_{t+1} + \frac{(1 - \theta)(1 - \beta \theta)}{\theta} (\mu + mc_t - p_t) + \varepsilon_t
\]

(13)

One way to justify the use of (13) is that, under the rational expectations hypothesis, the ex post realisations are unbiased predictors of their ex ante counterparts. If rational expectations are assumed, the realised inflation rate differs from the expected inflation rate by an unpredictable non-systematic error term (\( \varepsilon_t \)).

\(^5\) Roberts (1995) has shown that the staggered contract model of Taylor (1979, 1980) and the quadratic price adjustment cost model of Rotemberg (1982) also have the pure NKPC as a common representation. The NKPC specification has important implications for the conduct of monetary policy in that a fully credible central bank can bring about disinflation at no recessionary cost if inflation is a purely forward-looking phenomenon.

\(^6\) Recent papers have estimated the NKPC for the U.S. using data from the survey of professional forecasters to proxy expected inflation [see Adam and Padula (2003)]. However, using survey data to infer market expectations also has drawbacks. In general, surveys suffer from timing and small sample problems, which may render the forecasts unreliable.
The next section offers a brief survey of inflation, past and present, in mainland China, and attempts to indicate some of the important features that are relevant for our study.

2.2 Stylised Facts

Economists studying inflation dynamics in China face some thorny data issues. We use annual data because quarterly or even monthly data are not available for the 1970s and most of the 1980s. The sample is truncated in 1977 because prior data are not available. Furthermore, we need to remind ourselves of the inherently noisy nature of price data for mainland China. They are highly variable month-to-month and quarter-to-quarter, and they may be imperfect proxies for true changes in prices. In light of the short-term volatility, discerning the underlying trend of inflation using annual data is probably an easier task.7

Historical inflation rates for mainland China are plotted in Figure 1. Beginning at the end of the 1970s, a series of far-reaching economic reforms was undertaken in China, a key element of which was price liberalisation, introduced in a piecemeal fashion. Figure 1 indicates that inflation dynamics in mainland China conceal a marked cyclical pattern since the end of the 1970s. Price liberalisation led to sharp price increases in the 1980s, peaking at 18.5 per cent in 1988. Devaluation of the renminbi also contributed to increasing inflation. Retail price inflation (RPI) then moderated substantially at the beginning of the 1990s after progressive tightening of monetary policy. Finally, China experienced two episodes of mild deflation (1998-2000 and 2001-2002), even though real GDP growth was very high, averaging 8.6 per cent from 1998 to 2002. The inflation rate shows some persistence but still appears to be mean-reverting. Other measures of aggregate prices indicate movements similar to the RPI and do not deviate from RPI inflation for long periods.8

To make equation (13) empirically tractable, we need to get data on inflation in mainland China and to construct proxies for marginal costs. Following the literature, real marginal costs are measured by the output gap. The inflation-output gap relationship over the stretch of 26 years between 1977 and 2003 is plotted in Figure 2.9 As shown, the greater the excess capacity in the Chinese economy, the greater the

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7 Another reason for using annual data is that the quarterly GDP series is only available from 1994 onwards.
8 The RPI covers most of the prices that are included in the CPI, but excludes services. During the sample period, RPI inflation was slightly below CPI inflation, reflecting the fact that inflation in services was higher than inflation in other prices. One disadvantage of the CPI is that it also includes prices of services such as housing, transportation, health care and so on. For much of the sample period, these prices were administered by the government and did not reflect market conditions. Furthermore, potential measurement problems suggest that changes in the CPI may have to be interpreted with caution. Investigating the extent and impact of a bias in official inflation data would be a fruitful avenue for future research. Holz (2005) has recently provided an informative survey of data problems, institutional innovations and challenges for mainland China's Statistical System.
9 Calculating measures of the Chinese output gap presents a daunting challenge. A relatively straightforward way is to detrend GDP using the band-pass filter suggested by Baxter and King (1999). The band pass filter isolates the component of GDP that lies between 2 and 8 years. The filter uses a centred MA method. Thus, for the filter to work it was necessary to stretch the series at the beginning and end using AR backcasts and forecasts. See Christiano and Fitzgerald (2003) and Stock and Watson (1998). Artis et al. (2004) argue for the superiority of the band-pass version of the HP filter. The resulting cyclical pattern accords well with the demarcation of business cycles suggested by Oppers (1997).
rate of inflation, i.e. the output gap and inflation move in step. Another eye-ball observation is that since the mid-1990s, inflation seems to have become less responsive to the output gap. The traditional Phillips curve implies that demand-pull inflation occurs when the economy is overheated and declines when there is slack in the economy. However, rising growth rates in the 1990s and evidence that the Chinese economy is operating near potential, while inflation is falling, casts doubt on this model. Some industries, such as electricity and steel, are short of capacity. However, the economy as a whole is not as overheated as it was in the early 1990s because of the enormous increase in industrial capacity. In other words, China has been able to ward off significant price increases despite near double-digit economic growth, and therefore the recent inflation episode seems to require a different explanation. In the next section we therefore go beyond the traditional Phillips curve and gauge the explanatory power of the forward-looking NKPC.

In light of these facts, the purpose of the next section is to see what the data reveal, especially whether they favour (11) or (13).

### 2.3 Estimating and Testing the NKPC

The divergence of previous NKPC estimates warrants further studies using alternative econometric methods to derive accurate estimates. We therefore investigate the sensitivity of the NKPC estimation and testing results using alternative estimation approaches. Another notable feature mentioned above is that inflation may be measured with error (method errors and/or recording errors) and therefore we face potential measurement error in explanatory variables. The econometrics literature suggests that instrumental variables will almost always reduce bias and can therefore be used to ‘solve’ the error-in-variables problem. An additional estimation problem in rational expectations models with future period expectations is that some correction for serial correlation is required to account for $MA(q)$ error arising from the expectations variable.

We now turn to the estimation of equation (13). Table 1 below presents the estimation results using Hansen and Hodrick’s (1980) OLS estimator (HH), McCallum’s (1976) RE estimator (MC), Hayashi and Sims’ (1983) forward filter estimator (HS), and Cumby, Huizinga and Obstfeld’s (1983) two-step two-stage least squares estimator (CHO). The estimates use annual data for mainland China over the sample period 1982 - 2002.

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10 To fix ideas, the properties of the various rational expectations IV techniques are detailed in the Appendix for the econometrically oriented reader. Readers may go directly to the results below if they prefer to skip the econometric details. A critical review of the extensive range of econometric problems encountered when estimating the NKPC is available in Henry and Pagan (2004). The recent literature on ‘weak instruments’ has demonstrated that the performance of an IV estimator may be seriously flawed when the instruments are weak, i.e. have a low correlation with the variables they replace. We have assessed the set of instruments using the rule of thumb suggested by Staiger and Stock (1997) for the case of single equation estimation. According to their rule, instruments can be considered relevant if the first-stage regression of endogenous variable on instruments produces an $F$-test statistic larger than 10.

11 I am not considering GMM estimates. Because equation (13) is a linear model, the GMM procedure is the same as 2SLS. See Hamilton (1994, pp. 420-421) for a discussion.
The numbers in parenthesis denote $t$-statistics. Inflation dynamics are thus reasonably well explained, there is no misspecification detected, and the coefficients of explanatory variables are all significant at the 5 percent level. As expected, the sum of coefficients on the inflation terms is within a small neighbourhood of unity, which is consistent with the high persistence observed in the inflation series over the sample period. Point estimates of the forward-looking coefficient are in the range of $0.45 < \hat{\beta} < 0.48$. This implies that forward-looking behaviour is essential in explaining inflation dynamics in mainland China in the sense that other potential variables have had a chance to ‘knock the variable out of the equation’. On the other hand, the point estimate of the backward-looking coefficient is in the range of $0.52 < \alpha < 0.55$. Thus, the backward-looking behaviour is also necessary for the NKPC to match the Chinese data. Overall, the data clearly tend to assign similar weights to both components. Taken at face value, the results in Table 1 suggest that the New-Keynesian conjecture is essentially correct and reproduces the core of the dynamics behind China’s inflationary spells. These results hold across all estimators.

In the Chinese case, possible regime shifts since the inauguration of economic reform attract great attention and therefore stability analysis should be part of the model evaluation. We therefore take the analysis one step further by investigating the reliability of estimates and robustness of the parameters over time. How has the nature of the NKPC trade-off changed during China’s reform period? The recursive estimates modelling the parameters of the NKPC as time-varying are given in Figure 3. In the estimates the last observation increases by one year with every iteration. The results indicate that whereas the estimates of coefficients of lead and lagged inflation are fairly robust, the significance of the output gap is fragile.

Several papers have emphasised the fit of the hybrid NKPC. As a corollary, and to gain further insight, a goodness-of-fit evaluation is carried out. Figure 4 shows actual inflation vs. fitted inflation from the hybrid NKPC. Additionally, Figure 5 shows the scatter plot of actual inflation vs. the univariate benchmark specification of a simple random walk. Comparing the scatterplots descriptively indicates that the NKPC clearly outperforms the random walk specification. Thus, the results provide support for the view that forward-looking behaviour is an important determinant of inflation dynamics. This observation is both intriguing and reassuring. On the other hand, the graph counsels caution. There is much about the inflation process that we do not model and the hybrid NKPC leaves room for further variables that might help to approximate inflation dynamics empirically.

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12 One explanation is that price liberalisation in China followed a gradual pattern. It is therefore difficult to capture these gradual changes in a regression analysis.

13 Another obvious way of testing the NKPC specification is to check for other variables incorrectly omitted from the equation. One explanation for the rapid decline in inflation in recent years is that competition in domestic markets has intensified as China has lowered trade barriers and opened up its markets to foreign competition. We have therefore used openness variables as additional explanatory variables. Moreover, in order to capture the potential impact of money supply and therefore factors operating through the demand side, I have considered the growth rate of M2 as an additional regressor. Finally, I have considered the growth rate of labour productivity in manufacturing. Unfortunately, all these regressors are wiped out by the lead and lagged inflation terms, while the output gap measure turns out to be even more fragile.
We conclude with a thought that touches on the conduct of monetary policy. If expectations of future inflation matter for inflation dynamics in mainland China, then current inflation depends upon beliefs about future inflation and therefore the future course of monetary policy and credibility of the central bank. As recent literature has shown, with forward-looking price setting, establishing a credible commitment to maintaining price stability in the future reduces the cost of doing so in the present [see Woodford (2003)]. The issue is also relevant to current discussions of the liquidity trap [see Krugman (1998)]. With forward-looking price setting, an economy constrained by the zero interest rate bound may be able nonetheless to stimulate economic activity by committing to inflate in the future. Another obvious implication is that measures of inflation expectations can help to explain future inflation and therefore require careful scrutiny and a close watch.

3. Conclusions

This paper provides an empirical analysis of inflation dynamics in mainland China, with the aim of identifying and assessing the relative importance of factors underlying price movements. Given the conflicting evidence on the determinants of inflation in mainland China, we apply the reduced-form NKPC framework to characterise inflation dynamics in China. Building on yearly observations, we find support for the conclusion that the hybrid NKPC model performs quite well and delivers a reasonable approximation of the evolution of inflation in mainland China. While not intended as an inflation-forecasting model, we take this ability to characterise historical inflation behaviour as evidence that the rational expectations model of price setting with nominal rigidities does indeed provide a fairly good approximation to the actual dynamics of inflation.

We have relied on the NKPC model to obtain the estimates, but that also raises valid questions of robustness. A more theoretically complete model would incorporate explicit utility maximisation by a forward-looking policy maker, a richer structure of expectations formation by private agents, and lags in implementation of policy targets.\footnote{An obvious drawback of the approach is that the NKPC sheds little light on the determinants of inflation and/or inflation expectations.}

In conclusion, our goal in writing this paper was to clarify the relationship between forward inflation expectations and inflation dynamics. In doing so, we hope to provide guidance for future empirical investigations into the sources and propagation mechanisms of inflation in mainland China.
References:


Table 1. NKPC Estimation Results

<table>
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<th>HH</th>
<th>MC</th>
<th>HS</th>
<th>CHO</th>
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<tbody>
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<td>-0.04</td>
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<td>-0.005</td>
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<td>$\pi_{t-1}$</td>
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<td>0.52</td>
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<tr>
<td></td>
<td>(5.4)</td>
<td>(3.1)</td>
<td>(3.1)</td>
<td>(4.6)</td>
</tr>
<tr>
<td>$\pi_{t+1}$</td>
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<td>0.48</td>
<td>0.47</td>
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<tr>
<td></td>
<td>(4.3)</td>
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<td>(3.6)</td>
</tr>
<tr>
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<tr>
<td></td>
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<td>(2.6)</td>
<td>(2.6)</td>
<td>(2.2)</td>
</tr>
<tr>
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</tr>
<tr>
<td>$LB(4)$</td>
<td>0.91</td>
<td>0.02</td>
<td>0.01</td>
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</tr>
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</table>

Notes: The error term is assumed to be MA(1). $LB(4)$ is the Box-Ljung statistic for $k = 4$. One difficulty in using any IV estimation techniques is finding appropriate instruments and choosing their lag structure. In our model, the instrumental variables are needed for the lead of inflation, which is clearly endogenous, and possibly all other driving variables. Valid instruments are lags of the endogenous variables dated $t-2$ or earlier (see Appendix). In principle, there is no limit to the number of instruments to include, but one should be careful not to include too many in finite samples as this could yield biased results. One the other hand, more instruments and lags help to capture movements in the variables of interest. Tauchen (1986) found that the most credence should be placed on estimates obtained with small instrument sets because the confidence bands are more reliable. For the estimates above, the lag sequence on the instruments is set from $t-2$ to $t-3$. In addition to lagged inflation rates and output gaps, the real oil price and real effective exchange rate were used as instruments.
Figure 1. Alternative Annual Inflation Rates

Note: The solid line represents the retail price (RPI) inflation rate, the dashed line the consumer price (CPI) inflation rate, and the solid line with squares the GDP deflator inflation rate.

Figure 2. RPI Inflation Rate vs Output Gap

Note: The solid line represents the retail price (RPI) inflation rate (left scale), the dashed line the Baxter-King (1999) output gap measure (right scale).
Figure 3. Recursive Estimates ± 2 SE of the Hybrid NKPC estimated with CHO

Constant Term ± 2 SE

Coefficient of Future Inflation ± 2 SE

Coefficient of Lagged Inflation ± 2 SE

Output Gap Coefficient ± 2 SE

Figure 4. Actual Inflation and Fit of Hybrid NKPC

Figure 5. Actual Inflation and Fit of a Random Walk
Appendix. Estimation Issues in Models with Expectations Regarding the Endogenous Variable

In this section we set out the most basic ideas associated with estimators used for RE models with forward expectations, which is a hallmark of the NKPC. We will not present the underlying theory rigorously. The purpose of the section is rather to categorise some of the many RE estimators and to convey the overall structure of the methods, along with an application in the next section.\textsuperscript{15}

**OLS with Hansen-Hodrick Standard Errors**

One simple solution to deal with the serially correlated errors in RE models has been suggested by Hansen and Hodrick (1980). The serial correlation in RE models renders the OLS estimate of the covariance matrix invalid, although parameter estimates remain consistent but inefficient.\textsuperscript{16} To achieve correct inference, Hansen and Hodrick (1980) suggested one could estimate the parameters of the model consistently by OLS and modify the estimator for the covariance matrix. Consider the model

\[ y = X\beta + \epsilon \]  

(A1)

The Hansen and Hodrick (1980) estimator of $\beta$ and variance of $\beta$ are given by

\[ \hat{\beta}_{HH} = (XX^{-1}X') \]  

(A2)

and

\[ \hat{\sigma}^2(\hat{\beta}_{HH}) = (XX^{-1}X'\hat{\omega}X)(XX^{-1}) \]  

(A3)

where the lower-triangular part of the $n \times n$ matrix $\hat{\psi}$ is given by

\[
\hat{\psi} = \begin{bmatrix}
\hat{\sigma}^2(0) \\
\hat{\sigma}^2(1) & \hat{\sigma}^2(0) \\
\vdots & \ddots & \ddots \\
\hat{\sigma}^2(q) & \ddots & \ddots & \ddots \\
0 & \ddots & \ddots & \ddots & \ddots \\
0 & 0 & \ddots & \ddots & \ddots & \ddots \\
\end{bmatrix}
\]  

(A4)

\textsuperscript{15} Readers wanting more details on the estimation techniques may consult Pesaran (1987), who gives an extensive and thorough review of econometric issues related to the estimation of RE models.

\textsuperscript{16} A traditional response would be to deal with the serial correlation by using GLS. Flood and Garber (1980), however, have shown that GLS generates inconsistent estimates in RE models. The reason is that the transformation involved in using GLS results in the orthogonality condition between regressors and the disturbance term being violated.
in which

\[ \hat{\gamma}_e(s) = \frac{1}{T} \sum_{i=1}^{T} \hat{\gamma}_{1-i} \] (A5)

are the autocovariance coefficients for the error term and

\[ \hat{\epsilon} = y - X' \hat{\beta}_{HH} \] (A6)

are the OLS residuals.\(^{17}\)

**McCallum's (1976) Instrumental Variable Approach**

One of the earliest (limited information) IV approaches for estimating models with forward expectations formed rationally was McCallum's (1976) error-in-variables approach. To facilitate expositions, it is useful to express the model to be estimated in compact matrix notation as

\[ y = X^* \beta + \epsilon \] (A7)

\[ X = Z \kappa + \epsilon \] (A8)

where the matrix \( X^* \) is the (unobservable) expected value of \( X \) and \( Z \) is the matrix of appropriate instruments which have been lagged sufficiently to preclude correlation with the MA error. The residuals \( \epsilon \) are assumed to be independent and identically distributed with zero mean and constant variance. The random vector \( \epsilon \) is also taken to have the classical properties and to be uncorrelated with the information set. Accordingly, if the expectations in (A7) are formed rationally, we have

\[ X^* = Z \kappa = X - \epsilon \] (A9)

Equation (9) can be used to eliminate the unobservable matrix \( X^* \). We can thus restate (A7) as

\[ y = X \beta + \epsilon \] (A10)

where \( \epsilon = \epsilon - \epsilon \beta \). Because \( X \) is correlated with \( \epsilon \), the standard OLS estimation procedure is inappropriate, but IV is applicable with \( Z \) serving as instruments. To be more explicit, McCallum's (1976) consistent estimator of \( \beta \) is

\[ \hat{\beta}_{CA} = (X'Z(Z'Z)^{-1}Z'X)^{-1}X'Z(Z'Z)^{-1}Z'y \] (A11)

and the asymptotic variance-covariance matrix of \( \hat{\beta}_{CA} \) is given as

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\(^{17}\) \( q \) is the order of the MA process. Note that if \( q = 0 \) the matrix \( \psi \) is diagonal and therefore the Hansen and Hodrick (1980) procedure yields standard OLS.
\[
\hat{V}(\hat{\beta}_{CA}) = \frac{1}{T} \hat{\varepsilon}' \hat{\varepsilon} \left( X' Z (Z' Z)^{-1} Z' X \right)^{-1}
\]  

(A12)

where \( \hat{\varepsilon} = y - X \hat{\beta}_{CA} \). The feasible set of potential instruments consists of lagged endogenous and weakly exogenous variables set back \( t - 1 \) periods or earlier.\(^{18}\) It should be noted, however, that while the IV procedure is consistent it is not asymptotically efficient because it ignores the MA(q) error arising from the expectations variable.

**Hayashi and Sims’ (1983) Forward Filter Estimator**

The next estimator to be considered is the forward filter estimation procedure proposed by Hayashi and Sims (1983). Contrary to McCallum’s (1976) IV estimator presented above it removes the induced correlation between instruments and disturbances and therefore provides asymptotically efficient and consistent estimates. The basic equation to be estimated is again

\[
y = X\beta + \varepsilon
\]

(A13)

where \( \varepsilon \) has a MA(q) representation. Let \( V = E[\varepsilon \varepsilon'] \). Then there always exists an upper-triangular matrix \( W \) such that \( \sigma^2 I = WW' \), where \( \sigma^2 \) is the variance of \( \varepsilon \). Then \( W\varepsilon \) will have a covariance matrix with off-diagonal elements equal to zero, and so pre-multiplying equation (A13) by \( W' \), followed by an IV regression will remove the inconsistency in estimating \( \beta \). In other words, whereas the usual GLS transformation uses a lower triangular matrix to decompose the variance-covariance matrix, the major novelty of Hayashi and Sims (1983) is to choose \( W \) so that it is upper triangular, making \( W\varepsilon \) a linear combination of current and future values of \( \varepsilon \), which are orthogonal to the instrument set. Thus the asymptotically efficient Hayashi and Sims (1983) estimator is

\[
\hat{\beta}_{HS} = \left[ (WX)' Z (Z' Z)^{-1} Z' (WX) \right]^{-1} (WX)' Z (Z' Z)^{-1} Z' \varepsilon
\]

(A14)

and

\[
\hat{V}(\hat{\beta}_{HS}) = \sigma^2 \left[ (WX)' Z (Z' Z)^{-1} Z' (WX) \right]^{-1}
\]

(A15)

in which

\[
\hat{\sigma}^2 = \frac{1}{T} \left( y - X \hat{\beta}_{HS} \right)' \left( y - X \hat{\beta}_{HS} \right)
\]

(A16)

\(^{18}\) If some of the \( X \)’s are strictly exogenous, i.e. uncorrelated with the error term at all leads and lags, then there is no need to lag them. It is possible that the MA process is of an order greater than \( q \). It is well known that time aggregation can lead to MA error terms. Serial correlation due to time aggregation may thus lead to a higher order MA process than \( q \).
and where $Z$ is again the instrument matrix.\textsuperscript{19} As equations (A14) and (A15) reveal, the Hayashi-Sims (1983) estimator is simply IV, where the original dependent and independent variables (but not the instruments) have been forward filtered.\textsuperscript{20}

**Cumby, Huizinga and Obstfeld's (1983) Two-Step Two-Stage Least Squares Estimator**

The last estimator to be considered in this paper is Cumby, Huizinga and Obstfeld’s (1983) generalised IV estimator which they call the two step two stage least squares (2S2SLS) estimator. The model we consider is again

$$y = X\beta + \varepsilon$$  \hspace{1cm} (A17)

As it stands, $\varepsilon$ in (A17) is correlated with $X$. Multiplying equation (A17) by the instrument matrix $Z$ yields consistent estimates, but the matrix $E(Z'\varepsilon\varepsilon'Z)$ is not diagonal due to the serial correlation of $\varepsilon$. The correction made by the 2S2SLS estimator leads to

$$\Omega^{-1}Z'y = \Omega^{-1}Z'X\beta + \Omega^{-1}Z'\varepsilon$$  \hspace{1cm} (A18)

and therefore

$$\hat{\beta}_{CHO} = \left(X'Z\hat{\Omega}^{-1}Z'X\right)^{-1}X'Z\hat{\Omega}^{-1}Z'y$$  \hspace{1cm} (A19)

where $\hat{\Omega}$ is an estimate of $\lim_{T \to \infty} \frac{1}{T} E(Z'\varepsilon\varepsilon'Z)$ and the estimated parameter covariance matrix is given by

$$\hat{V}(\hat{\beta}_{CHO}) = T\left(X'Z\hat{\Omega}^{-1}Z'X\right)^{-1}$$  \hspace{1cm} (A20)

The first step in forming the 2S2SLS estimator involves an estimate of $\Omega$. If $\hat{\varepsilon}$ are the residuals from the first-step IV regression of (A17), then

$$\hat{\Omega} = \sum_{i=-q}^{q} \frac{1}{T} \sum_{t=1}^{T} Z'_{i} \hat{\varepsilon}_{t} \hat{\varepsilon}_{t-i} Z_{t-i}$$  \hspace{1cm} (A21)

where the disturbance has a $MA(q)$ representation.\textsuperscript{21} Unlike Hayashi and Sims’ (1983) estimator, the 2S2SLS estimator does not rely on homoscedasticity. Asymptotically, the Hayashi and Sims (1983) estimator and 2S2SLS have the same distribution. In finite samples their relative efficiency depends on the instruments used, as both estimators exploit a different orthogonality condition.\textsuperscript{22}

\textsuperscript{19}A consistent estimate for $V = E(\varepsilon'\varepsilon)$ is the matrix of sample autocovariance functions.

\textsuperscript{20}If, for example, we have an $MA(1)$ error $\varepsilon_t = (1-\phi L)u_t$ [where $u_t$ is white noise and $L(L-1)$ is the backward (forward) lag operator], then the backward filter $(1-\phi L)^{-1}$ applied to the variables removes the serial correlation but destroys the orthogonality between the information set and the error term. Hayashi and Sims’s (1983) suggestion therefore is to use the forward filter $-(1-\phi L)^{-1}$. Unlike in the case of backward filtering, the disturbances are now orthogonal to $Z$, as well as to all their past values; see Pesaran (1987), pp. 187 - 188.

\textsuperscript{21}The 2S2SLS estimator (like the other two RE model estimators) must produce an estimate of the variance-covariance matrix $\Omega$ which is positive definite. In a general formulation this is not necessarily the case. We have therefore used Newey and West’s (1987) modified Bartlett weights in the estimates.

\textsuperscript{22}The Hayashi and Sims (1983) estimator exploits the moment $E(Z'(W_\varepsilon))$, while the Cumby, Huizinga and Obstfeld (1983) estimator exploits $E(Z'\varepsilon)$.