

# A Simple Model of Optimal Monetary Policy with Financial Constraints

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## Abstract

Recent experience suggests that the operation of monetary policy in emerging market economies is severely limited by the presence of financial constraints. This is seen in the tendency to follow contractionary monetary policy during crises, and the observation that these countries pursue much more stable exchange rates than do high income advanced economies, despite having a more volatile external environment. This paper analyzes the use of monetary policy in an open economy in which exchange rate sensitive collateral constraints may bind in some states of the world. The appeal of the model is that it allows for a complete analytical description of the effects of collateral constraints, and admits a full characterization of welfare-maximizing monetary policy rules. The model can explain the two empirical features of emerging market monetary policy described above - in particular, that optimal monetary policy may be pro-cyclical under binding collateral constraints, and an economy with large external shocks may favor a fixed exchange rate, even though flexible exchange rates are preferred when external shocks are smaller.

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# 1 Introduction

In developed economies, monetary policy is generally counter-cyclical. For instance, there is a widespread consensus that policy should be eased in a recession. By contrast, in the recent experience of emerging market economies, monetary policy has often been pro-cyclical, raising interest rates during a crisis, usually in order to defend the exchange rate. As an example, after the Asian-Russian crisis of 1997-98, interest rates fell in the US, Australia, Canada, and most other developed economies, while they rose in almost all emerging market economies (Edwards 2001). Related evidence from Calvo and Reinhart (2002) indicates that many emerging economies place a high weight on exchange rate stability, even in face of large macroeconomic shocks which in principle would call for exchange rate adjustment <sup>1</sup>.

Why do we see such a contrast between the policy responses of developed economies and emerging markets? One explanation is market confidence. Many of these economies have a history of bad policy, so that in a crisis it is more important to raise interest rates to restore the confidence of international capital markets than to attempt to stabilize the domestic economy. But many economists (e.g. Krugman(1998), Stiglitz (2002)) have questioned this, arguing that tight monetary policies exacerbate the crisis rather than generating confidence.

An alternative explanation for the differences in policy is that emerging market economies are more financial vulnerable, and in the presence of a mismatch between domestic assets and foreign liabilities in balance sheets, an exchange rate depreciation may be more of a hindrance than a help.

Empirical evidence supports the view that financial vulnerability is an important constraint on macroeconomic policy in emerging markets (Goldstein, Kaminsky, and Reinhart (2000)). These countries can almost never issue external debt in their own

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<sup>1</sup>In face of external demand shocks, a fixed exchange rate is a pro-cyclical monetary policy rule.

currency, and weak domestic financial institutions mean that balance sheet effects are an important limitation on domestic production (Eichengreen and Hausmann (2003)).

A growing literature has developed models in which balance sheet constraints impinge upon the workings of monetary policy and exchange rates<sup>2</sup>. Many of these papers treat financial vulnerability as a collateral constraint that limit firm's investment or production financing. Due to foreign currency denominated debt, these collateral constraints are likely to be sensitive to movements in the exchange rate.

This paper develops a very simple model of monetary policy making in an open economy, in the presence of sometimes binding collateral constraints which are related to trade credit financing<sup>3</sup>. The main contribution of the paper is to construct an optimal, welfare maximizing monetary policy rule which takes collateral constraints into account. We find that an optimal rule calls for a conventional monetary policy in normal times, for small shocks. In this case, the exchange rate acts as a shock absorber, and helps to stabilize the real economy in face of external shocks.

But in face of large negative shocks which cause collateral constraints to bind, the optimal rule requires a counter-cyclical policy response. This is because when collateral constraints bind, exchange rate adjustment may be de-stabilizing. We can therefore rationalize why monetary policy should be pro-cyclical during a crisis, within the context of a welfare-maximizing optimal monetary policy problem. The key reason to tighten monetary policy in face of a negative shock is that this policy relaxes the collateral constraints facing the economy. In general however, we find that monetary policy should not be so pro-cyclical as to actually undo the collateral constraints entirely.

The model can help to explain why some countries might prefer exchange rate stability, even in face of large external shocks, which in the absence of financial constraints, would require substantial movement in exchange rates. Somewhat paradoxically, it is precisely when shocks are large and financial constraints may be binding

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<sup>2</sup>See references below

<sup>3</sup>IMF (2003) note that many recent emerging market crises were characterized by a very large decline in trade financing, and that this may have played a substantial role in exacerbating the crises.

that exchange rate stability may be desirable. With smaller shocks, which allow for adjustment without hitting collateral constraints, a flexible exchange rate is better.

An important consideration in the comparison of exchange rate regimes is the stock of outstanding foreign currency debt. When this is high, increasing the chances that the collateral constraint will bind, a fixed exchange rate is more likely to dominate a (non-optimal) floating exchange rate rule. This accords closely with the empirical evidence in Devereux and Lane (2003b).

There are a substantial number of papers that have explored different aspects of monetary policy in the presence of financial constraints<sup>4</sup>. Two sets of papers that are close in spirit to ours are Cook (2002) and Choi and Cook (2002), and Christiano, Gust and Roldos (2002), and Braggion, Christiano and Roldos (2003). Cook (2002) shows that a negative balance sheet effect of a devaluation can be enough to cause a fall in output when investment borrowing is limited by domestic firms net worth. Choi and Cook (2002) extend this model to allow for the role of banks, and show that a fixed exchange rate can enhance welfare by stabilizing banks balance sheets. Christiano, Gust and Roldos (2002) introduce collateral constraints in financing trade credit, as we do, and show that monetary policy may be contractionary with binding collateral constraints. Braggion, Christiano and Roldos (2003) compute an optimal interest rate rule as a response to a financial crisis.

Our paper differs from these in a number of dimensions. First, it is extremely simple. The assumption that trade credit financing determines the form of the collateral constraint allows us to represent the economy in a one-period environment, which may be illustrated in a single diagram. Secondly, due to this simple setup, we can compute welfare-maximizing monetary policy rules, with commitment, in a stochastic environment, using exactly the approach as used in the recent literature on optimal monetary policy in open economy models with nominal rigidities. Our model in fact

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<sup>4</sup>See for instance, Aghion, Bacchetta and Banerjee (2000), Braggion, Christiano, and Roldos, Burnside, Eichenbaum, and Rebelo (2000), Caballero and Krishnamurty (2001, 2003), Céspedes, Chang and Velasco (2002a, 2002b), Cook (2002), Choi and Cook (2002), Devereux and Lane (2002, 2003), Christiano, Gust and Roldos (2002), Mendoza (2002), Mendoza and Smith (2002).

necks a standard open economy sticky-wage environment, when collateral constraints are absent, or never binding. Thus, the analysis allows us to be precise about the nature and extent of differences in monetary policy stance between developed economies and financially vulnerable emerging market countries.

## 2 The Model

Consider a one-period model of a small open economy with consumer and firms<sup>5</sup>. There is a continuum of households along the unit interval, consuming both home and foreign produced goods, and providing heterogenous labour services to the final goods firms. Firms are competitive, and use both local labour and an intermediate imported good for production. Firms must finance their import purchases with trade credit extended by foreign exporters, which is repaid when they sell their output at the end of the period. However, as in Aghion et al. (2001), and Mendoza and Smith (2002), these importing firms may face collateral constraints related to their net worth. Their net worth comprises fixed domestic-currency denominated assets, less foreign currency denominated debt. When there is a large jump in the exchange rate, the collateral constraints may become binding, and firms then become rationed in their purchase of intermediates.

Households-workers set nominal wages in advance, before the realization of the state of the world.

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<sup>5</sup>Since the constraint on external financing falls on imported raw materials, rather than investment, a one-period decision making structure is sufficient to map out the key elements of our model. A similar model is presented in Devereux and Lane (2003b). In the case where investment finance is distorted by collateral constraints, as in Mendoza and Smith (2002), Céspedes et al. (2002a), or Aghion et al. (2001), it would be necessary to construct an explicitly dynamic model. The virtue of the present analysis is that all features of the model be explicitly characterized in a simple analytical way, and we may also conduct a standard optimal monetary policy analysis. See IMF (2003) for evidence on the importance of trade financing disruptions in exacerbating the effects of crises in emerging markets.

## 2.1 Firms

Final goods are produced using labour and an intermediate imported good. Labour is differentiated across households, so that households have market power in wage setting.

We can define the aggregate labour composite as:

$$H = \left[ \int_0^1 H(i)^{1-\frac{1}{\rho}} di \right]^{\frac{1}{1-\frac{1}{\rho}}} \quad (2.1)$$

where  $H(i)$  is employment of household  $i$ , and  $\rho > 1$  is the elasticity of substitution between labour varieties.

The production function for final goods is given by:

$$Y = AH^\omega I^{1-\omega} \quad (2.2)$$

where  $A$  is a constant productivity term, and  $I$  represents the imported intermediate input.

Firms' profits are defined as:

$$\Pi = P_h Y - WH - Sq^* I \quad (2.3)$$

Firms maximize profits taking the nominal wage  $W$  and the foreign currency price of intermediate imports  $q^*$  as given.  $S$  is the nominal exchange rate.

In addition, firms are assumed to face a collateral constraint, related to net worth. The constraint is represented as

$$Sq^* I \leq N - SD^* \quad (2.4)$$

where  $N$  is the domestic currency denominated assets and  $D^*$  is the pre-existing foreign currency liabilities of the importers <sup>6</sup>. This collateral constraint may limit access to intermediate imported goods when the economy faces a large devaluation. In this

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<sup>6</sup>As in Mendoza and Smith (2002), we can motivate this collateral constraint by the difficulties of enforcing international contracts. It could be argued that the model would be more realistic were there a separate group of intermediate importing firms who purchased intermediates from abroad, subject to trade-credit related collateral constraints, and sold intermediates to final goods firms. But in fact, the aggregate results in that case would be identical to those in this paper, so to avoid excess notation, we simply assume that final goods firms are subject to these constraints.

sense, it captures the importance of *currency mismatch* between assets and liabilities, a phenomenon that has been emphasized by many commentators on emerging market crises (e.g. Eichengreen, and Hausmann (2003)). The constraint is *not always binding*. We might think that in normal times, the collateral constraint holds with strict inequality, and firms can freely import intermediates at the world price. On the other hand, following a large devaluation, this constraint may be binding, and firms will be constrained.

### 2.1.1 Constraints not binding

In normal times, firms are unconstrained. Assuming free entry, profit maximization problem gives the price of home produced goods:

$$P_h = \kappa \frac{W^\omega (Sq^*)^{1-\omega}}{A} \quad (2.5)$$

where  $\kappa = \left(\frac{1}{1-\omega}\right)^{1-\omega} \left(\frac{1}{\omega}\right)^\omega$ .

### 2.1.2 Binding constraints

In times of financial stress, the collateral constraint is binding,  $Sq^*I = N - SD^*$ . Then we have  $I = \frac{N - SD^*}{Sq^*}$ . Firms choose employment to maximize profits, and we get the implicit labour demand function:

$$W(i) = \frac{\omega AH^\omega I^{1-\omega}}{H} \left(\frac{H(i)}{H}\right)^{-\frac{1}{\rho}} P_h. \quad (2.6)$$

In a symmetric equilibrium as described below,  $H(i) = H$  and  $W(i) = W$ , where  $W(i)$  is the nominal wage set by household  $i$ . We therefore get the optimal employment condition:

$$P_h \omega \frac{Y}{H} = W \quad (2.7)$$

and output:

$$Y = AH^\omega \left(\frac{N - SD^*}{Sq^*}\right)^{1-\omega} \quad (2.8)$$

## 2.2 Households

Household  $i$ ,  $i \in [0, 1]$ , have preferences given by:

$$\ln(C(i)) + \chi \ln\left(\frac{M(i)}{P}\right) - \eta \frac{H(i)^{1+\psi}}{1+\psi} \quad (2.9)$$

$C(i)$  is a composite of the consumption of home and foreign goods, given by:

$$C(i) = C_h(i)^\alpha C_f(i)^{1-\alpha} \quad (2.10)$$

and  $P$  is the price index, given by  $P = \left(\frac{P_h}{\alpha}\right)^\alpha \left(\frac{SP_f^*}{1-\alpha}\right)^{1-\alpha}$ , where  $P_f^*$  is the foreign currency price of foreign goods.  $\alpha$  represents the relative preference for home goods.  $M(i)$  is the quantity of domestic money held, and  $\psi$  is the elasticity of labour supply. Households face the budget constraint:

$$PC(i) + M(i) = W(i)H(i) + M_0(i) + T(i) + \Pi \quad (2.11)$$

where  $M_0(i)$  is initial money holdings,  $T$  is total transfer from the monetary authority, and  $\Pi$  is total profits of the final good firms.

Households choose money balances and consumption of each good to maximize utility, subject to their budget constraint. We get the demand for each good,  $C_h(i)$  and  $C_f(i)$ , and that of money balances:

$$C_h(i) = \frac{\alpha PC(i)}{P_h} \quad (2.12)$$

$$C_f(i) = \frac{(1-\alpha)PC(i)}{P_f} \quad (2.13)$$

$$M(i) = \chi PC(i) \quad (2.14)$$

We assume that nominal wages are pre-set ex ante, and cannot adjust to shocks within the period. Each household  $i$  faces a downward-sloping labour demand curve with elasticity  $\rho$ , given in equation (2.6). The expected utility maximizing wage is:

$$W(i) = \eta \frac{\rho}{\rho-1} \frac{E\{H(i)^{1+\psi}\}}{E\left\{\frac{H(i)}{PC(i)}\right\}} \quad (2.15)$$



## 2.3 Equilibrium

We assume that foreign demand for the home good is unit elastic, and is given by:

$$X^d = \tilde{X} \frac{S}{P_h}, \quad (2.16)$$

where  $\tilde{X}$  is an exogenous stochastic foreign demand shift term.

We focus on symmetric equilibria in the sense that:  $C(i) = C$ ,  $H(i) = H$ ,  $W(i) = W$ ,  $M(i) = M$ ,  $M_0(i) = M_0$  and  $T(i) = T$ ,  $\forall i \in [0, 1]$ . Define a symmetric, imperfectly competitive equilibrium, given any monetary policy rule, as the set of allocations,  $\Theta = \{C, H, M\}$  and the set of prices,  $\wp = \{W^s, S, P_h\}$  given  $P_f^*, q^*$  such that:

1. Firms maximize profits;
2. The wage is set by households to maximize expected utility;
3. Households maximize their utility over consumption and real balances subject to ex-post budget constraints;
4. The money market clears:

$$M = M_0 + T \quad (2.17)$$

5. The home goods market clears:

$$Y = \alpha \frac{PC}{P_h} + \tilde{X} \frac{S}{P_h}. \quad (2.18)$$

The equilibrium conditions must be characterized separately under the two regimes, depending upon whether the collateral constraints bind or not.

### 2.3.1 Equilibrium Conditions without collateral constraints

When the collateral constraint is not binding, the equilibrium conditions are characterized as follows. Money market clearing and profit maximization imply:

$$PC = WH = P_h Y - S q^* I = P_h Y - (1 - \omega) P_h Y = \omega P_h Y \quad (2.19)$$

which implies:

$$M = \chi PC = \chi \omega P_h Y \quad (2.20)$$

The market clearing condition can be written as:

$$Y = \alpha \omega Y + \tilde{X} \frac{S}{P_h}. \quad (2.21)$$

Along with the optimal pricing equation,

$$P_h = \kappa \frac{W^\omega (Sq^*)^{1-\omega}}{A}, \quad (2.22)$$

equations (2.20) - (2.22) may be solved for  $\{P_h, S, Y\}$ , conditional on  $\tilde{X}$  and the pre-set wage  $W$ . Equation (2.15) then determines the wage, given the distribution of employment, prices and consumption.

### 2.3.2 Equilibrium Conditions with collateral constraints

When the collateral constraint is binding, the household's budget constraint becomes:

$$PC = P_h Y - N + SD^* \quad (2.23)$$

Then the money market equilibrium is given by:

$$M = \chi PC = \chi (P_h Y - N + SD^*). \quad (2.24)$$

The goods market clearing condition can then be written as:

$$Y = \alpha \left( Y - \frac{N - SD^*}{P_h} \right) + \tilde{X} \frac{S}{P_h}. \quad (2.25)$$

Together with the profit maximization condition and the production function:

$$P_h \omega \frac{Y}{H} = W \quad (2.26)$$

$$Y = AH^\omega \left( \frac{N - SD^*}{Sq^*} \right)^{1-\omega}, \quad (2.27)$$

equations (2.24) - (2.27) can be solved for the variables  $\{P_h, H, S, Y\}$ , conditional on  $\tilde{X}$  and the nominal wage. Again, equation (2.15) determines the nominal wage.

## 2.4 The nature of the collateral constraint

What determines whether the collateral constraint binds? From the properties of the economy in the *unconstrained region*, we have:

$$Sq^*I = (1 - \omega)P_hY = \frac{1 - \omega}{\omega} \frac{M}{\chi} \quad (2.28)$$

Hence, total spending on intermediate imports in the unconstrained economy depends only on the domestic money supply. We can therefore write the collateral constraint as:

$$\frac{1 - \omega}{\omega} \frac{M}{\chi} \leq N - SD^* \quad (2.29)$$

We can then define the cut-off exchange rate  $\bar{S}$ , at which the collateral constraint will just bind, as:

$$\bar{S} = \frac{1}{D^*} \left[ N - \frac{1 - \omega}{\omega} \frac{M}{\chi} \right] \quad (2.30)$$

When the nominal exchange rate is below  $\bar{S}$  ( $S < \bar{S}$ ), the constraint doesn't bind. When  $S \geq \bar{S}$ , however, firms are restricted by the collateral constraint.

In the analysis below, we will make the regularity assumption  $\frac{1 - \omega}{\omega} \frac{M}{\chi} < N$ . This implies that the collateral constraint will not bind in an economy without foreign currency debt. Note that there exists a monetary policy rule for which the collateral constraint does not bind in any state of the world. To see this, note from (2.30), the cut-off exchange rate,  $\bar{S}$ , is negatively related to  $M$ , so that the monetary authority can always choose a money supply low enough so that the constraint is not binding.

This highlights a particular property of the model. By following a contractionary monetary policy, the collateral constraint becomes less binding on two counts. First, as we see below, the equilibrium exchange rate appreciates, so it is more likely that  $S < \bar{S}$ . At the same time, the contractionary monetary policy also raises the cut-off exchange rate  $\bar{S}$ , because it reduces the total domestic currency demand for intermediate imported goods. This too reduces the likelihood that the collateral constraint binds. As we see below, it is this joint property of monetary contraction that leads to our results regarding unconventional monetary policy rules.

### 3 A diagrammatic analysis

Given a fixed nominal wage, the behavior of the model under each regime can be illustrated in a very simple fashion.

#### 3.1 Unconstrained Regime

In the unconstrained regime, the economy behaves as a simple Mundell-Fleming type model. This is described by equations (2.20)-(2.22). Substituting from (2.22) into (2.20) and (2.21) gives the two equations

$$M = \chi\omega\kappa \frac{W^\omega (Sq^*)^{1-\omega}}{A} Y \quad (3.1)$$

The market clearing condition can be written as:

$$Y = \frac{1}{1-\alpha\omega} A \tilde{X} \frac{S^\omega}{\kappa W^\omega (q^*)^{1-\omega}}. \quad (3.2)$$

Equation (3.1) gives the money market clearing condition, while (3.2) gives the goods market clearing condition. The first equation describes a downward sloping schedule in  $S, Y$  space, while the second describes an upward sloping schedule. These are described as the LM and IS curves in the lower half of Figure 1, respectively. A fall in foreign demand  $\tilde{X}$  will shift the IS schedule back to the left, while a rise in the money supply shifts the LM schedule to the right.

#### 3.2 Constrained Regime

When the collateral constraint binds, the conditions analogous to (3.1) and (3.2) must be amended. From (2.24)-(2.27), we can combine profit maximizing behaviour of firms with the money market clearing condition to get:

$$M = \chi \left( W^\omega A^{-\frac{1}{\omega}} Y^{\frac{1}{\omega}} \left[ \frac{N - SD^*}{q^* S} \right]^{\frac{\omega-1}{\omega}} - (N - SD^*) \right) \quad (3.3)$$

The equivalent condition for the goods market equilibrium condition may be derived as:

$$Y^{\frac{1}{\omega}} = \frac{1}{1-\alpha} \frac{[\tilde{X} S - \alpha(N - SD^*)]}{W^\omega A^{-\frac{1}{\omega}}} \left[ \frac{N - SD^*}{Sq^*} \right]^{\frac{1-\omega}{\omega}} \quad (3.4)$$

Equation (3.3) represents the money market clearing condition when the economy is in the collateral constrained region. As before it is represented by a downward sloping relationship in  $S, Y$  space, illustrated as the constrained LM schedule in the top half of Figure 1. An exchange rate depreciation first of all reduces nominal purchases of intermediate imports, and *ceteris paribus*, raises nominal income and the demand for money. But there is a secondary effect of a depreciation, coming from a rise in the home good price, which also raises the demand for money. In both cases, for a given money stock, output must fall to allow the money market to clear. In the region where the collateral constraint is just binding (point C in Figure 1), the constrained LM schedule is always flatter than that in the unconstrained region <sup>7</sup>

Equation (3.4) describes the goods market clearing relationship between output and the exchange rate. Again, there are two effects to take into account. First, an exchange rate depreciation directly increases demand for the home good, because it raises foreign demand, and increases home nominal income by a reduction in payments on intermediate imports (since  $N - SD^*$  must fall). But the depreciation also raises the home goods price  $P_h$ , which reduces demand. The extent of the rise in the price of the home good depends on two features of the model; a), the ratio of foreign currency debt to net worth,  $\frac{SD^*}{N - SD^*}$ , which we define as the *leverage ratio*, and b) the share of intermediate imports in production,  $1 - \omega$ . The higher is the leverage ratio, and the smaller is  $\omega$ , the more likely it is that an exchange rate depreciation has a negative impact on total demand, through equation (3.4). Unlike the unconstrained economy, it is possible that (3.4) implies a *negative* relationship between output and the exchange rate. The constrained IS schedule is illustrated in the top part of Figure 1. It is possible to show that if the constrained IS schedule is negatively sloped, it is always steeper than that of the constrained LM schedule. This ensures that equilibria are locally unique, even in the presence of sometimes binding collateral constraints.

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<sup>7</sup>We can show this by taking a log linear approximation of (3.1) and (3.3) around the point where the collateral constraint is just binding. The slope of the unconstrained LM schedule at this point is  $-\frac{1}{1-\omega}$ , while the slope of the constrained LM schedule is  $\frac{-\omega^2}{(1-\omega)(1+l(1+\omega))}$ , where  $l = \frac{SD^*}{N - SD^*}$  is the leverage ratio (see below).

The threshold between the unconstrained and the constrained regimes in Figure 1 is given by the cut-off exchange rate  $\bar{S}$ . When there is a very low value of foreign currency debt  $D^*$ ,  $\bar{S}$  will be very high, and the constrained regime is less likely to be operative.

We can use (3.1)-(3.2) and (3.3)-(3.4) and Figure 1 to discuss the implications of the model for the response to world demand shocks and the conduct of monetary policy.

### 3.2.1 Cushioning world demand shocks

The response of the economy to fluctuations in world demand  $\tilde{X}$  depends critically upon whether the collateral constraint binds or not. Take the case where the collateral constraint never binds (e.g. where there is a very low value of  $D^*$ , and thus a low leverage ratio, so  $\bar{S}$  is very high), and equilibrium is characterized by the lower part of Figure 1. Furthermore, assume that monetary policy targets nominal income, which is equivalent in this case to a fixed money supply. Then, after a fall in  $\tilde{X}$ , the exchange rate will depreciate, hence mitigating the fall in GDP. If the exchange rate were to be kept fixed, then GDP would fall by the full amount of the leftward shift in the IS curve. But with a fixed money supply, the depreciation of the exchange rate cushions the economy from foreign demand shocks.

On the other hand, if fluctuations in  $\tilde{X}$  occur in the constrained region, so that equilibrium is characterized by the upper part of Figure 1, then the characteristics of adjustment are substantially different. Assume that the leverage ratio is high enough so that the IS schedule is negatively sloped. Then a fall in  $\tilde{X}$  shifts the IS curve back to the left. At a given exchange rate, output will fall. Again, a monetary rule which keeps  $M$  constant ensures the LM curve does not shift, so the exchange rate will depreciate. In contrast to the unconstrained case, now the depreciation is *destabilizing*. GDP falls by more after the depreciation, and the fall in output would be mitigated by preventing the depreciation.

This comparison illustrates a critical difference between the workings of flexible

exchange rates in economies with and without financial frictions. Without frictions, the exchange rate acts as a shock absorber in the standard way, and in response to external shocks, a flexible exchange rate is stabilizing. But with severe balance-sheet related financial frictions, arising from large foreign-currency debt positions, the exchange rate no longer acts as a macro-economic shock absorber, and endogenous movements in the exchange rate may de-stabilize the economy.

### 3.2.2 Large versus small shocks

So far we have simply assumed that the economy was either in the unconstrained or the constrained region. But it is clear from Figure 1 that shocks can push the equilibrium from one region to another. For instance, a negative shock to world demand can shift the IS curve back so that the economy moves from the unconstrained region to the constrained region. Since the cut-off exchange rate  $\bar{S}$  is independent of  $\tilde{X}$ , this happens if the required nominal depreciation would entail an exchange rate greater than  $\bar{S}$ .

This has two implications. First, the economy may behave quite differently for large versus small shocks. In response to moderate macro-economic shocks, the fluctuation in the real economy may quite modest and movements in exchange rates are stabilizing. But when shocks are large, exchange rate adjustment is de-stabilizing, and can produce much larger movements in real income (in the case of negative shocks). In this sense, our model carries some implications for the ‘fear of floating’ debate initiated by Calvo and Reinhart (2002). This is discussed further below.

Secondly, the model predicts a distinct non-linearity. Large positive world demand shocks have a smaller positive effect on output than the negative effect of large negative shocks. Again, this carries implications for monetary policy.

### 3.2.3 Monetary policy and fear of floating

The discussion until now assumed that the country followed a constant money supply rule. But if the economy is in the constrained region, it is doubtful whether such a rule

is desirable. In the next section, we compute the exact utility maximizing monetary policy rule. Before doing this however, we can make some immediate observations about the model's implications for monetary policy.

Calvo and Reinhart (2002) uncover the puzzling fact that some developing and emerging market economies seem to prevent their exchange rate adjusting to shocks, even though they experience bigger shocks than the high income countries. This 'fear of floating' seems clearly at variance with the standard Mundell-Fleming intuition on the value of exchange rate adjustment. But we can see that such an aversion to exchange rate adjustment with large shocks may be precisely what a model with sometimes-binding collateral constraints implies. If a monetary authority had to choose either a commitment to a stable exchange rate, or a fully flexible exchange rate, then in the presence of large shocks, and with a high foreign currency debt position, the fixed exchange rate may dominate a flexible exchange rate. Paradoxically, this is *less* likely to be the case when the country is subject to a lower volatility of external shocks. Thus, with financial frictions of the type described here, the relationship between the volatility of macro shocks and the benefits of flexible exchange rate becomes complicated. It is likely to be different for countries with large shocks and high foreign debt positions than for countries without these attributes.

If a country wishes to use monetary policy to prevent exchange rate adjustment in the collateral constrained region, it can do so by shifting the LM curve *back* to the left. With a high leverage ratio, and when we are in the constrained region, monetary policy is *contractionary*, and GDP can be stabilized by a monetary policy tightening after a negative shock. Fixing the exchange rate at  $\bar{S}$  prevents the economy from entering the constrained region, and leads to a lower output loss after a negative shock than allowing the exchange rate to float with a constant money supply. But in order to undo the collateral constraint, the monetary authority does not need to keep the exchange rate fixed at  $\bar{S}$ . This is because, as is clear from (2.30), the cut-off exchange rate depends upon the monetary policy rule itself. To see how this affects the workings of monetary policy, take Figure 2, where the economy is at point D in the constrained



region after a negative world demand shock. By reducing the money supply, the central bank can shift the LM curve down and raise GDP. But at the same time, the reduction in the money supply raises  $\bar{S}$ , and reduces the area over which the collateral constraint binds. In this sense, a pro-cyclical monetary policy has two benefits when a negative shock pushes the economy into a financially constrained equilibrium - it raises output, and raises the threshold exchange rate at which the economy becomes financially vulnerable.

Figure 1: *IS - LM* Diagram

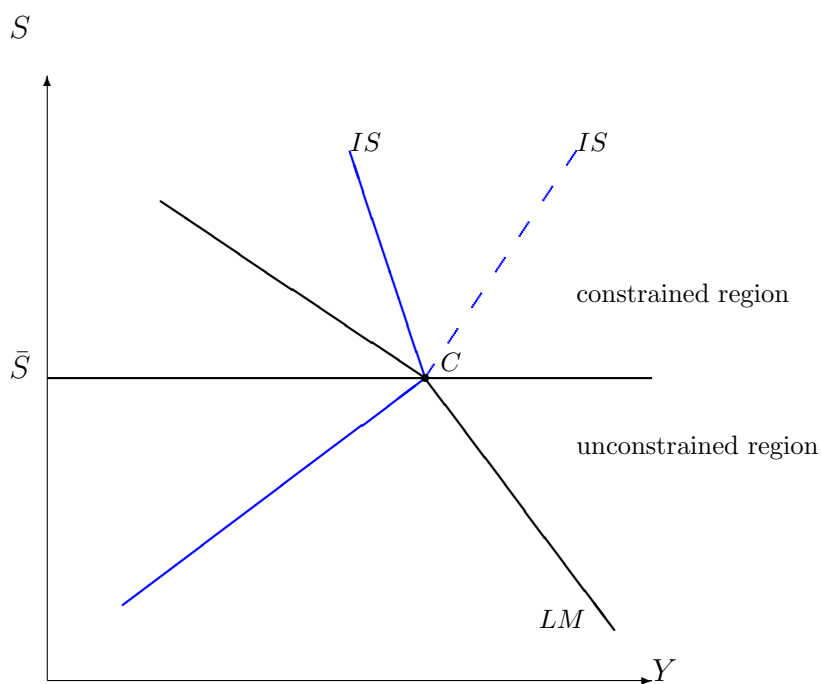
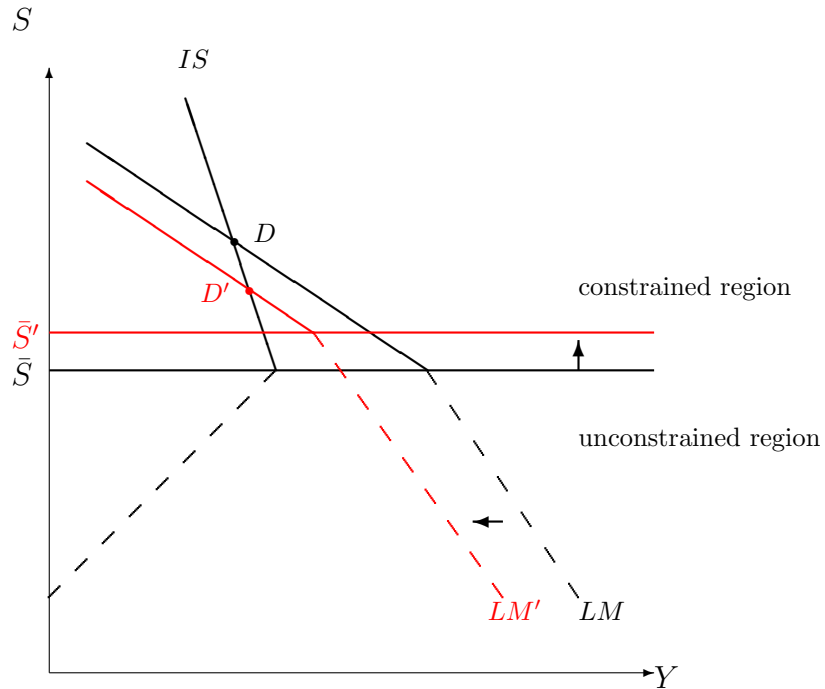


Figure 2: A Contractionary Monetary Policy



## 4 Optimal Monetary Policy

What is the optimal monetary policy to follow in this economy? Here we define an optimal monetary policy with commitment as that which maximizes expected utility of the representative home agent, taking into account the way in which the wage is set (c.f. equation (2.15)). In the absence of collateral constraints the optimal monetary policy follows as a simple application of the results of recent literature (e.g. Obstfeld and Rogoff (2000)), and is very easy to describe.

### 4.1 Optimal monetary policy without collateral constraints

When the collateral constraint does not bind, optimal monetary policy (with commitment) is the one that replicates the *flexible* wage equilibrium allocation.

**Proposition 1 (Optimal Monetary Policy in the Unconstrained Economy).**

*When the collateral constraint does not bind, the optimal monetary policy is a fixed level of the money stock,  $M = \bar{M}$ .*

*Proof.* See Appendix. □

The intuition behind this proposition is in two parts. First, the model has the property that in the unconstrained region, the optimal pre-set wage is independent of the the distribution of  $\tilde{X}$ . To see this note from the profit maximizing employment condition;  $WH = \omega P_h Y$ , and also that  $P_h Y = \frac{M}{\chi}$ . Then from (2.15), it follows that

$$W = \left[ \frac{\eta \rho}{\rho - 1} \right]^{\frac{1}{1+\psi}} \frac{\omega^{\frac{\psi}{1+\psi}}}{\chi} [EM^{1+\psi}]^{\frac{1}{1+\psi}} \quad (4.1)$$

If the money stock is constant, then the pre-set nominal wage is the same as the wage that would obtain in the flexible wage economy. Hence a fixed money stock supports the flexible wage allocation.

The second part of the reasoning behind proposition 1 relates to the optimality of the flexible wage allocation. The flexible wage allocation is inefficient, due to a) monopoly wage setting, and b) the economy has international market power over the sale of its good, as evidenced by the foreign demand schedule (2.16). A social planner would wish to exploit this market power. Nevertheless, in the economy without collateral constraints, these other inefficiencies dichotomize from the inefficiency due to nominal wage setting. Monetary policy under commitment cannot systematically influence the equilibrium real wage, or the equilibrium terms of trade faced by the economy. Hence, the best that the monetary authority can do, under commitment, and without collateral constraints, is to achieve the flexible wage allocation. It does this by following a constant money supply (or equivalently, constant nominal income) rule. <sup>8</sup>

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<sup>8</sup>Of course, it would follow the same rule even if an optimal package of taxes and subsidies sustained the fully efficient real wage and terms of trade.

## 4.2 Optimal monetary policy with sometimes binding constraints

When the collateral constraints may bind, the nature of monetary policy is substantially altered from proposition 1. In order to explore this however, we must take care to frame the problem in an appropriate way.

There are two features of the collateral constrained economy that alter the monetary policy problem in an artificial sense. The first is that we have defined financial assets  $N$  in nominal terms. This introduces a nominal non-neutrality into the economy which would allow the monetary authority to systematically alter real magnitudes, even when monetary policy is chosen with commitment. To avoid this, we assume that financial assets are set proportional to the expected money supply. Hence, we assume that  $N = \bar{N}E(M)$ , where  $\bar{N}$  is constant. This means that the monetary authority cannot systematically alter the real value of  $N$ <sup>9</sup>.

The second feature of the model that must be addressed is the presence of the multiple distortions discussed in the previous sub-section. The distortions due to monopoly wage setting and optimal tariff considerations dichotomize from the monetary policy problem in the economy without binding collateral constraints. But this is not true in the economy where the constraints may occasionally bind. The reason is that average employment and output is in general influenced by the fraction of the total state space over which the constraint will bind, and this fraction is itself affected by the monetary policy rule. In order to avoid the convolution of the optimal monetary stabilization rule with these monopoly distortions, we assume that a set of optimal taxes and subsidies is set so that in the absence of nominal wage setting, the equilibrium allocation in the unconstrained economy is socially optimal (first-best). This ensures that the monetary authority has no incentive to push average output down by forcing the economy to

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<sup>9</sup>We could think  $N$  as being determined in a ex ante contract between imported intermediate suppliers and final goods firms, where  $N$  can adjust to expected changes in the money supply, but is not state-contingent.

operate for more time in the constrained region, thereby raising the average terms of trade. In the appendix, the following result is shown:

**Proposition 2.** *The social planning optimal allocation of the unconstrained economy is supported by a) a tax on intermediate imports in the amount  $\tau_I = -1 + \frac{(1-\omega)}{(1-\alpha)\phi_I}$ , and b) a tax on employment for firms in the amount  $\tau_H = -1 + \frac{(\rho-1)}{\rho} \frac{1}{\alpha(1-\frac{1-\omega}{1+\tau_I})}$ , where*

$$\phi_I = \left[ \frac{1}{2} \frac{\omega \alpha^2 - \alpha - 1 + \alpha \omega + \sqrt{\omega^2 \alpha^4 + 2\omega \alpha^3 - 2\omega^2 \alpha^3 - 3\alpha^2 + 2\alpha + 1 - 2\alpha\omega + \alpha^2 \omega^2}}{\alpha(-1 + \alpha\omega)} \right]$$

*represents the socially optimal value of intermediate imports, as a fraction of the normalized foreign demand  $\frac{X}{q^*}$ .*

*Proof.* See Appendix. □

The combination of a tax on employment and tax on purchases on intermediate imports ensures that the monopoly distortion in wage setting is eliminated, and the optimal-tariff level of the terms of trade is attained. Note that with respect to employment, the tax may be positive or negative, depending on the strength of the monopoly distortion in wage setting (which tends to reduce employment below the optimal level), and the optimal-tariff level of employment (which tends to reduce employment below the price-taking competitive level, in order to improve the terms of trade). The tax on intermediate imports is always positive however.

When the economy may move between the constrained and unconstrained regions, depending on external shocks and the monetary rules followed, the optimal monetary policy rule must be derived numerically. Assume a discrete distribution of  $\tilde{X}$ . Then, let monetary policy be represented as a state contingent response vector  $M_i = M_i(\tilde{X}_i)$ , where we solve for the values of  $M_i$  that maximize expected utility of the home representative individual. Since the response of the economy to any systematic monetary rule is neutral, we normalize so that  $M_i(\tilde{X}_1) = 1$ , where  $\tilde{X}_1$  is the highest value of the foreign demand shock. In an economy without binding collateral constraints, Proposition 1 would then ensure that  $M_i = 1$  would obtain for all  $i$ . The influence of collateral constraints is then seen to the extent that  $M_i \neq 1$  for some  $i$ .

### 4.2.1 Calibration

Let  $\tilde{X}$  take on three values,  $\tilde{X}_1, \tilde{X}_2, \tilde{X}_3$ . We think of  $\tilde{X}_1$  and  $\tilde{X}_2$  as high and low values, but giving variation in a range which does not lead to a binding collateral constraint. On the other hand  $\tilde{X}_3$  is a ‘crash’ state, in the sense that it represents a sharp fall in foreign demand. This will generally lead the collateral constraint to bind. Thus, a situation in which all shock variance was concentrated over  $\tilde{X}_1$  and  $\tilde{X}_2$  would be typical of a high income advanced economy, whereas realizations of  $\tilde{X}_3$  would be characteristic of an emerging market economy.

Our model has only a small number of parameters that need be calibrated. First, we set  $\omega$  so that the share of intermediate goods in production is 40 percent. Thus,  $\omega = 0.6$ . This is consistent with the estimates given for intermediate imports as a fraction of GDP in Braggion et al (2003) for Thailand. In other countries the share of intermediate imports is higher, and would imply a lower value of  $\omega$ . This would strengthen the case for a pro-cyclical monetary policy. Hence we see  $\omega = 0.6$  as a conservative estimate. We set  $\alpha = 0.5$ , on the calculation that for Asian economies, imports are up to half of GDP (e.g. Thailand), and half of these imports go to consumption goods. Since about half of GDP is in the non-traded sector, which is absent from our model, it is appropriate to make consumption of imports equal to 50 percent of traded sector GDP. We assume that state 1 and state 2 occur with equal probability, equal to 0.475, and a ‘crash’ occurs with probability 0.05<sup>10</sup>. The values of the external shock  $\tilde{X}$  are set so that, separately,  $\tilde{X}_1$  and  $\tilde{X}_2$  would generate a 2 percent standard deviation in GDP, for a fixed money stock. The ‘crash’ state is determined so that GDP in this state falls by 10 percent, for a given money stock. This is roughly the fall in GDP seen in emerging market crises.

An empirical counterpart for the leverage ratio at the aggregate level could be seen as the ratio of short term debt to usable foreign exchange rate reserves. Radelet and

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<sup>10</sup>Typically, the empirical models of crisis probabilities in emerging markets have predicted very low crisis probabilities, in the range 5-10 percent, even as the economies get very close to crises. See Berg and Patillo (1999).

Sachs (1998) report estimates for this (see also Chang and Velasco (2000)) for emerging market countries prior to the Asian crisis. The estimates vary considerably across countries. Many countries have leverage ratio's exceeding unity. In the experiment below, we vary the leverage ratio between 0.25 and 3.

#### 4.2.2 Optimal Monetary Rules

Table 1 reports the the distribution of output, consumption, employment, and the exchange rate for the model, under a fixed money rule, a fixed exchange rate rule, and the optimal monetary policy rule, for a range of values for the leverage ratio. In addition, the table reports the values of expected utility and the optimal monetary rule in each case. The appendix describes the solution for the optimal rule in more detail.

For a low leverage ratio, i.e.  $l = 0.25$ , the collateral constraint does not bind, under the fixed money rule (which is equivalent to a nominal income target). In this case, this is the optimal rule. Most of the time, output will fluctuate between state 1 and 2, with a low variance. In the rare state 3, output will fall substantially. But because the collateral constraint never binds, the exchange rate acts as a 'shock absorber' for all states. This rule clearly welfare-dominates a fixed exchange rule.

As the leverage ratio rises however, the optimal rule prescribes a monetary contraction in the low state of the world. Now the exchange rate adjusts freely only when output fluctuates between states 1 and 2, but in the crash state, the monetary rule limits the exchange rate depreciation. For a leverage ratio of 0.5, the optimal rule requires a 1 percent money contraction. This contraction slightly reduces output, since, in terms of Figure 1, the IS curve is still upward sloping for a leverage ratio of 0.5. A monetary contraction is desirable nonetheless, because by limiting the exchange rate depreciation, it relaxes the collateral constraint. This allows for an increase in the quantity of intermediate imports in production. In this way, the monetary rule is designed to undo the effects of the trade credit financing constraints that hit the collateral constrained economy in a crisis <sup>11</sup>.

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<sup>11</sup>In utility terms, the monetary contraction has conflicting effects. It directly reduces consumption,

As the leverage ratio rises even more, there is a larger fall in output in the ‘crash’ state. Then the optimal monetary rule calls for an even greater monetary contraction in this state. Hence, we find that the greater is the output loss in the crash, the *more pro-cyclical should be the monetary response*. But in this case, the monetary contraction is actually expansionary. When the leverage ratio is above 0.6 (for this calibration), the IS curve is negatively sloped in the constrained region, and the monetary contraction actually raises output in the crash state. Although the contraction still reduces employment, the rise in intermediate imports more than offsets this, so that GDP rises.

The optimal monetary policy in this model can therefore rationalize the observation that emerging economies tend to follow pro-cyclical monetary policies. In our model, they do so not to shore up credibility - we are focusing solely on optimal monetary policy with commitment, so that full credibility is assumed. Rather, the monetary contraction in a crisis is an optimal response to the financing constraint - by limiting the size of the nominal exchange rate depreciation, it relaxes the constraint. If the collateral constraint is severe enough, the monetary contraction will in fact raise output, but this is not a pre-requisite for a pro-cyclical monetary rule. With a less severe collateral constraint, a monetary contraction is still optimal, even though it reduces output.

The optimal monetary policy in the crash state trades off the benefits of relaxing the financing constraint against the costs of the monetary contraction - in all cases the contraction reduces consumption. It is always feasible for the monetary contraction to be great enough so as to eliminate this constraint. We find however that this is not optimal - for our calibration (and all other experiments we conducted) an optimal rule will not be so great as to entirely undo the collateral constraint.

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but also reduces employment. The gain in utility comes about because, due to the rise in intermediate imports, the utility cost of the fall in consumption is offset by the benefit of a fall in labour supply.



### 4.2.3 Hard peg vs. flexible exchange rates

A general property of the optimal monetary rule is that as long as the collateral constraint binds, there is less exchange rate depreciation than would occur under a fixed money stock rule. But note that the optimal monetary rule does not fix the exchange rate. In the unconstrained region, it allows the exchange rate to adjust freely to shocks, and narrows the range of exchange rate movement in the bad state.

But what if this optimal monetary response is infeasible? Is it possible that the monetary policy maker would prefer to operate under a fully fixed exchange (e.g. a currency board, or dollarized system), rather than a nominal income targeting rule? This question underlies the debate between the argument for emerging economies to move towards ‘inflation targeting’ regimes, allowing the exchange rate to float freely<sup>12</sup>, or alternatively adopt a ‘hard peg’. In our context, there is a trade off between the benefits of exchange rate adjustment in the unconstrained economy, and the costs of exchange rate depreciation in the constrained economy. While the optimal monetary rule exploits this trade off in the best way possible, in light of this recent debate, a relevant comparison is between the nominal income target and a fixed exchange rate. In Figure 3 we illustrate expected utility under each regime, as a function of the leverage ratio. Note that under our calibration, a crash state occurs only with 5 percent probability. For relatively low leverage ratios, this ensures that a nominal income target will dominate a fully fixed exchange rate. But as the leverage ratio rises above 2.2, the expected utility cost of exchange rate depreciation in a crash (even though it occurs with low probability) offsets the benefits of exchange rate adjustment in the high states, and a fixed exchange rate actually dominates.

The empirical evidence in Devereux and Lane (2003b) closely accords with this theoretical finding. They find that countries with a larger stock of portfolio liabili-

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<sup>12</sup>A constant nominal income rule followed in the unconstrained model above is the exact theoretical counterpart of the argument for inflation targeting in dynamic sticky price models such as those of Woodford (2003). In both cases (absent cost-push shocks), the rule sustains the flexible price or wage allocation.

ties against a given trading partner tend to minimize bilateral exchange rate volatility against that partner. Thus, for emerging market countries with weak financial constraints, the net debt position must be taken into account, as well as the standard optimal currency area factors, in the assessment of optimal exchange rate policy.

We find this result of interest, because it in fact gives a precise sense in which an emerging market economy may display a ‘fear of floating’, in the Calvo and Reinhart (2002) terminology. An observer may feel that a fixed exchange rate is inefficient, by preventing the economy from adjusting to small shocks, which occur frequently. But in fact there is a ‘peso problem’ in making judgements based on small samples. In reality the authorities may be choosing a fixed exchange rate optimally (if the choice is only between fixed and floating), with knowledge of the consequences of a very costly, but low probability crash event.

An alternative perspective on the fear of floating may be seen in the different magnitude of shocks. Table 2 shows the comparison of fixed and flexible exchange rates between two economies with equal leverage ratios ( $l=1.5$ ). In the first comparison, the crash state is associated with a 12 percent fall in output for a fixed money stock rule, while in the second, the fall in output is 26 percent. For this example, the second economy displays a larger ‘fear of floating’, in the sense that it would choose a fixed exchange rate over a free float, despite the fact that it is subjected to a more volatile external environment. The key driving force is that the consequences of exchange rate depreciation in the crash state is more damaging for the more volatile economy.

## 5 Concluding Remarks

We have developed a very simple macroeconomic model of optimal monetary policy in an emerging market economy, where the economy is subject to occasionally binding collateral constraints. The model can rationalize why it makes sense for monetary policy to be tight during a financial crisis, because the channels of monetary policy are very different when collateral constraints bind in a crisis than in normal times, without financial constraints. The model can also help to explain a predominance of the ‘fear of floating’ in emerging market economies, and the fact that empirically, this phenomenon seems to be greater among more heavily indebted countries, and that paradoxically, this tendency is greater among countries that are experiencing larger external shocks.

In developing a very simple framework which is analogous to the monetary policy analysis carried out in the recent literature on general equilibrium sticky price models, we see our paper as a complement rather than a substitute for the large literature on financial constraints in emerging market economies. An obvious drawback of our model is that we have nothing to say about the implications of capital flows for monetary policy. But an advantage is that we can be specific about the design of optimal monetary policy that takes account of financial vulnerabilities. We leave for future work an extension of our approach to deal with capital flows.

**Table 1: Distribution of Variables (at optimal M)**

(a)  $l = 0.25$

	Fixed M	Fixed ER	Optimal M
<i>Max EU</i>	-0.8318	-0.8321	-0.8318
<i>Optimal M</i>	–	–	[1, 1, 1]
<i>Y</i>	[0.2573, 0.2521, 0.2353]	[0.2627, 0.2495, 0.2101]	[0.2573, 0.2521, 0.2353]
$E(Y)$	0.2537	0.2538	0.2537
$\sigma_Y$	0.0133	0.0317	0.0133
<i>C</i>	[0.6024, 0.5811, 0.5152]	[0.6086, 0.5782, 0.4869]	[0.6024, 0.5811, 0.5152]
$E(C)$	0.5879	0.5881	0.5879
$\sigma_C$	0.0526	0.0733	0.0526
<i>H</i>	[0.3000, 0.3000, 0.3000]	[0.3105, 0.2950, 0.2484]	[0.3000, 0.3000, 0.3000]
$E(H)$	0.3	0.3	0.3
$\sigma_H$	0	0.0374	0
<i>S</i>	[0.6285, 0.6615, 0.7856]	[0.6000, 0.6000, 0.6000]	[0.6285, 0.6615, 0.7856]
$E(S)$	0.6520	0.6	0.6520
$\sigma_S$	0.0961	0	0.0961

( $\tilde{X} = [1, 0.95, 0.8]$ ,  $\pi = [0.475, 0.475, 0.05]$ )

(b)  $l = 0.5$

	Fixed M	Fixed ER	Optimal M
<i>Max EU</i>	-0.8318	-0.8321	-0.8318
<i>Optimal M</i>	–	–	[1, 1, 0.99]
<i>Y</i>	[0.2573, 0.2521, 0.2327]	[0.2627, 0.2495, 0.2101]	[0.2574, 0.2522, 0.2327]
$E(Y)$	0.2536	0.2538	0.2537
$\sigma_Y$	0.0150	0.0317	0.0151
<i>C</i>	[0.6024, 0.5811, 0.5147]	[0.6086, 0.5782, 0.4869]	[0.6024, 0.5812, 0.5135]
$E(C)$	0.5879	0.5881	0.5879
$\sigma_C$	0.0530	0.0733	0.0538
<i>H</i>	[0.3000, 0.3000, 0.2990]	[0.3105, 0.2950, 0.2484]	[0.3001, 0.3001, 0.2967]
$E(H)$	0.3	0.3	0.3
$\sigma_H$	0.0007	0.0374	0.0024
<i>S</i>	[0.6285, 0.6615, 0.7810]	[0.6000, 0.6000, 0.6000]	[0.6285, 0.6615, 0.7754]
$E(S)$	0.6518	0.6	0.6515
$\sigma_S$	0.0931	0	0.0894

( $\tilde{X} = [1, 0.95, 0.8]$ ,  $\pi = [0.475, 0.475, 0.05]$ )

(c)  $l = 1$ 

	Fixed M	Fixed ER	Optimal M
<i>Max EU</i>	-0.8318	-0.8321	-0.8318
<i>Optimal M</i>	–	–	[1, 1, 0.96]
<i>Y</i>	[0.2573, 0.2521, 0.2246]	[0.2627, 0.2495, 0.2101]	[0.2576, 0.2524, 0.2257]
$E(Y)$	0.2532	0.2538	0.2535
$\sigma_Y$	0.0205	0.0317	0.0199
<i>C</i>	[0.6024, 0.5811, 0.5126]	[0.6086, 0.5782, 0.4869]	[0.6027, 0.5815, 0.5084]
$E(C)$	0.5878	0.5881	0.5879
$\sigma_C$	0.0543	0.0733	0.0574
<i>H</i>	[0.3000, 0.3000, 0.2961]	[0.3105, 0.2950, 0.2484]	[0.3006, 0.3006, 0.2870]
$E(H)$	0.2998	0.3	0.2999
$\sigma_H$	0.0026	0.0374	0.0091
<i>S</i>	[0.6285, 0.6615, 0.7673]	[0.6000, 0.6000, 0.6000]	[0.6285, 0.6615, 0.7470]
$E(S)$	0.6511	0.6	0.6501
$\sigma_S$	0.0840	0	0.0707

( $\tilde{X} = [1, 0.95, 0.8]$ ,  $\pi = [0.475, 0.475, 0.05]$ )

(d)  $l = 1.5$ 

	Fixed M	Fixed ER	Optimal M
<i>Max EU</i>	-0.8319	-0.8321	-0.8318
<i>Optimal M</i>	–	–	[1, 1, 0.93]
<i>Y</i>	[0.2573, 0.2521, 0.2176]	[0.2627, 0.2495, 0.2101]	[0.2579, 0.2526, 0.2218]
$E(Y)$	0.2528	0.2538	0.2536
$\sigma_Y$	0.0251	0.0317	0.0227
<i>C</i>	[0.6024, 0.5811, 0.5103]	[0.6086, 0.5782, 0.4869]	[0.6030, 0.5817, 0.5038]
$E(C)$	0.5877	0.5881	0.5879
$\sigma_C$	0.0558	0.0733	0.0606
<i>H</i>	[0.3000, 0.3000, 0.2937]	[0.3105, 0.2950, 0.2484]	[0.3010, 0.3010, 0.2785]
$E(H)$	0.2997	0.3	0.2999
$\sigma_H$	0.0042	0.0374	0.0152
<i>S</i>	[0.6285, 0.6615, 0.7561]	[0.6000, 0.6000, 0.6000]	[0.6285, 0.6615, 0.7238]
$E(S)$	0.6506	0.6	0.6489
$\sigma_S$	0.0767	0	0.0556

( $\tilde{X} = [1, 0.95, 0.8]$ ,  $\pi = [0.475, 0.475, 0.05]$ )

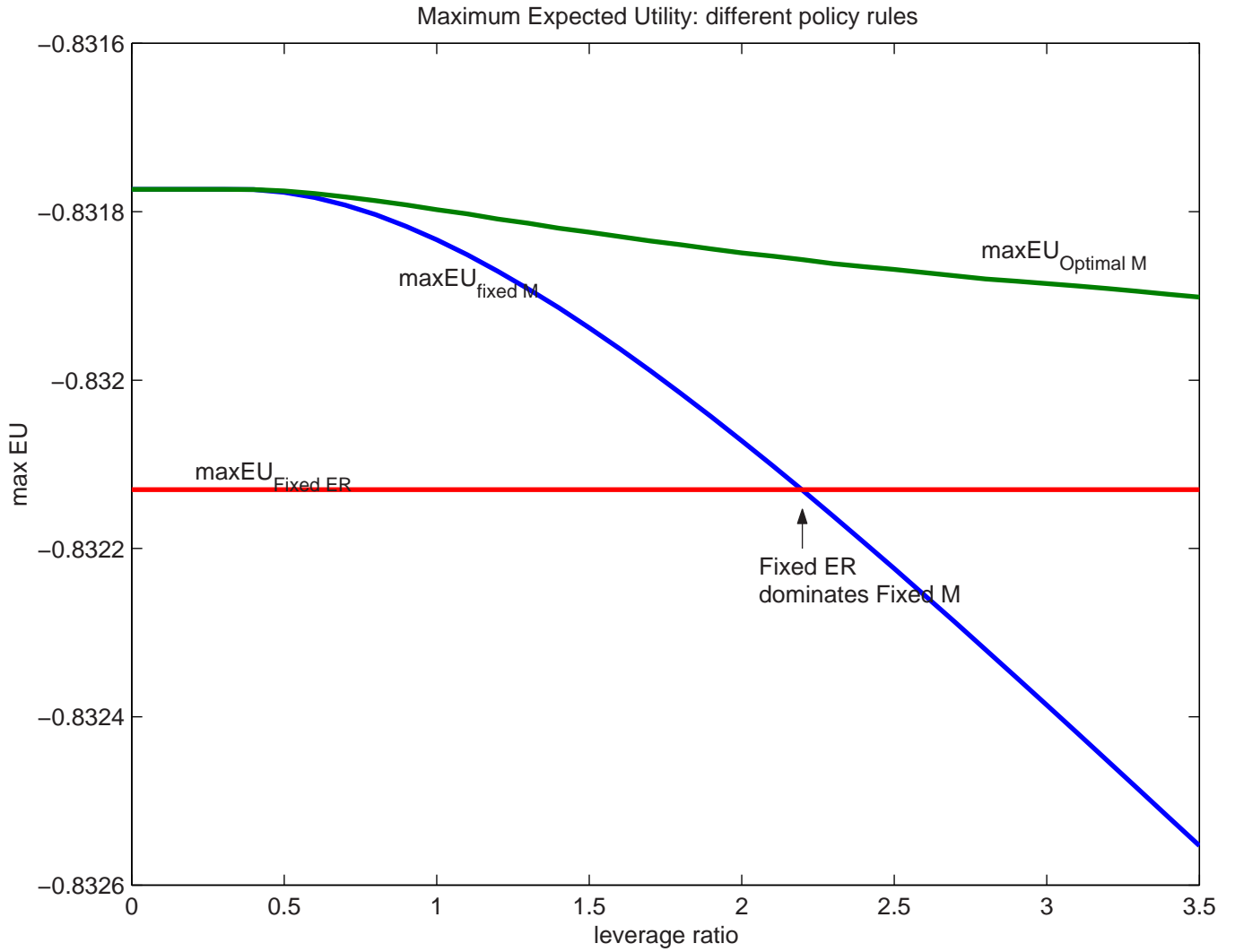
(e)  $l = 3$ 

	Fixed M	Fixed ER	Optimal M
<i>Max EU</i>	-0.8324	-0.8321	-0.8319
<i>Optimal M</i>	–	–	[1, 1, 0.89]
<i>Y</i>	[0.2573, 0.2521, 0.2015]	[0.2627, 0.2495, 0.2101]	[0.2582, 0.2529, 0.2160]
<i>E(Y)</i>	0.2520	0.2538	0.2536
$\sigma_Y$	0.0359	0.0317	0.0267
<i>C</i>	[0.6024, 0.5811, 0.5033]	[0.6086, 0.5782, 0.4869]	[0.6033, 0.5821, 0.4975]
<i>E(C)</i>	0.5873	0.5881	0.5879
$\sigma_C$	0.0605	0.0733	0.0650
<i>H</i>	[0.3000, 0.3000, 0.2887]	[0.3105, 0.2950, 0.2484]	[0.3017, 0.3017, 0.2670]
<i>E(H)</i>	0.2994	0.3	0.2999
$\sigma_H$	0.0076	0.0374	0.0233
<i>S</i>	[0.6285, 0.6615, 0.7326]	[0.6000, 0.6000, 0.6000]	[0.6285, 0.6615, 0.6923]
<i>E(S)</i>	0.6494	0.6	0.6474
$\sigma_S$	0.0613	0	0.0359

 $(\tilde{X} = [1, 0.95, 0.8], \pi = [0.475, 0.475, 0.05])$

Figure 3: Maximum Expected Utility, different policy rules

$$(\tilde{X} = [1, 0.95, 0.8]; \pi = [0.475, 0.475, 0.05])$$



**Table 2: Different magnitudes of shocks,  $l = 1.5$**

$\tilde{X} = [1, 0.95, 0.82]$			
	Fixed M	Fixed ER	Optimal M
<i>Max EU</i>	-0.831	-0.8312	-0.8309
<i>Optimal M</i>	–	–	[1, 1, 0.94]
<i>Y</i>	[0.2573, 0.2521, 0.2230]	[0.2625, 0.2494, 0.2153]	[0.2578, 0.2525, 0.2264]
$E(Y)$	0.2531	0.2539	0.2537
$\sigma_Y$	0.0215	0.0282	0.0196
<i>C</i>	[0.6024, 0.5811, 0.5204]	[0.6084, 0.5780, 0.4989]	[0.6029, 0.5816, 0.5144]
$E(C)$	0.5882	0.5885	0.5884
$\sigma_C$	0.0492	0.0653	0.0535
<i>H</i>	[0.3000, 0.3000, 0.2948]	[0.3102, 0.2946, 0.2543]	[0.3009, 0.3009, 0.2818]
$E(H)$	0.2997	0.3	0.2999
$\sigma_H$	0.0035	0.0333	0.0129
<i>S</i>	[0.6285, 0.6615, 0.7427]	[0.6000, 0.6000, 0.6000]	[0.6285, 0.6615, 0.7154]
$E(S)$	0.6499	0.6	0.6485
$\sigma_S$	0.0678	0	0.0502

$\tilde{X} = [1, 0.95, 0.67]$			
	Fixed M	Fixed ER	Optimal M
<i>Max EU</i>	-0.8391	-0.8390	-0.8382
<i>Optimal M</i>	–	–	[1, 1, 0.84]
<i>Y</i>	[0.2573, 0.2521, 0.1773]	[0.2637, 0.2506, 0.1767]	[0.2585, 0.2533, 0.1913]
$E(Y)$	0.2508	0.2531	0.2527
$\sigma_Y$	0.0522	0.0546	0.0436
<i>C</i>	[0.6024, 0.5811, 0.4388]	[0.6098, 0.5793, 0.4086]	[0.6038, 0.5825, 0.4313]
$E(C)$	0.5841	0.5853	0.5851
$\sigma_C$	0.1036	0.1262	0.1095
<i>H</i>	[0.3000, 0.3000, 0.2856]	[0.3126, 0.2970, 0.2094]	[0.3024, 0.3024, 0.2514]
$E(H)$	0.2993	0.3	0.2998
$\sigma_H$	0.0097	0.0647	0.0343
<i>S</i>	[0.6285, 0.6615, 0.8573]	[0.6000, 0.6000, 0.6000]	[0.6285, 0.6615, 0.7735]
$E(S)$	0.6556	0.6	0.6514
$\sigma_S$	0.1440	0	0.0881



# Appendix A: Solving the Model

## A.1 Solutions for the unconstrained economy

The unconstrained economy is described by the following equations:

$$P_h = \kappa \frac{W^\omega (Sq^*)^{1-\omega}}{A} \quad (\text{A.1})$$

$$PC = WH = \omega P_h Y \quad (\text{A.2})$$

$$M = \chi PC \quad (\text{A.3})$$

$$W = \eta \frac{\rho}{\rho - 1} \frac{E\{H^{1+\psi}\}}{E\left\{\frac{H}{PC}\right\}} \quad (\text{A.4})$$

$$Y = \alpha\omega Y + \tilde{X} \frac{S}{P_h} \quad (\text{A.5})$$

These five equations are used to solve for the five variables:  $\{W, H(\sigma), P_h(\sigma), S(\sigma), C(\sigma)\}$ , where  $\sigma$  is the ex-post state of the world, depending on the foreign demand,  $\tilde{X}$ .

Equations (A.2) and (A.5) are used to derive the consumption expression. We combine equations (A.2) and (A.3) to get the employment solution. The resulting solutions are:

$$C^u = \frac{\omega}{1 - \alpha\omega} \frac{\tilde{X} S^u}{P^u} \quad (\text{A.6})$$

$$H^u = \frac{\omega M}{\chi W^u} \quad (\text{A.7})$$

We substitute this employment solution into equation (A.4) to solve for the wage, given by:

$$W^u = \left[ \eta \frac{\rho}{\rho - 1} \right]^{\frac{1}{1+\psi}} \frac{\omega^{\frac{\psi}{1+\psi}}}{\chi} [E(M^{1+\psi})]^{\frac{1}{1+\psi}} \quad (\text{A.8})$$

(A.1) gives the price of home produced goods, and the nominal exchange rate can be solved by using equations (A.2), (A.3) and (A.5).

Using these, we may prove Proposition 1.

## Proof of Proposition 1

*Proof.* From (A.6), (A.7), and (A.8), we may write out expected utility in the unconstrained economy as:

$$EU = \Gamma + E\omega \ln \left( \frac{S}{W} \right) \quad (\text{A.9})$$

where  $\Gamma$  is a constant function of parameters. Without loss of generality, assume that  $\tilde{X}$  takes on a discrete distribution  $\tilde{X} \in \{X(1)..X(N)\}$ , with probabilities  $\{\pi_1..\pi_N\}$ . Let the monetary policy be defined as  $M_i = M(X_i)$ ,  $i = 1..N$ . Since there are  $N$  states, there are only  $N - 1$  degrees of freedom with respect to monetary policy. Hence we normalize so that  $M_1 = 1$ . Then (A.9) may be re-written as:

$$EU = \Gamma' + E\omega \ln \left( \frac{M}{E(M^{1+\psi})^{\frac{1}{1+\psi}}} \right) \quad (\text{A.10})$$

where  $\Gamma'$  is again a constant function of parameters. The first order condition for an expected utility maximizing choice of monetary policy is

$$\frac{\pi_i}{M_i} = \pi_i \frac{M_i^\psi}{E(M^{1+\psi})^{\frac{1}{1+\psi}}} \quad (\text{A.11})$$

From (A.11), it is clear that  $M_i = M = 1$ , for all  $i$ . This establishes the proof. □

## A.2 Solutions for the constrained economy

The corresponding representation for the *constrained* economy is as follows:

$$P_h \omega \frac{Y}{H} = W \quad (\text{A.12})$$

$$PC = P_h Y - Sq^* I = P_h Y - N + SD^* \quad (\text{A.13})$$

$$M = \chi PC = \chi [P_h Y - N + SD^*] \quad (\text{A.14})$$

$$W = \eta \frac{\rho}{\rho - 1} \frac{E\{H^{1+\psi}\}}{E\left\{\frac{H}{PC}\right\}} \quad (\text{A.15})$$

$$Y = \alpha \left( Y - \frac{N - SD^*}{P_h} \right) + \tilde{X} \frac{S}{P_h} \quad (\text{A.16})$$

$$Y = AH^\omega \left( \frac{N - SD^*}{Sq^*} \right)^{1-\omega} \quad (\text{A.17})$$

These six equations can be solved for the six variables:  $\{W, H(\sigma), P_h(\sigma), S(\sigma), C(\sigma), Y(\sigma)\}$ , where  $\sigma$  is the ex-post state of the world.

Using (A.13), (A14), and (A16), we can solve for the nominal exchange rate:

$$S^c = \frac{1 - \alpha}{\chi} \frac{M}{\tilde{X} + D^*} + \frac{N}{\tilde{X} + D^*} \quad (\text{A.18})$$

From (A12), (A13) and (A14), we can obtain the solution for consumption and employment as:

$$C^c = \frac{\omega}{1 - \alpha} \left[ \frac{\tilde{X} S^c}{P^c} - \alpha \left( \frac{N}{P^c} - \frac{S^c D^*}{P^c} \right) \right] \quad (\text{A.19})$$

$$H^c = \frac{\omega \left[ \frac{M}{\chi} + N - S^c D^* \right]}{W^c} \quad (\text{A.20})$$

## Proof of Proposition 2

*Proof.* Take a social planner who wishes to maximize utility of the home agent, facing the foreign demand function (2.16). The constraints faced by the planner are:

$$P_h C_h + C_m = P_h A H^\omega I^{1-\omega} - I \quad (\text{A.21})$$

$$P_h A H^\omega I^{1-\omega} = C_h + \tilde{X} \quad (\text{A.22})$$

The planner will choose the consumption allocation so that  $C_m = \frac{1-\alpha}{\alpha} P_h C_h$ . Using this in (A.21) and (A.22), we can express the problem of the planner as the choice of  $I$  and  $H$  so as to maximize:

$$EU = \ln(\tilde{X} - I) - \alpha \ln(\tilde{X} - \alpha I) + \alpha \omega \ln(H) + \alpha(1 - \omega) \ln(I) - \eta \frac{H^{1+\psi}}{1 + \psi} \quad (\text{A.23})$$

The planner's optimal choice of  $H$  and  $I$  are represented as

$$H = \left( \frac{\alpha \omega}{\eta} \right)^{\frac{1}{1+\psi}}$$

$$I = \tilde{X} \left[ \frac{1}{2} \frac{\omega \alpha^2 - \alpha - 1 + \alpha \omega + \sqrt{\omega^2 \alpha^4 + 2\omega \alpha^3 - 2\omega^2 \alpha^3 - 3\alpha^2 + 2\alpha + 1 - 2\alpha \omega + \alpha^2 \omega^2}}{\alpha(-1 + \alpha \omega)} \right]$$

Using the taxes described in the Proposition, we may ensure that employment and intermediate imports satisfies these two equations.  $\square$

## Appendix B: Numerical Solution

We look for the optimal monetary policy in the constrained region numerically. We assume the monetary policy rule is some function of the state of the world,  $\tilde{X}$ . Denote this policy rule as  $\mathbf{M}(\tilde{X})$ . In particular, assume  $\tilde{X}$  takes on three realizations:  $\tilde{X} = \{\tilde{X}_1, \tilde{X}_2, \tilde{X}_3\}$  with probabilities  $\{\pi_1, \pi_2, \pi_3\}$ . We choose  $\tilde{X}$  such that, for given leverage ratio, collateral constraint becomes binding when the foreign demand for home good is  $\tilde{X}_3$ .

The optimal monetary policy rule in the constrained region is the vector of state-contingent money response,  $M_i = M_i(\tilde{X}_i)$ ,  $i = \{1, 2, 3\}$  that maximizes the expected utility of home households. The optimal policy is solved in the following steps.

1. Set the initial state-contingent money response as:

$$\mathbf{M}^0 = [M_1^0, M_2^0, M_3^0] = [1, 1, 1] \quad (\text{B.1})$$

2. Solve for the preset optimal nominal wage, given  $\mathbf{M}^0$ :

$$W(\mathbf{M}^0) = \Lambda \frac{E\{H(\mathbf{M}^0)^{1+\psi}\}}{E\left\{\frac{H(\mathbf{M}^0)}{P(\mathbf{M}^0)C(\mathbf{M}^0)}\right\}} = \Lambda \frac{\sum_{i=1}^3 \{H(M_i^0)^{1+\psi}\}}{\sum_{i=1}^3 \left\{\frac{H(M_i^0)}{P(M_i^0)C(M_i^0)}\right\}} \quad (\text{B.2})$$

where  $\Lambda$  is a function of constant parameters. Given this wage, we can solve for the other variables.

3. Use  $W(\mathbf{M}^0)$  to solve for other variables and compute the expected utility. Denote the resulting expected utility as  $EU_0$ .
4. Define a vector  $\delta_{\mathbf{M}}^j = [\delta_{M_1}^j, \delta_{M_2}^j, \delta_{M_3}^j]$ ,  $j \in \mathcal{J}$ , where  $\mathcal{J}$  is the policy space. This vector represents the exogenous money change of policy  $j$ , carried out by the policymaker. Denote the new state-contingent money response as  $\mathbf{M}^1$ , given by:

$$\mathbf{M}^1 = \mathbf{M}^0 - \delta_{\mathbf{M}}^j \quad (\text{B.3})$$

5. Given  $\mathbf{M}^1$ , solve for the preset optimal nominal wage  $W(\mathbf{M}^1)$  as in Step 2. Then compute the expected utility as in Step 3, denoted it by  $EU_1$ .

6. Repeat Steps 4 and 5  $n$  times. The state-contingent money response at the  $k^{th}$  iteration is:

$$\mathbf{M}^k = \mathbf{M}^0 - k\delta_{\mathbf{M}}^j, \quad (\text{B.4})$$

and the corresponding expected utility is  $EU_k$ .

7. The optimal money supply under policy  $j$  is the  $\mathbf{M}^k$  that gives the highest expected utility. Call the highest expected utility under policy  $j$  as  $EU_{max}^j$
8. Repeat Steps 4 to 7 for different money policies (characterized by  $\delta_{\mathbf{M}}^j, j \in \mathcal{J}$ ).
9. Compare  $EU_{max}^j, \forall j \in \mathcal{J}$ . The optimal policy rule is characterized by the  $\delta_{\mathbf{M}}^j$  that gives the highest  $EU_{max}^j$ .

We choose the following parameters values to illustrate our numerical results:

$\psi$	1	Elasticity of labour supply
$\chi$	1	Coefficient on money balance in utility function
$\omega$	0.6	Share of labour in Cobb-Douglas production function
$\alpha$	0.5	Consumption share in home goods
$\Lambda$	1	Coefficient on the optimal nominal wage
$A$	1	Productivity in Cobb-Douglas function
$q^*$	1	Foreign price of imported intermediate goods
$P_f^*$	1	Foreign price of foreign produced final goods

Table A.1: Parameter Values for the Unconstrained Model.

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